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Difference Cordial Labeling of Graphs Obtained from Double Snakes

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Abstract

Let G be a (p, q) graph. Let $f: V(G) \to \{1, 2, \ldots, p\}$ be a function. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a difference cordial if f is a one to one map and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with admits a difference cordial labeling is called a difference cordial graph. In this paper, we investigate the difference cordial labeling behavior of several graphs which are obtained from triangular snake and quadrilateral snake.

Keywords: corona, comb, tree, cycle, wheel.

2010 Subject Classification: 05C78

Introduction:

Throughout this paper we have considered only simple and undirected graph. Let $G = (V, E)$ be a (p, q) graph. The cardinality of V is called the order of G and the cardinality of E is called the size of G. Labeled graphs are used in several areas such as astronomy, radar and circuit design and database management [1]. The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. The concept of difference cordial labeling has been introduced by R. Ponraj, S. Sathish

Narayanan, R. Kala in [3]. In [3, 4, 5] difference cordial labeling behavior of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web, some snake graphs, crown $C_n \odot K_1$, comb $P_n \odot K_1$, $P_n \odot C_m$, $C_n \odot C_m$, $W_n \odot K_2$, $W_n \odot 2K_1$ and some more standard graphs have been investigated. In this paper we investigate the difference cordial labeling behavior of $DT_n \odot K_1$, $DT_n \odot 2K_1$, $DT_n \odot K_2$, $DQ_n \odot K_1$, $DQ_n \odot 2K_1$, $DQ_n \odot K_2$ where DT_n and DQ_n are double triangular snake and double quadrilateral snake respectively. Let x be any real number. Then $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [2].

Difference Cordial Labeling

Definition 2.1:
Let G be a (p, q) graph. Let f be a map from $V(G)$ to $\{1, 2, \ldots, p\}$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called difference cordial labeling if f is $1-1$ and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.

A double triangular snake DT_n consists of two triangular snakes that have a common path. That is, a double triangular snake is obtained from a path $u_1, u_2 \ldots u_n$ by joining u_i and u_{i+1} to a new vertex v_i $(1 \leq i \leq n-1)$ and to a new vertex w_i $(1 \leq i \leq n-1)$.

Theorem 2.2: $DT_n \odot K_1$ is difference cordial.

Proof: Let $V(DT_n \odot K_1) = V(DT_n) \cup \{x_i : 1 \leq i \leq n\} \cup \{v_i', w_i' : 1 \leq i \leq n-1\}$ and $E(DT_n \odot K_1) = E(DT_n) \cup \{u_i x_i : 1 \leq i \leq n\} \cup \{v_i v_i', w_i w_i' : 1 \leq i \leq n-1\}$. Define $f: V(DT_n \odot K_1) \to \{1, 2 \ldots 6n-4\}$ by

$$f(u_i) = 4i - 2 \quad 1 \leq i \leq n$$
$$f(x_i) = 4i - 3 \quad 1 \leq i \leq n$$
$$f(v_i) = 4i - 1 \quad 1 \leq i \leq n-1$$
$$f(v_i') = 4i \quad 1 \leq i \leq n-1$$
$$f(w_i) = 4n + 2i - 3 \quad 1 \leq i \leq n-1$$
$$f(w_i') = 4n + 2i - 2 \quad 1 \leq i \leq n-1$$

Since $e_f(1) = 4n-3$ and $e_f(0) = 4n-4$, $DT_n \odot K_1$ is difference cordial. ■

Theorem 2.3: $DT_n \odot 2K_1$ is difference cordial.

Proof: Let $V(DT_n \odot 2K_1) = V(DT_n) \cup \{x_i, y_i : 1 \leq i \leq n\} \cup \{v_i', v_i'', w_i', w_i'' : 1 \leq i \leq n-1\}$ and $E(DT_n \odot 2K_1) = E(DT_n) \cup \{u_i x_i, u_i y_i : 1 \leq i \leq n\} \cup \{v_i v_i', v_i v_i'', w_i w_i', w_i w_i'' : 1 \leq i \leq n-1\}$. Define $f: V(DT_n \odot 2K_1) \to \{1, 2 \ldots 9n-6\}$ by

$$f(u_i) = 6i - 4 \quad 1 \le i \le n$$
$$f(x_i) = 6i - 5 \quad 1 \le i \le n$$
$$f(y_i) = 6i - 3 \quad 1 \le i \le n$$
$$f(v_i) = 6i - 1 \quad 1 \le i \le n - 1$$
$$f(v_i') = 6i - 2 \quad 1 \le i \le n - 1$$
$$f(v_i'') = 6i \quad 1 \le i \le n - 1$$
$$f(w_i) = 6n + 3i - 4 \quad 1 \le i \le \left\lfloor \frac{n-2}{2} \right\rfloor$$
$$f(w_i') = 6n + 3i - 5 \quad 1 \le i \le \left\lfloor \frac{n-2}{2} \right\rfloor$$
$$f(w_i'') = 6n + 3i - 3 \quad 1 \le i \le \left\lfloor \frac{n-2}{2} \right\rfloor$$
$$f(w_i) = 6n + 3\left\lfloor \frac{n-2}{2} \right\rfloor + 3i - 5 \quad 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil$$
$$f\left(w'_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 6n + 3\left\lfloor \frac{n-2}{2} \right\rfloor + 3i - 4 \quad 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil$$
$$f\left(w''_{\left\lfloor \frac{n-2}{2} \right\rfloor + i}\right) = 6n + 3\left\lfloor \frac{n-2}{2} \right\rfloor + 3i - 3 \quad 1 \le i \le \left\lceil \frac{n+2}{2} \right\rceil$$

The following table (i) proves that f is a difference cordial labeling of $DT_n \odot 2K_1$.

Table (i)

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 (mod\ 2)$	$\frac{11n - 10}{2}$	$\frac{11n - 8}{2}$
$n \equiv 1 (mod\ 2)$	$\frac{11n - 9}{2}$	$\frac{11n - 9}{2}$

■

Theorem 2.4: $DT_n \odot K_2$ is difference cordial.

Proof: Let $V(DT_n \odot K_2) = V(DT_n) \cup \{x_i, y_i : 1 \le i \le n\} \cup \{v_i', v_i'', w_i', w_i'' : 1 \le i \le n - 1\}$ and $E(DT_n \odot K_2) = E(DT_n) \cup \{u_i x_i, u_i y_i, x_i y_i : 1 \le i \le n\} \cup \{v_i v_i', v_i v_i'', w_i w_i', w_i w_i'' : 1 \le i \le n - 1\}$.
Define $f: V(DT_n \odot K_2) \to \{1, 2 \dots 9n - 6\}$ by

$$f(u_i) = 6i - 3 \quad 1 \le i \le n - 1$$
$$f(x_i) = 6i - 4 \quad 1 \le i \le n - 1$$
$$f(y_i) = 6i - 5 \quad 1 \le i \le n - 1$$
$$f(v_i) = 6i - 2 \quad 1 \le i \le n - 1$$
$$f(v_i') = 6i \quad 1 \le i \le n - 1$$

$$f(v_i'') = 6i - 1 \; 1 \leq i \leq n-1$$
$$f(w_i) = 6n + 3i - 5 \; 1 \leq i \leq n-1$$
$$f(w_i') = 6n + 3i - 4 \; 1 \leq i \leq n-1$$
$$f(w_i'') = 6n + 3i - 3 \; 1 \leq i \leq n-1$$

$f(u_n) = 6n - 5$, $f(x_n) = 6n - 4$ and $f(y_n) = 6n - 3$. Since $e_f(1) = 7n - 5$ and $e_f(0) = 7n - 6$, f is a difference cordial labeling of $DT_n \odot K_2$. ■

A double quadrilateral snake DQ_n consists of two triangular snakes that have a common path. Let $V(DQ_n) = \{u_i: 1 \leq i \leq n\} \cup \{v_i, w_i, x_i, y_i : 1 \leq i \leq n-1\}$ and $E(DQ_n) = \{u_iu_{i+1}, v_iw_i, x_iy_i, w_iu_{i+1}, y_iu_{i+1} : 1 \leq i \leq n-1\}$.

Theorem 2.5: $DQ_n \odot K_1$ is difference cordial.

Proof: Let $V(DQ_n \odot K_1) = V(DQ_n) \cup \{u_i': 1 \leq i \leq n\} \cup \{v_i', w_i', x_i', y_i': 1 \leq i \leq n-1\}$ and $E(DQ_n \odot K_1) = E(DQ_n) \cup \{u_iu_i': 1 \leq i \leq n\} \cup \{v_iv_i', w_iw_i', x_ix_i', y_iy_i': 1 \leq i \leq n-1\}$. Define a map $f: V(DQ_n \odot K_1) \to \{1, 2 \dots 10n - 8\}$ by

$$f(u_i) = 4n + 2i - 5 \; 1 \leq i \leq n$$
$$f(u_i') = 4n + 2i - 4 \; 1 \leq i \leq n$$
$$f(v_i) = 4i - 2 \; 1 \leq i \leq n-1$$
$$f(v_i') = 4i - 3 \; 1 \leq i \leq n-1$$
$$f(w_i) = 4i - 1 \; 1 \leq i \leq n-1$$
$$f(w_i') = 4i \; 1 \leq i \leq n-1$$
$$f(x_i) = 6n + 4i - 7 \; 1 \leq i \leq n-1$$
$$f(x_i') = 6n + 4i - 6 \; 1 \leq i \leq n-1$$
$$f(y_i) = 6n + 4i - 5 \; 1 \leq i \leq n-1$$
$$f(y_i') = 6n + 4i - 4 \; 1 \leq i \leq n-1$$

Since $e_f(1) = 6n - 5$ and $e_f(0) = 6n - 6$, f is a difference cordial labeling of $DQ_n \odot K_1$ ■

Theorem 2.6: $DQ_n \odot 2K_1$ is difference cordial.

Proof: Let $V(DQ_n \odot 2K_1) = V(DQ_n) \cup \{v_i', v_i'', w_i', w_i'', x_i', x_i'', y_i', y_i'': 1 \leq i \leq n-1\} \cup \{u_i', u_i'': 1 \leq i \leq n\}$,
$E(DQ_n \odot 2K_1) = E(DQ_n) \cup \{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'', x_ix_i', x_ix_i'', y_iy_i'': 1 \leq i \leq n-1\} \cup \{u_iu_i', u_iu_i'': 1 \leq i \leq n\}$. Define a map $f: V(DQ_n \odot 2K_1) \to \{1, 2, \dots 15n - 12\}$ by

$$f(u_i) = 9i - 7 \; 1 \leq i \leq n$$
$$f(u_i') = 9i - 8 \; 1 \leq i \leq n$$
$$f(u_i'') = 9i - 6 \; 1 \leq i \leq n$$
$$f(v_i) = 9i - 4 \; 1 \leq i \leq n-1$$
$$f(v_i') = 9i - 5 \; 1 \leq i \leq n-1$$
$$f(v_i'') = 9i - 3 \; 1 \leq i \leq n-1$$

$$f(w_i) = 9i - 1 \quad 1 \le i \le n-1$$
$$f(w_i') = 9i - 2 \quad 1 \le i \le n-1$$
$$f(w_i'') = 9i \quad 1 \le i \le n-1$$
$$f(x_i) = 9n + 6i - 10 \quad 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor$$
$$f(x_i') = 9n + 6i - 11 \quad 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor$$
$$f(x_i'') = 9n + 6i - 9 \quad 1 \le i \le \left\lfloor \frac{n}{4} \right\rfloor$$
$$f\left(x_{\left\lfloor \frac{n}{4} \right\rfloor + i}\right) = 9n + 6\left\lfloor \frac{n}{4} \right\rfloor + 6i - 11 \quad 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil - 1$$
$$f\left(x'_{\left\lfloor \frac{n}{4} \right\rfloor + i}\right) = 9n + 6\left\lfloor \frac{n}{4} \right\rfloor + 6i - 10 \quad 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil - 1$$
$$f\left(x''_{\left\lfloor \frac{n}{4} \right\rfloor + i}\right) = 9n + 6\left\lfloor \frac{n}{4} \right\rfloor + 6i - 9 \quad 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil - 1$$

$f(y_i) = 9n + 6i - 7 \; 1 \le i \le \left\lfloor \frac{n-4}{4} \right\rfloor$ if $n \equiv 0, 1 \ (mod\ 4)$ $1 \le i \le \left\lceil \frac{n-4}{4} \right\rceil$ if $n \equiv 2, 3 \ (mod\ 4)$. $f(y_i') = 9n + 6i - 8 \; 1 \le i \le \left\lfloor \frac{n-4}{4} \right\rfloor$ if $n \equiv 0, 1 \ (mod\ 4)$ $1 \le i \le \left\lceil \frac{n-4}{4} \right\rceil$ if $n \equiv 2, 3 \ (mod\ 4)$. $f(y_i'') = 9n + 6i - 6 \; 1 \le i \le \left\lfloor \frac{n-4}{4} \right\rfloor$ if $n \equiv 0, 1 \ (mod\ 4)$ $1 \le i \le \left\lceil \frac{n-4}{4} \right\rceil$ if $n \equiv 2, 3 \ (mod\ 4)$.

Case (i) : $n \equiv 0, 1 \ (mod\ 4)$.
$f\left(y_{\left\lfloor \frac{n-4}{4} \right\rfloor + i}\right) = 9n + 6\left\lfloor \frac{n-4}{4} \right\rfloor + 6i - 8 \; 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil$ $f\left(y'_{\left\lfloor \frac{n-4}{4} \right\rfloor + i}\right) = 9n + 6\left\lfloor \frac{n-4}{4} \right\rfloor + 6i - 7 \; 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil$ $f\left(y''_{\left\lfloor \frac{n-4}{4} \right\rfloor + i}\right) = 9n + 6\left\lfloor \frac{n-4}{4} \right\rfloor + 6i - 6 \; 1 \le i \le \left\lceil \frac{3n}{4} \right\rceil$.

Case (ii) : $n \equiv 2, 3 \ (mod\ 4)$.
$f\left(y_{\left\lceil \frac{n-4}{4} \right\rceil + i}\right) = 9n + 6\left\lceil \frac{n-4}{4} \right\rceil + 6i - 8 \; 1 \le i \le \left\lfloor \frac{3n}{4} \right\rfloor$ $f\left(y'_{\left\lceil \frac{n-4}{4} \right\rceil + i}\right) = 9n + 6\left\lceil \frac{n-4}{4} \right\rceil + 6i - 7 \; 1 \le i \le \left\lfloor \frac{3n}{4} \right\rfloor$ $f\left(y''_{\left\lceil \frac{n-4}{4} \right\rceil + i}\right) = 9n + 6\left\lceil \frac{n-4}{4} \right\rceil + 6i - 6 \; 1 \le i \le \left\lfloor \frac{3n}{4} \right\rfloor$.

The following table (ii) proves that f is a difference cordial labeling of $DQ_n \odot 2K_1$.

Table (ii)

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 (mod\ 2)$	$\frac{17n-16}{2}$	$\frac{17n-14}{2}$
$n \equiv 1 (mod\ 2)$	$\frac{17n-15}{2}$	$\frac{17n-15}{2}$

■

Theorem 2.7: $DQ_n \odot K_2$ is difference cordial.

Proof: Let $V(DQ_n \odot K_2) = V(DQ_n) \cup \{v_i', v_i'', w_i', w_i'', x_i', x_i'', y_i', y_i'': 1 \le i \le n-1\} \cup \{u_i', u_i'': 1 \le i \le n\}$ and $E(DQ_n \odot K_2) \cup \{u_iu_i', u_iu_i'', u_i'u_i'': 1 \le i \le n\} \cup \{v_iv_i', v_iv_i'', v_i'v_i'', w_iw_i', w_iw_i'', w_i'w_i'', x_ix_i', x_ix_i'', x_i'x_i'', y_iy_i', y_iy_i'', y_i'y_i'': 1 \le i \le n-1\}$. Define a map $f: V(DQ_n \odot K_2) \rightarrow \{1, 2 \ldots 15n - 12\}$ by

$$f(u_i) = 6n + 3i - 8 \quad 1 \le i \le n$$
$$f(u_i') = 6n + 3i - 7 \quad 1 \le i \le n$$
$$f(u_i'') = 6n + 3i - 6 \quad 1 \le i \le n$$
$$f(v_i) = 6i - 3 \quad 1 \le i \le n-1$$
$$f(v_i') = 6i - 4 \quad 1 \le i \le n-1$$
$$f(v_i'') = 6i - 5 \quad 1 \le i \le n-1$$
$$f(w_i) = 6i - 2 \quad 1 \le i \le n-1$$
$$f(w_i') = 6i \quad 1 \le i \le n-1$$
$$f(w_i'') = 6i - 1 \quad 1 \le i \le n-1$$
$$f(x_i) = 9n + 6i - 11 \quad 1 \le i \le n-1$$
$$f(x_i') = 9n + 6i - 10 \quad 1 \le i \le n-1$$
$$f(x_i'') = 9n + 6i - 9 \quad 1 \le i \le n-1$$
$$f(y_i) = 9n + 6i - 8 \quad 1 \le i \le n-1$$
$$f(y_i') = 9n + 6i - 7 \quad 1 \le i \le n-1$$
$$f(y_i'') = 9n + 6i - 6 \quad 1 \le i \le n-1$$

Since $e_f(1) = 11n - 9$ and $e_f(0) = 11n - 10$, $DQ_n \odot K_2$ is a difference cordial ■

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