International Journal of Mathematics Research.
ISSN 0976-5840 Volume 5, Number 2 (2013), pp. 289-298

http://www.irphouse.com

Second Order Strong F-Pseudoconvexity in a Class of Non-Differentiable Scalar Nonlinear Programming Problems

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Abstract

A new class of functions namely, second order strongly F-pseudoconvex functions is introduced which is a generalization of both strongly pseudoconvex functions and strongly pseudoinvex functions. Second order optimality conditions and second order duality results for a class of nonlinear non-differentiable scalar programming problems with a square root term in the objective function as well as in the constraints are obtained under the assumptions of second order F-pseudoconvexity and second order strong F-pseudoconvextiy on the functions involved.

Key words: Second order F-pseudoconvexity, Second order strong F-pseudoconvexity, Nondifferentiable nonlinear programming, Optimality, Duality**.**

1. INTRODUCTION

The literature of the mathematical programming is crowded with the necessary and sufficient optimality conditions for a point to be an optimal solution to the optimization problem. The role of optimality criteria in mathematical programming is important both from theoretical and computational point of view. The concept of generalized convexity is of great importance in the study of optimization problems. The best known and frequently used optimality conditions for nonlinear programming problems where all the functions involved are differentiable. The various forms of mathematical programming problems involving square root of positive semidefinite form $(x^tBx)^{1/2}$ has been of interest to many researchers.

Weir [8] introduced new class of functions namely, strongly pseudoconvex functions which is weaker than the class of convex functions and established various duality theorems for differentiable scalar nonlinear programming problem under assumption of pseudoconvexity and strong pseudoconvexity. Kumar and Agarwal [5] proved sufficient optimality conditions and various duality results for nondifferentiable scalar nonlinear programming problems with a square root in the objective function is obtained under the assumptions of pseudoconvexity and strong pseudoconvexity. Kanniappan and Pandian [2] introduced a new class of functions namely, strongly pseudoinvex functions which is a generalization of strongly pseudoconvex functions and established duality theorems for differentiable scalar nonlinear programming problems with a square root term in the objective function is obtained under the assumptions of pseudoinvexity and strong pseudoinvexity. Sinha and Aylawadi [7] established the optimality conditions for mathematical programs under the convexity assumptions where some of the constraints are non-diferentiable but the objective function is differentiable. Kanniappan and Pandian [3] established sufficient conditions for optimality and duality theorems for a class of non-differentiable scalar nonlinear programming problems with square term in the objective function under assumptions of pseudoinvexity and strong pseudoinvexity. Kumar and Ghosh [6] obtained necessary and sufficient conditions of optimality conditions and various duality results for a class of nondifferentiable scalar nonlinear programmng problems with a square root term in the objective function as well as in the constraints under the assumptions of pseudoinvexity and strong pseudoinvexity.

In this paper, we introduce a new class of functions, called second order strongly F-pseudoconvex functions which is a generalization of strongly pseudoconvex functions [8] and strongly pseudoinvex functions [2]. Further, we obtain second order optimality conditions and also establish second order duality results for a class of nonlinear non-differentiable programming problems with a square root term in the objective function as well as in the constraints under the assumptions of second order F-pseudoconvexity and second order strong F-pseudoconvextiy on the functions involved.

2. PRELIMINARIES

Let X be an open convex set in R^{n}, an Euclidean n-dimensional space and R_{+} denote the set of all positive real numbers. Let $\mathrm{f}:\mathrm{X}\to R$ be a twice differentiable functions on X, $\phi:\mathrm{XxX}\to R_{+}$, $\mathrm{p,q}$ and $\mathrm{r}:\mathrm{XxX}\to R^{\mathrm{n}}$, $\mathrm{F}:\mathrm{XxXx}R^{\mathrm{n}}\to R^{\mathrm{n}}$ and $\mathrm{g}:R^{\mathrm{n}}\to R^{\mathrm{m}}$ are twice differentiable functions and B_1 and B_2 are an nxn positive semidefinite symmetric matrices with $\mathrm{B}_2\neq 0$ and $\mathrm{s}\in R^{\mathrm{n}}$.

Let $x, y\in \mathrm{R}^{\mathrm{n}}$. Then,

$x \leqq y \Leftrightarrow x_{\mathrm{i}} \leqq y_{\mathrm{i}}$, for all i, i=1, 2, …, n

$x \le y \Leftrightarrow x_{\mathrm{i}} \leqq y_{\mathrm{i}}$, for all i, i = 1, 2, …, n and $x_{\mathrm{r}} < y_{\mathrm{r}}$ for some r, $1\le \mathrm{r}\le \mathrm{n}$

and

$x < y \Leftrightarrow x_i < y_i$, for all i, i=1, 2, …, n.

We need the following definitions which can be found in [4].

DEFINITION 2.1: A function $F : XxXxR^n \to R^n$ is said to be ***sublinear*** **on** R^n if for each $x, u \in X$,

(i) $F(x,u;(a+b)) \le F(x,u;a) + F(x,u;b)$, for all $a, b \in R^n$ and

(ii) $F(x,u;\alpha a) = \alpha F(x,u;a)$, for all $a \in R^n$ and $\alpha \ge 0$ in R.

Note: From (ii), it follows that $F(x,u;0) = 0$.

DEFINITION 2.2 : The function f is said to be ***second order F-convex*** at $u \in X$ with respect to the functions $p(x,u)$, $q(x,u)$ and $r(x,u)$ if for all $x \in X$,

$$f(x) - f(u) \ge F(x,u;\nabla f(u) + \nabla^2 f(u)p(x,u)) - \frac{1}{2} q(x,u)\nabla^2 f(u)\, r(x,u).$$

DEFINITION 2.3 : The function f is said to be ***second order F-quasiconvex*** at $u \in X$ with respect to the functions $p(x,u)$, $q(x,u)$ and $r(x,u)$ if for all $x \in X$,

$$f(x) \le f(u) - \frac{1}{2} q(x,u)\nabla^2 f(u)\, r(x,u) \Rightarrow F(x,u;\nabla f(u) + \nabla^2 f(u)p(x,u)) \le 0.$$

DEFINITION 2.4 : The function f is said to be ***second order F-pseudoconvex*** at $u \in X$ with respect to the functions $p(x,u)$, $q(x,u)$ and $r(x,u)$) if for all $x \in X$,

$$F(x,u;\nabla f(u) + \nabla^2 f(u)p(x,u)) \ge 0 \Rightarrow f(x) \ge f(u) - \frac{1}{2} q(x,u)\nabla^2 f(u)\, r(x,u).$$

Consider the following nondifferentiable nonlinear programming problem

(P) Minimize $f(x) + (x^t B_1 x)^{1/2}$

subject to

$g(x) \le 0$ (1)

$s^t + (x^t B_2 x)^{1/2} \le 1$ (2)

$x \in X$.

We propose following dual problem (D) to the problem (P)

(D) Maximize $f(u) + u^t B_1 w_1$

subject to

$$\nabla(f(u) + y^t g(u)) + B_1 w_1 + B_2 w_2 + \alpha s + \nabla^2(f(u) + y^t g(u))p(x,u) = 0 \quad (3)$$

$$y^t g(u) - \frac{1}{2} q(x,u)\nabla^2(y^t g(u))\, r(x,u) \ge 0 \quad (4)$$

$$q(x,u)\nabla^2(f(u))\, r(x,u) \le 0 \quad (5)$$

$$\alpha(s^t u + (u^t B_2 u)^{1/2} - 1) \ge 0 \quad (6)$$

$$w_1^t B_1 w_1 \le 1 \tag{7}$$

$$w_2^t B_2 u = \alpha(u^t B_2 u)^{1/2} \tag{8}$$

$y \ge 0$ (9)

$\alpha \ge 0$ (10)

where $\alpha \in R$, $u \in R^n, w_1, w_2 \in R^n, y \in R^m$

We need the following lemma which can be found in [1].

LEMMA 2.1: Let B_1 be an nxn positive semi definite symmetric matrix and $x, w_1 \in R^n$. Then

$$x^t B_1 w_1 \le (x^t B_1 w_1)^{1/2} (w_1^t B_1 w_1)^{1/2}$$

where equality holds if $B_1 x = t B_1 w_1$ for some $t \ge 0$.

For solving strong duality theorem, we need the following necessary optimality condition for a feasible point x_o to be an optimal solution of (P) which can be found in Sinha and Aylawadi [6].

Let x_o be the solution of (P). Define the set z_o as follows

$$z_o = \left[\begin{array}{l} (\nabla g_i(x_o))^t z \le 0, \ for\ all\ i \in \{i : g_i(x_o) = 0\} \\ s^t + \dfrac{z^t B_2 x_o}{(x_o^t B_1 x_o)^{1/2}} \le 0, if\ x_o^t B_2 x_o = 0\ and\ s^t x_o + (x_o^t B_2 x_o)^{1/2} = 1 \\ z : s^t z + (z^t B_2 z)^{1/2} \le 0, if\ x_o^t B_2 x_o = 0\ and\ s^t x_o + (x_o^t B_2 x_o)^{1/2} = 1 \\ (\nabla f(x_o))^t z - \dfrac{z^t B_1 x_o}{(x_o^t B_1 x_o)^{1/2}} > 0, if\ x_o^t B_1 x_o > 0\ and \\ (\nabla f(x_o))^t\ z^t B_1 x_o \ge 0, if\ x_o^t B_1 x_o = 0 \end{array}\right]$$

THEOREM 2.1: (Necessary Optimality Conditions)
If x is an optimal solution of (P) and the set z_o is empty, then there exist $w_1, w_2 \in R^n, y \in R^m$ and a scalar α such that

$$\left.\begin{array}{l} \nabla(f(x) + B_1 w_1 + B_2 w_2 + \alpha s + \nabla(y^t g(x)) = 0 \\ y^t g(x) \ge 0 \\ \alpha(s^t x + (x^t B_2 x)^{1/2} - 1) = 0 \\ w_1^t B_1 w_1 \le 1 \\ w_1^t B_1 x = (x^t B_1 x)^{1/2} \\ w_2^t B_2 x = \alpha(x^t B_2 x)^{1/2} \\ y \ge 0,\ \alpha \ge 0. \end{array}\right. \tag{11}$$

3. SECOND ORDER STRONGLY F-PSEUDOCONVEX FUNCTIONS

Now, we introduce a new class of functions namely, second order strongly F-pseudoconvex function which is a generalization of first order strongly pseudoconvex functions and also, first order strongly η-pseudoinvex functions.

DEFINITION 3.1 : The function f is said to be ***second order strongly F-pseudoconvex*** at $u \in X$ if there exist a real positive function $\phi(x,u)$ on XxX with respect to the functions $p(x,u)$, $q(x,u)$ and $r(x,u)$ such that

$$f(x) - f(u) \;\geq\; \phi(x,u)\, F(x,u;\nabla f(u) + \nabla^2 f(u)p(x,u)) - \frac{1}{2} q(x,u)\nabla^2 f(u)\, r(x,u) \qquad (12)$$

In this case, we say that f is ***second order strongly F-pseudoconvex*** at $u \in X$ with respect to the vector functions $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(\mathrm{x,u})$. The function $\phi(\mathrm{x,u})$ is known as the ***proportional function*** of *f*.

REMARK 3.1: From(12), we can conclude that that every second order strongly F-pseudoconvex function at $u \in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(\mathrm{x,u})$ is second order $\overline{F}$-convex function at $u \in X$ with respect to the same $p(x,u)$, $q(x,u)$ and $r(x,u)$ where $\overline{F}(x,u;z) = \phi(x,u)\, F(x,u;z)$.

REMARK 3.2: From(12), we can conclude that that every second order strongly F-pseudoconvex function $u \in X$ at with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(\mathrm{x,u})$ is second order η-convex function $u \in X$ at with respect to the same $p(x,u)$, $q(x,u)$ and $r(x,u)$ where $\eta^t(x,u)z = \phi(x,u)\, F(x,u;z)$.

REMARK 3.4: If we take $p(x,u) = q(x,u) = r(x,u) = 0$, and $F(x,u;z) = (x-u)z$, then (12) reduces to the definition of strongly pseudoconvex function.

REMARK 3.5: If we take $p(x,u) = q(x,u) = r(x,u) = 0$, and $F(x,u;z) = \eta(\mathrm{x,u})\mathrm{z}$, then (12) reduces to the definition of strongly pseudoinvex function.

REMARK 3.6: Every second order F-convex function at $u \in X$ with respect to $p(x,u)$, $q(x,u)$ and $r(x,u)$ is second order strongly F-pseudoconvex function at $u \in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$ =1. The converse is not true. This is demonstrated by the following example.

EXAMPLE 3.1: Let X = $(0,\infty)$. Define $f : X \to R$, $p,q,r : XxX \to R$, $\phi : XxX \to R_+$ and $F : XxXxR \to R$ as follows

$$f(x) = \frac{1}{x^3}\ ;\ p(x,u) = \frac{3u}{12}\left(1 - \frac{u^4}{3}\left(\frac{1}{x} + \frac{1}{u}\right)\right)\ ;\ q(x,u) = \frac{u^4}{6x};\ r(x,u) = \left(\frac{1}{x} - \frac{1}{u}\right)\ ;$$

$$\phi(x,u) = \left(\frac{1}{x} - \frac{1}{u}\right) \text{ and } F(x,u;a) = \left(\frac{1}{x} - \frac{1}{u}\right)|a|$$

Then, f is second order strongly F-pseudoconvex at $u \in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$ on X, but not second order F-convex function on X with respect to the same $p(x,u)$, $q(x,u)$ *and* $r(x,u)$ for $x = \frac{3}{2}$ *and* $u = 1$,

$$f(x) - f(u) + \frac{1}{2}q(x,u)\nabla^2 f(u)r(x,u) < F(x,u;\nabla f(u) + \nabla^2 f(u)p(x,u)) .$$

LEMMA 3.1: Let g be a vector function from $X \to R^m$ and $\sigma = (\sigma_1, \sigma_2, ..., \sigma_m)$ be a non-negative vector in R^n. If each component of $g_i, i = 1,2,....,m$ is second order strongly F-pseudoconvex at $u \in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$, then $\sigma^t g$ is also second order strongly F-pseudoconvex at $u \in X$ with respect to the same functions $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$.

Proof: Let $x \in X$. Now, since g_i is second order strongly F-pseudoconvex at $u \in X$ with respect to $p(x, u)$, $q(x, u)$, $r(x, u)$ and $\phi(x,u)$, we have

$$g_i(x) - g_i(u) + \frac{1}{2}q(x,u)\nabla^2(g_i(u))r(x,u) \ge \phi(x,u)\,F(x,u;\nabla g_i(u) + \nabla^2 g_i(u)p(x,u))$$

for all $x,u \in X$ and all i, i=1, 2, …, m

Since $\sigma_i \ge 0$, for all i =1, 2, .., m and the sublinearity of F, it follows that

$$\sigma^t g(x) - \sigma^t g(u) + \frac{1}{2}q(x,u)\nabla^2(\sigma^t g(u))r(x,u) \ge \phi(x,u)F(x,u;\nabla(\sigma^t g(u)) + \nabla^2(\sigma^t g(u))p(x,u))$$

Thus, $\sigma^t g$ is also second order strongly F-pseudoconvex at $u \in X$ with respect to the same $p(x, u)$, $q(x, u)$, $r(x, u)$ and $\phi(x,u)$.

4. SUFFICIENT OPTIMALITY CONDITIONS

We, now prove the sufficient optimality conditions for a feasible point x_o to be an optimal solution of (P) under second-order F-pseudoconvexity of $f(\cdot) + \cdot^t (B_1 w_1 + B_2 w_2 + \alpha s)$ and second order strong F-pseudoconvexity of g.

THEOREM 4.1: (Sufficient Optimality Conditions) If $f(\cdot) + \cdot^t (B_1 w_1 + B_2 w_2 + \alpha s)$ is second order F-pseudoconvex at $x_o \in$ X with respect to $p(x, x_o)$, $q(x, x_o)$ and $r(x,$

x_o) and each component of $g_i, i = 1,2,....,m$ is second order strongly F-pseudoconvex at $x_o \in X$ with respect to $p(x, x_o)$, $q(x, x_o)$ and $r(x, x_o)$ and $\phi(x, x_o)$ and there exists $(x_o, y, w_1, w_2, \alpha, p, q, r)$ satisfying (1) to (10) and also, (11) and further if $w_2^t B_2 w_2 \leq \alpha^2$, then x_o is an optimal solution for (P).

Proof: Let x be feasible for (P).

Now, from (1), (4) and (9), it follows that,

$$y^t g(x) - y^t g(x_o) + \frac{1}{2} q(x,x_o) \nabla^2 (y^t g(x_o)) r(x,x_o) \leq 0 \tag{13}$$

Since $g_i, i = 1,2,....,m$ are second order strongly F-pseudoconvex at $x_o \in X$ with respect to $p(x, x_o)$, $q(x, x_o)$ and $r(x, x_o)$ and $\phi(x, x_o)$, and $y \geq 0$ and by lemma 3.1., then $y^t g$ is also second order strongly F-pseudoconvex with respect to $\phi(x,x_o)$, $p(x,x_o)$, $q(x,x_o)$ and $r(x,x_o)$.

By the second order strongly F-pseudoconvexity of $y^t g$ at $x_o \in X$ and from (13), we have

$$\phi(x,x_o) F(x,x_o; \nabla(y^t g(x_o)) + \nabla^2 (y^t g(x_o)) p(x,x_o)) \leq 0 .$$

Since $\phi(x,x_o) > 0$ and from (3) and (7), we can conclude that

$$F(x,x_o; \nabla f(x_o) + Bw + \nabla^2 f(x_o) p(x,x_o)) \geq 0 .$$

By the second order F-pseudoconvexity of $f(\cdot) + \cdot^t (B_1 w_1 + B_2 w_2 + \alpha s)$ at $x_o \in X$, we have

$$f(x) + x^t (B_1 w_1 + B_2 w_2 + \alpha s) \geq (f(x_o) + x_o^t (B_1 w_1 + B_2 w_2 + \alpha s)) .$$

Using (6), (7), (8), (11), (2) and the lemma 2.1., we get

$$\begin{aligned} f(x) + (x^t B_1 x)^{1/2} &\geq f(x_o) + (x_o^t B_1 x_o)^{1/2} + \alpha(s^t x_o) + \alpha(x_o^t B_2 x_o)^{1/2} \\ &\qquad - \alpha(s^t x) - \alpha(w_2^t B_2 w_2)^{1/2} (x^t B_2 x)^{1/2} \\ &\geq f(x_o) + (x_o^t B_1 u)^{1/2} + \alpha(s^t x_o + (x_o^t B_2 x_o)^{1/2} - 1) \\ &\qquad + \alpha(1 - s^t x - (x^t B_2 x)^{1/2} \\ &= f(x_o) + (x_o^t B_1 x_o)^{1/2} + \alpha(1 - s^t x - (x^t B_2 x)^{1/2}) \\ &\geq f(x_o) + (x_o^t B_1 x_o)^{1/2} . \end{aligned}$$

Thus, x_o is an optimal solution for (P).

Hence the theorem.

5. DUALITY THEOREMS

We, now prove the following duality theorems between the problem (P) and its dual problem (D) under the assumptions of second-order F-pseudoconvexity of $f(\cdot)+\cdot^t(B_1w_1+B_2w_2+\alpha s)$ and second order strong F-pseudoconvexity of g.

THEOREM 5.1: (Weak Duality Theorem) Let x be feasible for (P) and $(u,y,w_1,w_2,\alpha,p,q,r)$ be feasible for (D). If $f(\cdot)+\cdot^t(B_1w_1+B_2w_2+\alpha s)$ is second order F-pseudoconvex at $u\in X$ with respect to $p(x,u)$, $q(x,u)$ and $r(x,u)$ and each component of $g_i, i=1,2,....,m$ is second order F-pseudoconvex at $u\in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$, then

$$f(x)+(x^tB_1x)^{1/2}\geq f(u)+(u^tB_1w_1).$$

Proof: Suppose that $f(x)+(x^tB_1x)^{1/2}< f(u)+(u^tB_1w_1)$.

Since $f(\cdot)+\cdot^t(B_1w_1+B_2w_2+\alpha s)$ is second order F-pseudoconvex at $u\in X$ with respect to $p(x,u)$, $q(x,u)$ and $r(x,u)$, it follows that

$$F(x,u;\nabla f(u)+B_1w_1+B_2w_2+\alpha s+\nabla^2 f(u)p(x,u))<0 \tag{14}$$

Now, since $g_i, i=1,2,....,m$ are second order strongly F-pseudoconvex at $u\in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$ and $y\geq 0$ and by lemma 3.1., y^tg is also second order strongly F-pseudoconvex at $u\in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$.

Now, since x is feasible for (P) and (u, y, w, p, q, r) is feasible for the problem (D), we have

$$y^tg(x)-y^tg(u)+\frac{1}{2}q(x,u)\nabla^2(y^tg(u))r(x,u)\leq 0.$$

Since y^tg is also second order strongly F-pseudoconvex at $u\in X$ with respect to $p(x,u)$, $q(x,u)$, $r(x,u)$ and $\phi(x,u)$ and $\phi(x,u)>0$, it follows that

$$F(x,u;\nabla(y^tg(u))+\nabla^2(y^tg(u))p(x,u))\leq 0 \tag{15}$$

Now, from (14) and (15) and by sublinearity of F, we can conclude that

$$F(x,u;\nabla(f(u)+y^tg(u))+B_1w_1+B_2w_2+\alpha s+\nabla^2(f(u)+y^tg(u))p(x,u))<0 \tag{16}$$

Now, from (3) and by the sublinearity of F, we have

$$F(x,u;\nabla(f(u)+y^tg(u))+B_1w_1+B_2w_2+\alpha s+\nabla^2(f(u)+y^tg(u))p(x,u))=0$$

which contradicts (16).

Thus, $f(x)+(x^tB_1x)^{1/2}\geq f(u)+(u^tB_1w_1)$.

Hence the theorem.

THEOREM 5.2: (Strong Duality) Let x_o be optimal for the problem (P) and the set z_o be empty. Then, there exist a scalar $\alpha \geq 0$ and vectors $u \in R^n, w_1, w_2 \in R^n, y_o \in R^m$ such that $(x_o, y_o, w_1, w_2, \alpha, \bar{p}=0, \bar{q}=0, \bar{r}=0)$ is feasible for (D) where $p(x_\circ, y_\circ) = \bar{p}$, $q(x_\circ, y_\circ) = \bar{q}$ and $r(x_\circ, y_\circ) = \bar{r}$ and the objective value of (P) at x_o and the objective value of (D) at $(x_o, y_o, w_1, w_2, \alpha, \bar{p}=0, \bar{q}=0, \bar{r}=0)$ are the same. If $f(\cdot) + \cdot^t (B_1 w_1 + B_2 w_2 + \alpha s)$ is second order *F*-pseudoconvex at $x_o \in X$ with respect to *p*(*x*, x_o), *q*(*x*, x_o) and *r*(*x*, x_o) and each component of $g_i, i = 1,2,....,m$ is second order strongly *F*- pseudoconvex at $x_o \in X$ with respect to *p*(*x*, x_o), *q*(*x*, x_o) and *r*(*x*, x_o) and $\phi(x, x_o)$, for all feasible $(x, u, y, w_1, w_2, \alpha, p, q, r)$, then $(x_o, y_o, w_1, w_2, \alpha, \bar{p}=0, \bar{q}=0, \bar{r}=0)$ is optimal for (D).

Proof: Since x_o is optimal for (P) and z_o is empty, there exists $\alpha \geq 0$, $u \in R^n, w_1, w_2 \in R^n, y_o \in R^m$ such that (1) to (10) are satisfied.

Thus, $(x_o, y_o, w_1, w_2, \alpha, \bar{p}=0, \bar{q}=0, \bar{r}=0)$ where $p(x_\circ, y_\circ) = \bar{p}$, $q(x_\circ, y_\circ) = \bar{q}$ and $r(x_\circ, y_\circ) = \bar{r}$ is feasible for (D).

From (11), we conclude that the objective value of (P) at x_o and the objective value of (D) at $(x_o, y_o, w_1, w_2, \alpha, \bar{p}=0, \bar{q}=0, \bar{r}=0)$ are the same.

Suppose that $(x_o, y_o, w_1, w_2, \alpha, \bar{p}=0, \bar{q}=0, \bar{r}=0)$ is not optimal for (D).

Then there exists a feasible $(u, y, w_1^1, w_2^1, \alpha_1, p, q, r)$ of (D) such that

$$f(u) + u^t B_1 w_1^1 > f(x_o) + (x_o^t B_1 w_1^1).$$

From (11), it follows that

$$f(u) + u^t B_1 w_1^1 > f(x_o) + (x_o^t B_1 x_o)^{1/2}$$

which is a contradiction to theorem 5.1.

Thus, $(x_o, y_o, w_1, w_2, \alpha, \bar{p}=0, \bar{q}=0, \bar{r}=0)$ is optimal for (D).

Hence the theorem.

References

[1] E.Eisenburg, Support of a convex function, Bulletin of American Mathematical Society, 688(1962), 192-195.

[2] P.Kanniappan and P.Pandian, Duality for nonlinear programming problems with strong pseudoinvexity constraints, Opsearch, 32(1995), 95-104.

[3] P.Kanniappan and P.Pandian, A class of non-differentiable programming problems with strong pseudoinvexity constraints, International Journal of Management and Systems, 14(1998), 311-316.

[4] P.Kanniappan and P.Pandian, Optimality and duality in generalized pseudolinear multiobjective programming, Indian J. pure. appl. Math., 31(2000), 1151-1160.

[5] A.Kumar and J.P.Agarwal, A note on a class of non-differentiable programming problems, International Journal of Management and Systems, 10(1994), 189-194.

[6] A. Kumar and S.Ghosh, A non-differentiable programming problems under strong pseudoinvexity, Opsearch, 40(2003), 299-304.

[7] S.M.Sinha and D.R.Aylawadi, Optimality conditions for a class of non-differentiable mathematical programming problems, Opsearch., 19(1982), 225-237.

[8] T.Weir, On strong pseudoconvexity in nonlinear programming duality, Opsearch, 27(1990), 117-121.