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Some Properties on Intuitionistic Fuzzy Soft Matrices

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Abstract

In this work, we prove the properties of necessity and possibility and other related properties with examples. We introduce the operator of implication and their properties . Some examples are given.

Key words – Soft sets, fuzzy soft matrices, intuitionistic fuzzy soft matrices, necessity and possibility operator, operator of implication.

1. Introduction

In 1999, Molodtsov [1] initiated the novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. Soft set theory is a new mathematical tool for dealing with uncertainties which traditional mathematical tool can not handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science etc. in his pioneer work [1], and so forth Maji et al.[2, 3] have further studied the soft sets and used this theory to solve some decision making problems.

2. Definition and Preliminaries:

Soft Matrices

Definition 2.1 [1] Let U be an initial universe, P(U) be the power set of U, E be the set of all parameters and A⊆ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs

$(f_A, E) = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U) \}$

where $f_A : E \rightarrow P(U)$ such that $f_A(e) = \phi$ if $e \notin A$.

Here f_A is called an approximate function of the soft set (f_A, E). The set f_A(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter e ∈ E .

Definition 2.2 [7] A pair (F, A) is called a fuzzy set over U where F : A → $\tilde{P}$(U) is a mapping from A into $\tilde{P}$(U) .

3. Fuzzy Soft Matrices (FSM)

Definition 3.1[5] Let (f_A, E) be fuzzy soft set over U. Then a subset of U x E is uniquely defined by

R_A = { (u, e) : e∈ A, u ∈ f_A(e) }

which is called relation form of (f_A, E) . The characteristic function of R_A is written by

χ_{R_A} : U x E → [0, 1], where μ_{R_A}(u, e) ∈ [0, 1] is the membership value of u∈U for each e ∈U.

If $\mu_{ij} = \chi_{R_A}(u_i, e_j)$, we can define a matrix

$$[\mu_{ij}]_{mXn} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

which is called an m x n soft matrix of the soft set (f_A, E) over U.

4. Intuitionistic Fuzzy Soft Matrices (IFSMs)

Definition 4.1 [6] Let U be an initial universe, E be the set of parameters and A⊆ E. Let] Let (f_A, E) be an Intuitionistic fuzzy soft set (IFSS) over U. Then a subset of U x E is uniquely defined by

R_A = { (u, e) : e∈ A, u ∈ f_A(e) }

which is called relation form of (f_A, E) . The membership and non membership functions of are written by

μ_{R_A} : U x E → [0, 1] and ν_{R_A} : U x E → [0, 1] where μ_{R_A}: (u, e) ∈ [0, 1] and ν_{R_A} : (u, e) ∈ [0, 1] are the membership value and non membership value of u∈U for each e ∈ E.

If (μ_{ij}, ν_{ij}) = ($\mu_{R_A}(u_i, e_j)$, $\nu_{R_A}(u_i, e_j)$), we can define a matrix

$$[\mu_{ij}, \nu_{ij}]_{m\,X\,n} = \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \cdots & (\mu_{1n}, \nu_{1n}) \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \cdots & (\mu_{2n}, \nu_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{m1}, \nu_{m1}) & \mu_{m2}, \nu_{m2}) & \cdots & (\mu_{mn}, \nu_{mn}) \end{bmatrix}$$, which is called

an m x n IFSM of the IFSS(f_A, E) over U. Therefore, we can say that IFSS(f_A, E) is uniquely characterized by the matrix $[\mu_{ij}, \nu_{ij}]_{m\,X\,n}$ and both concepts are interchangeable. The set of all m x n IFS matrices will be denoted by IFSM$_{m\,X\,n}$.

Definition 4.2 [6]Let $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)] ∈ IFSM$_{m\,X\,n}$.
Then $\tilde{A}$ is called
a) a zero IFSM denoted by $\tilde{0}$ = [(0, 0)], if $\mu_{ij}^{\tilde{A}}$ = 0 and $\nu_{ij}^{\tilde{A}}$ = 0 for all i and j .
b) a μ-universal IFSM, denoted by $\tilde{I}$ = [(1, 0)] if $\mu_{ij}^{\tilde{A}}$ = 1 and $\nu_{ij}^{\tilde{A}}$ = 0 for all i and j.
c) a ν- universal IFSM, denoted by $\underbrace{I}$ = = [(0, 1)] if $\mu_{ij}^{\tilde{A}}$ = 0 and $\nu_{ij}^{\tilde{A}}$ = 1 for all i and j.

Definition 4.3 [6]Let $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)], $\tilde{B}$ = [($\mu_{ij}^{\tilde{B}}$, $\nu_{ij}^{\tilde{B}}$)] ∈ IFSM$_{m\,X\,n}$. Then
a) $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)] is said to be intuitionistic fuzzy soft sub matrix of $\tilde{B}$ = [($\mu_{ij}^{\tilde{B}}$, $\nu_{ij}^{\tilde{B}}$)] denoted by $\tilde{A} \subseteq \tilde{B}$ if $\mu_{ij}^{\tilde{A}} \leq \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{A}} \geq \nu_{ij}^{\tilde{B}}$ for all i and j .
b) $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)] is said to be intuitionistic fuzzy soft super matrix of $\tilde{B}$ = [($\mu_{ij}^{\tilde{B}}$, $\nu_{ij}^{\tilde{B}}$)] denoted by $\tilde{A} \supseteq \tilde{B}$ if $\mu_{ij}^{\tilde{A}} \geq \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{A}} \leq \nu_{ij}^{\tilde{B}}$ for all i and j .
c) $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)], $\tilde{B}$ = [($\mu_{ij}^{\tilde{B}}$, $\nu_{ij}^{\tilde{B}}$)] are said to be intuitionistic fuzzy soft equal matrices denoted by $\tilde{A} = \tilde{B}$ if $\mu_{ij}^{\tilde{A}} = \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{A}} = \nu_{ij}^{\tilde{B}}$ for all i and j .

Definition 4.4 [6]Let $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)], $\tilde{B}$ = [($\mu_{ij}^{\tilde{B}}$, $\nu_{ij}^{\tilde{B}}$)] ∈ IFSM$_{m\,X\,n}$. Then
IFSM $\tilde{C}$ =[($\mu_{ij}^{\tilde{C}}$, $\nu_{ij}^{\tilde{C}}$)] is called
a) Union $\tilde{A}$ of $\tilde{B}$ = denoted by $\tilde{A} \mathbin{\tilde{\cup}} \tilde{B}$ if $\mu_{ij}^{\tilde{C}}$ = max {$\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}$ } and
$\nu_{ij}^{\tilde{C}}$ = min {$\nu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{B}}$} for all i and j .

b) Intersection $\tilde{A}$ of $\tilde{B}$ = denoted by $\tilde{A} \mathbin{\tilde{\cap}} \tilde{B}$ if $\mu_{ij}^{\tilde{C}}$ = min {$\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}$ } and
$\nu_{ij}^{\tilde{C}}$ = max{$\nu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{B}}$} for all i and j .

c) Complement of $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)] denoted by $\tilde{A}^0$ = [($\nu_{ij}^{\tilde{A}}$, $\mu_{ij}^{\tilde{A}}$)] for all i and j .

Definition 4.5 . [8] Let $\tilde{A}$ = [($\mu_{ij}^{\tilde{A}}$, $\nu_{ij}^{\tilde{A}}$)], $\tilde{B}$ = [($\mu_{ij}^{\tilde{B}}$, $\nu_{ij}^{\tilde{B}}$)] ∈ IFSM$_{m\,X\,n}$. Then
IFSM $\tilde{C}$ =[($\mu_{ij}^{\tilde{C}}$, $\nu_{ij}^{\tilde{C}}$)] is called

a) the "**.**"**(product)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} \,.\, \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{C}} = \nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}}$ for all i and j.
b) the "+"**(Probabilistic sum)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} + \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}} - \mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}$ and $\nu_{ij}^{\tilde{C}} = \nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}}$ for all i and j.
c) the "@"**(Arithmetic Mean)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} \,@\, \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \frac{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}}}{2}$ and $\nu_{ij}^{\tilde{C}} = \frac{\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}}}{2}$ for all i and j.
d) the "$@^w$" **(Weighted Arithmetic Mean)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} \,@^w\, \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \frac{w_1\mu_{ij}^{\tilde{A}} + w_2\mu_{ij}^{\tilde{B}}}{w_1 + w_2}$, $\nu_{ij}^{\tilde{C}} = \frac{w_1\nu_{ij}^{\tilde{A}} + w_2\,\nu_{ij}^{\tilde{B}}}{w_1 + w_2}$, for all i and j. $w_1 > 0, w_2 > 0$

e) the "\$" **(Geometric Mean)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A}\ \$\ \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = \sqrt{\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}}$ and $\nu_{ij}^{\tilde{C}} = \sqrt{\nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}}}$ for all i and j.

f) the "$\$^w$" **(Weighted Geometric Mean)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A}\ \$^w\ \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = ((\mu_{ij}^{\tilde{A}})^{w_1} . (\mu_{ij}^{\tilde{B}})^{w_2})^{\frac{1}{w_1 + w_2}}$ and $\nu_{ij}^{\tilde{C}} = ((\nu_{ij}^{\tilde{A}})^{w_1} . (\mu_{ij}^{\tilde{B}})^{w_2})^{\frac{1}{w_1 + w_2}}$ for all i and j. $w_1 > 0, w_2 > 0$

g) the "⋈"**(Harmonic Mean)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} \bowtie \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = 2 . \frac{\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}}{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}}}$ and $\nu_{ij}^{\tilde{C}} = 2 . \frac{\nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}}}{\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}}}$ for all i and j. $w_1 > 0, w_2 > 0$

h) the "$\bowtie^w$" **(Weighted Harmonic Mean)** operation of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{C} = \tilde{A} \bowtie^w \tilde{B}$ if $\mu_{ij}^{\tilde{C}} = (\frac{w_1 + w_2}{\frac{w_1}{\mu_{ij}^{\tilde{A}}} + \frac{w_2}{\mu_{ij}^{\tilde{B}}}}, \nu_{ij}^{\tilde{C}} = \frac{w_1 + w_2}{\frac{w_1}{\nu_{ij}^{\tilde{A}}} + \frac{w_2}{\nu_{ij}^{\tilde{B}}}}$ for all i and j.

Proposition 1.1 : Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{m\,X\,n}$. Then

i) $(\tilde{A} \tilde{\cap} \tilde{B}) + (\tilde{A} \tilde{\cup} \tilde{B}) = \tilde{A} + \tilde{B}$

ii) $(\tilde{A} \tilde{\cap} \tilde{B}) . (\tilde{A} \tilde{\cup} \tilde{B}) = \tilde{A} . \tilde{B}$

iii) $(\tilde{A} \tilde{\cap} \tilde{B})\ @\ (\tilde{A} \tilde{\cup} \tilde{B}) = \tilde{A}\ @\ \tilde{B}$

iv) $(\tilde{A} \tilde{\cap} \tilde{B})\ \$\ (\tilde{A} \tilde{\cup} \tilde{B}) = \tilde{A}\ \$\ \tilde{B}$

v) $(\tilde{A} \tilde{\cap} \tilde{B}) \bowtie (\tilde{A} \tilde{\cup} \tilde{B}) = \tilde{A} \bowtie \tilde{B}$

vi) $(\tilde{A} + \tilde{B})\ @\ (\tilde{A} . \tilde{B}) = \tilde{A}\ @\ \tilde{B}$

Proof : For all i and j,

i) $(\tilde{A} \tilde{\cap} \tilde{B})+(\tilde{A} \tilde{\cup} \tilde{B})$

$= [(\min\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})] + [(\max\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \min\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})] = [(\min\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \} + \max\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \} - \min\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \} . \max\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \} . \min\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})]$

$= [(\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}} - \mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}}]$ [∵ min(a, b) + max(a, b) = a + b & min(a, b) . max(a, b) = a . b]

$= [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] + [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})]$

$= \tilde{A} + \tilde{B}$ □

ii) $(\tilde{A} \tilde{\cap} \tilde{B}) . (\tilde{A} \tilde{\cup} \tilde{B})$

$= [(\min\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})] . [(\max\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \min\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})]$

$= [(\min\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \} . \max\{ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \} + \min\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \} - \max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \} . \min\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})]$

$= [(\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}})]$

$= [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] . [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})]$

$= \tilde{A} . \tilde{B}$ □

iii) $(\tilde{A}\,\tilde{\cap}\,\tilde{B})\ @\ (\tilde{A}\,\tilde{\cup}\,\tilde{B})$

$= [\,(\min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \max\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\})\,]\ @\ [\,(\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\})\,]$

$= [\,(\frac{\min\{\mu_{ij}^{\tilde{A}},\mu_{ij}^{\tilde{B}}\}+\max\{\mu_{ij}^{\tilde{A}},\mu_{ij}^{\tilde{B}}\}}{2}, \frac{\max\{\nu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{B}}\}+\min\{\nu_{ij}^{\tilde{A}},\nu_{ij}^{\tilde{B}}\}}{2})]$

$= [\,(\frac{\mu_{ij}^{\tilde{A}}+\mu_{ij}^{\tilde{B}}}{2}, \frac{\nu_{ij}^{\tilde{A}}+\nu_{ij}^{\tilde{B}}}{2})]$

$= \tilde{A}\ @\ \tilde{B}$ □

Similar proof for others.

Example 2 : Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{2\,X\,2}$, where

$\tilde{A} = \begin{bmatrix}(.3,.2) & (.4,.5)\\(.5,.3) & (.7,.2)\end{bmatrix}$, $\tilde{B} = \begin{bmatrix}(.4,.3) & (.6,.2)\\(.7,.2) & (.3,.4)\end{bmatrix}$. Then

$(\tilde{A}\,\tilde{\cap}\,\tilde{B}) + (\tilde{A}\,\tilde{\cup}\,\tilde{B})$

$= \left(\begin{bmatrix}(.3,.2) & (.4,.5)\\(.5,.3) & (.7,.2)\end{bmatrix}\tilde{\cap}\begin{bmatrix}(.4,.3) & (.6,.2)\\(.7,.2) & (.3,.4)\end{bmatrix}\right) +$

$\left(\begin{bmatrix}(.3,.2) & (.4,.5)\\(.5,.3) & (.7,.2)\end{bmatrix}\tilde{\cup}\begin{bmatrix}(.4,.3) & (.6,.2)\\(.7,.2) & (.3,.4)\end{bmatrix}\right)$

$= \begin{bmatrix}(.3,.3) & (.4,.5)\\(.5,.3) & (.3,.4)\end{bmatrix} + \begin{bmatrix}(.4,.2) & (.6,.2)\\(.7,.2) & (.7,.2)\end{bmatrix}$

$= \begin{bmatrix}(.58,.06) & (.76,.1)\\(.85,.06) & (.79,.08)\end{bmatrix} = \tilde{A} + \tilde{B}$ □

Other results can be verified similarly.

Definition4.6 : Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m\,X\,n}$. Then

i) The necessity operation of $\tilde{A}$ is denoted by $\square\,\tilde{A}$ and defined as

$\square\,\tilde{A} = [\,(\mu_{ij}^{\tilde{A}}, 1-\mu_{ij}^{\tilde{A}})\,]$ for all i and j .

ii) The possibility operation of $\tilde{A}$ is denoted by $\lozenge\tilde{A}$ and defined as

$\lozenge\,\tilde{A} = [\,(1-\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})\,]$ for all i and j .

Proposition 1.2: Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] \in \text{IFSM}_{m\,X\,n}$. Then

i) $[\square\,[\,\tilde{A}^0]]^0 = \lozenge\,\tilde{A}$ iv) $\square\,\lozenge\,\tilde{A} = \lozenge\,\tilde{A}$

ii) $[\,\lozenge\,[\,\tilde{A}^0\,]]^0 = \square\,\tilde{A}$ v) $\lozenge\,\square\,\tilde{A} = \square\,\tilde{A}$

iii) $\square\,\square\,\tilde{A} = \square\,\tilde{A}$ vi) $\lozenge\,\lozenge\,\tilde{A} = \lozenge\,\tilde{A}$

Proof : For all i and j,

i) $[\square\,[\,\tilde{A}^0]]^0$ iv) $\square\,\lozenge\,\tilde{A}$

$= [\square\,[\,\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}}\,]]^0 = \square\,\lozenge[(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$

$= [\,(\nu_{ij}^{\tilde{A}}, 1-\nu_{ij}^{\tilde{A}})]^0 = \square\,[\,(1-\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})\,]$

$= [\,(1-\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})\,] = [\,(1-\nu_{ij}^{\tilde{A}}, 1-(1-\nu_{ij}^{\tilde{A}})\,]$

$= \Diamond \tilde{A}$ □ = [$(1 - \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})$]

ii) $[\Diamond [\tilde{A}^0]]^0 = \Diamond \tilde{A}$ □
$= [\Diamond [\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}}]]^0$ v) $\Diamond \square \tilde{A}$
$= [(1 - \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}})]^0 = \Diamond \square [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$
$= [(\mu_{ij}^{\tilde{A}}, 1 - \mu_{ij}^{\tilde{A}})] = \Diamond [(\mu_{ij}^{\tilde{A}}, 1 - \mu_{ij}^{\tilde{A}})]$
$= \square \tilde{A}$ □ $= [(1 - (1 - \mu_{ij}^{\tilde{A}}), 1 - \mu_{ij}^{\tilde{A}})]$

iii) $\square \square \tilde{A} = [(\mu_{ij}^{\tilde{A}}, 1 - \mu_{ij}^{\tilde{A}})]$
$= \square \square [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] = \square \tilde{A}$ □
$= \square [(\mu_{ij}^{\tilde{A}}, 1 - \mu_{ij}^{\tilde{A}})]$ vi) $\Diamond \Diamond \tilde{A}$
$= [(\mu_{ij}^{\tilde{A}}, 1 - \mu_{ij}^{\tilde{A}})] = \Diamond \Diamond [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$
$= \square \tilde{A}$ □ $= \Diamond [(1 - \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$
$= [(1 - \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$
$= \Diamond \tilde{A}$ □

Example 1: Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})] \in \mathrm{IFSM}_{2 X 2}$, where

$$\tilde{A} = \begin{bmatrix} (.3, .2) & (.4, .5) \\ (.5, .3) & (.7, .2) \end{bmatrix}. \text{ Then}$$

$$\tilde{A}^0 = \begin{bmatrix} (.2, .3) & (.5, .4) \\ (.3, .5) & (.2, .7) \end{bmatrix}$$

$$\square [\tilde{A}^0] = \begin{bmatrix} (.2, .8) & (.5, .5) \\ (.3, .7) & (.2, .8) \end{bmatrix}$$

$$[\square [\tilde{A}^0]]^0 = \begin{bmatrix} (.8, .2) & (.5, .5) \\ (.7, .3) & (.8, .2) \end{bmatrix} = \Diamond \tilde{A}$$ □

Other results can be verified similarly .

Proposition 1.3 : Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \mathrm{IFSM}_{m X n}$. Then
i) $\square [\tilde{A} \cap \tilde{B}] = \square \tilde{A} \cap \square \tilde{B}$ vii) $\square [\tilde{A} \bowtie \tilde{B}] \supset \square \tilde{A} \bowtie \square \tilde{B}$
ii) $\square [\tilde{A} \cup \tilde{B}] = \square \tilde{A} \cup \square \tilde{B}$ viii) $\Diamond [\tilde{A} \cap \tilde{B}] = \Diamond \tilde{A} \cap \Diamond \tilde{B}$
iii) $[\square[\tilde{A}^0 + \tilde{B}^0]]^0 = \Diamond \tilde{A} . \Diamond \tilde{B}$ ix) $\Diamond [\tilde{A} \cup \tilde{B}] = \Diamond \tilde{A} \cup \Diamond \tilde{B}$
iv) $[\square [\tilde{A}^0 . \tilde{B}^0]]^0 = \Diamond \tilde{A} + \Diamond \tilde{B}$ x) $[\Diamond [\tilde{A}^0 + \tilde{B}^0]]^0 = \square \tilde{A} . \square \tilde{B}$
v) $\square [\tilde{A} @ \tilde{B}] = \square \tilde{A} @ \square \tilde{B}$ xi) $[\Diamond [\tilde{A}^0 . \tilde{B}^0]]^0 = \square \tilde{A} + \square \tilde{B}$
vi) $\square [\tilde{A} \$ \tilde{B}] \supset \square \tilde{A} \$ \square \tilde{B}$ xii) $\Diamond [\tilde{A} @ \tilde{B}] = \Diamond \tilde{A} @ \Diamond \tilde{B}$

Some of the proofs are given :

Proof : For all i and j,
i) $\square [\tilde{A} \cap \tilde{B}]$
$= \square [(\min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \max\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\})]$
$= [(\min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, 1 - \min\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\})]$

$= [\ (\min\{\ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \max\ \{\ 1-\mu_{ij}^{\tilde{A}}, 1-\mu_{ij}^{\tilde{B}}\ \}\ )\ ]$
$= [\ (\ \mu_{ij}^{\tilde{A}}, 1-\mu_{ij}^{\tilde{A}})]\ \tilde{\cap}\ [(\ \mu_{ij}^{\tilde{B}}, 1-\mu_{ij}^{\tilde{B}}\ )\ ]$
$= \Box\ \tilde{A}\ \tilde{\cap}\ \Box\ \tilde{B}$ □

ii) $\Box\ [\tilde{A}\ \tilde{\cup}\ \tilde{B}\ ]$
$= \Box\ [\ (\max\{\ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\ \{\ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\}\ )\ ]$
$= [\ (\max\{\ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, 1-\max\ \{\ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\}\ )\ ]$
$= [\ (\max\ \mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\ \{\ 1-\mu_{ij}^{\tilde{A}}, 1-\mu_{ij}^{\tilde{B}}\ \}\ )\ ]$
$= [\ (\ \mu_{ij}^{\tilde{A}}, 1-\mu_{ij}^{\tilde{A}})]\ \tilde{\cup}\ [(\ \mu_{ij}^{\tilde{B}}, 1-\mu_{ij}^{\tilde{B}}\ )\ ]$
$= \Box\ \tilde{A}\ \tilde{\cup}\ \Box\ \tilde{B}$ □

iii) $[\Box[\tilde{A}^0 + \tilde{B}^0\ ]]^0$
$= [\ \Box\left[\left(\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}}\right)\right] + [(\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{B}})]]^0$
$= [\Box\ [\left(\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{A}}.\ \mu_{ij}^{\tilde{B}}\right)]]^0$
$= [\ \left(\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}, 1 - \left(\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}\right)\right)\ ]^0$
$= [\ (1 - (\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}), \nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}} - \nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}\ )\ ]$
$= [\ (1 - \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}}\ )\ ]\ .\ [(\ 1 - \nu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})\ ]$
$= \Diamond\ \tilde{A}\ .\ \Diamond\ \tilde{B}$ □

iv) $[\Box\ [\tilde{A}^0\ .\tilde{B}^0\ ]]^0$
$= [\ \Box\left[\left(\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{A}}\right)\right].[(\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{B}})]]^0$
$= [\Box\ [\left(\nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}} - \mu_{ij}^{\tilde{A}}.\ \mu_{ij}^{\tilde{B}}\ \right)]]^0$
$= [(\nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}, 1 - \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}})]^0$
$= [\ (1 - \nu_{ij}^{\tilde{A}}.\nu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{A}}\ .\nu_{ij}^{\tilde{B}}\ )\ ]$
$= [\ (1 - \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})\ ] + [(\ 1 - \nu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})\ ]$
$= \Diamond\ \tilde{A} + \Diamond\ \tilde{B}$ □

v) $\Box\ [\tilde{A}\ @\ \tilde{B}\ ]$
$= \Box\ (\ [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]\ @\ [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})])$
$= \Box\ [\ (\ \frac{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}}}{2}, \frac{\nu_{ij}^{\tilde{A}} + \nu_{ij}^{\tilde{B}}}{2}\ )\ ]$
$= [(\ \frac{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}}}{2}, 1 - \frac{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}}}{2}\ )\ ]$
$= [\ (\mu_{ij}^{\tilde{A}}, 1-\mu_{ij}^{\tilde{A}}\ )\ ]\ @\ [\ (\mu_{ij}^{\tilde{B}}, 1-\mu_{ij}^{\tilde{B}}\ )\ ]$
$= \Box\ \tilde{A}\ @\ \Box\ \tilde{B}$ □

vi) $\Box\ (\tilde{A}\ \$\ \tilde{B}\ )$
$= \Box\ (\ [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]\ \$\ [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})\ ]\ )$

$= \square\ (\sqrt{\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}}, \sqrt{\nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}}})$

$= (\sqrt{\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}}, 1 - \sqrt{\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}})$

$\square\ \tilde{A}\ \$\ \square\ \tilde{B}$

$= [(\mu_{ij}^{\tilde{A}}, 1 - \mu_{ij}^{\tilde{A}})]\ \$\ [(\mu_{ij}^{\tilde{B}}, 1 - \mu_{ij}^{\tilde{B}})]$

$=[(\sqrt{\mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}}, \sqrt{(1 - \mu_{ij}^{\tilde{A}}).(1 - \mu_{ij}^{\tilde{B}})})]$

Clearly $\square\ (\tilde{A}\ \$\ \tilde{B}) \supset \square\ \tilde{A}\ \$\ \square\ \tilde{B}$ □

Other results can be proved similarly.

Definition 4.7: Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})] \in \text{IFSM}_{m\ X\ n}$. Then the operation '↦' **(Implication)** denoted by $\tilde{A} \mapsto \tilde{B}$ is defined by

$\tilde{A} \mapsto \tilde{B} = [(\max\{ \nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \min\{ \mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})]$

Proposition 1.4 : Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})]$, $\tilde{C} = [(\mu_{ij}^{\tilde{C}}, \nu_{ij}^{\tilde{C}})] \in \text{IFSM}_{m\ X\ n}$. Then

i) $[\tilde{A} \tilde{\cap} \tilde{B}] \mapsto \tilde{C} \supseteq [\tilde{A} \mapsto \tilde{C}] \tilde{\cap} [\tilde{B} \mapsto \tilde{C}]$

ii) $[\tilde{A} \tilde{\cup} \tilde{B}] \mapsto \tilde{C} \subseteq [\tilde{A} \mapsto \tilde{C}] \tilde{\cup} [\tilde{B} \mapsto \tilde{C}]$

iii) $[\tilde{A} \tilde{\cap} \tilde{B}] \mapsto \tilde{C} = [\tilde{A} \mapsto \tilde{C}] \tilde{\cup} [\tilde{B} \mapsto \tilde{C}]$

iv) $[\tilde{A} + \tilde{B}] \mapsto \tilde{C} \supseteq [\tilde{A} \mapsto \tilde{C}] + [\tilde{B} \mapsto \tilde{C}]$

v) $[\tilde{A} . \tilde{B}] \mapsto \tilde{C} \subseteq [\tilde{A} \mapsto \tilde{C}] .[\tilde{B} \mapsto \tilde{C}]$

vi) $\tilde{A} \mapsto [\tilde{B} + \tilde{C}] \subseteq [\tilde{A} \mapsto \tilde{B}] + [\tilde{A} \mapsto \tilde{C}]$

vii) $\tilde{A} \mapsto [\tilde{B} . \tilde{C}] \supseteq [\tilde{A} \mapsto \tilde{B}] .[\tilde{A} \mapsto \tilde{C}]$

viii) $\tilde{A} \mapsto \tilde{A}^0 = \tilde{A}^0$

Proof: Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})]$, $\tilde{C} = [(\mu_{ij}^{\tilde{C}}, \nu_{ij}^{\tilde{C}})] \in \text{IFSM}_{m\ X\ n}$. Then for all i and j

i) $[\tilde{A} \tilde{\cap} \tilde{B}] \mapsto \tilde{C}$

$= [(\min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \})] \mapsto [(\mu_{ij}^{\tilde{C}}, \nu_{ij}^{\tilde{C}})]$

$= [(\max\{ \max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \}, \mu_{ij}^{\tilde{C}} \}, \min\{ \min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \nu_{ij}^{\tilde{C}} \})] \ldots$ (1)

$[\tilde{A} \mapsto \tilde{C}] \tilde{\cap} [\tilde{B} \mapsto \tilde{C}]$

$= [(\max\{ \nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}} \}, \min\{\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{C}} \})] \tilde{\cap} [(\max\{ \nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}} \}, \min\{\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}} \})]$

$= [(\min\{\max\{ \nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}} \}, \max\{ \nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}} \}\}, \max\{\min\{\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{C}} \},$
$\min\{\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}} \}\})]$

$=[(\min\{\max\{ \nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}} \}, \mu_{ij}^{\tilde{C}} \}, \max\{\min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}} \}, \nu_{ij}^{\tilde{C}} \})] \ldots\ldots\ldots\ldots\ldots$ (2)

From (1) and (2) it is clear that

$[\tilde{A} \tilde{\cap} \tilde{B}] \mapsto \tilde{C} \supseteq [\tilde{A} \mapsto \tilde{C}] \tilde{\cap} [\tilde{B} \mapsto \tilde{C}]$ □

ii) $[\tilde{A} \tilde{\cup} \tilde{B}] \mapsto \tilde{C}$

$= [(\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \min\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\}] \mapsto [(\mu_{ij}^{\tilde{C}}, \nu_{ij}^{\tilde{C}})]$

$= [(\max\{\min\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\}, \mu_{ij}^{\tilde{C}}\}, \min\{\max\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \nu_{ij}^{\tilde{C}}\})]$ ……………. (3)

$[\tilde{A} \mapsto \tilde{C}] \tilde{\cup} [\tilde{B} \mapsto \tilde{C}]$

$= [(\max\{\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}}\}, \min\{\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{C}}\})] \tilde{\cup} [(\max\{\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}}\}, \min\{\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}}\})]$

$[(\max\{\max\{\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}}\}, \max\{\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}}\}\}, \min\{\min\{\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{C}}\}, \min\{\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}}\}\})]$

$=[(\max\{\max\{\nu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{B}}\}, \mu_{ij}^{\tilde{C}}\}, \min\{\min\{\mu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{B}}\}, \nu_{ij}^{\tilde{C}}\})]$ ……………….. (4)

From (3) and (4), it is clear that

$[\tilde{A} \tilde{\cup} \tilde{B}] \mapsto \tilde{C} \subseteq [\tilde{A} \mapsto \tilde{C}] \tilde{\cup} [\tilde{B} \mapsto \tilde{C}]$ □

iii) $[\tilde{A} \tilde{\cap} \tilde{B}] \mapsto \tilde{C} = [\tilde{A} \mapsto \tilde{C}] \tilde{\cup} [\tilde{B} \mapsto \tilde{C}]$

Proof: Proof follows from (1) and (4).

iv) $[\tilde{A} + \tilde{B}] \mapsto \tilde{C} = [(\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}} - \mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}})] \mapsto [(\mu_{ij}^{\tilde{C}}, \nu_{ij}^{\tilde{C}})]$

$= [(\max\{\nu_{ij}^{\tilde{A}} . \nu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}}\}, \min\{\mu_{ij}^{\tilde{A}} + \mu_{ij}^{\tilde{B}} - \mu_{ij}^{\tilde{A}} . \mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}}\})]$ …….. (5)

$[\tilde{A} \mapsto \tilde{C}] + [\tilde{B} \mapsto \tilde{C}]$

$= [(\max\{\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}}\}, \min\{\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{C}}\})] + [(\max\{\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}}\}, \min\{\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}}\})]$

$= [(\max\{\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}}\} + \max\{\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}}\} - \max\{\nu_{ij}^{\tilde{A}}, \mu_{ij}^{\tilde{C}}\} . \max\{\nu_{ij}^{\tilde{B}}, \mu_{ij}^{\tilde{C}}\},$

$\min\{\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{C}}\} . \min\{\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{C}}\})]$ …….. (6)

From (5) and (6), it follows that

$[\tilde{A} + \tilde{B}] \mapsto \tilde{C} \supseteq [\tilde{A} \mapsto \tilde{C}] + [\tilde{B} \mapsto \tilde{C}]$ □

Similar proof for others.

Example 3: Let $\tilde{A} = [(\mu_{ij}^{\tilde{A}}, \nu_{ij}^{\tilde{A}})]$, $\tilde{B} = [(\mu_{ij}^{\tilde{B}}, \nu_{ij}^{\tilde{B}})]$, $\tilde{C} = [(\mu_{ij}^{\tilde{C}}, \nu_{ij}^{\tilde{C}})] \in \text{IFSM}_{2 \times 2}$, where

$\tilde{A} = \begin{bmatrix} (.3,.2) & (.4,.5) \\ (.5,.3) & (.7,.2) \end{bmatrix}, \tilde{B} = \begin{bmatrix} (.4,.5) & (.3,.4) \\ (.7,.2) & (.6,.3) \end{bmatrix}, \tilde{C} = \begin{bmatrix} (.3,.6) & (.4,.6) \\ (.6,.3) & (.5,.4) \end{bmatrix}$

$[\tilde{A} + \tilde{B}] \mapsto \tilde{C} = \begin{bmatrix} (.3,.58) & (.4,.58) \\ (.6,.3) & (.5,.4) \end{bmatrix}$

$[\tilde{A} \mapsto \tilde{C}] + [\tilde{B} \mapsto \tilde{C}] = \begin{bmatrix} (.8,.12) & (.7,.12) \\ (.84,.09) & (.75,.58) \end{bmatrix}$

Obviously, $[\tilde{A} + \tilde{B}] \mapsto \tilde{C} \supseteq [\tilde{A} \mapsto \tilde{C}] + [\tilde{B} \mapsto \tilde{C}]$ □

Conclusion

In this paper, we have proved the properties of necessity and possibility and other related properties with examples. We have introduced the operator of implication and their properties . Some of the properties of implication have been proved.

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