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Slow Steady Flow of a Second Order Thermo-Viscous Fluid between Two Rotating Spheres without Dissipation Term

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Abstract

In this paper, the steady rotary flow of a second-order thermo-viscous fluid between two concentric spheres is examined. The viscous dissipation term in the energy equation is neglected. The velocity and temperature fields are obtained analytically from the momentum and energy equations using appropriate boundary conditions. The drag force, viscous couple and the Nussult number on the inner and outer boundaries of the spheres are calculated. The pressure and the stress components are calculated and these are influenced by the thermo-viscous parameters α_6, α_8. It is noted that a force is generated in transverse direction due to the thermo-stress and thermo-stress-viscous nature of the fluid. The existence of ρF_θ is a remarkable feature of thermo-viscous fluid. Such a force is not needed in sustaining the motion when the fluid is classical Newtonian Fourier heat conducting fluid Lamb [15].

Key words: Thermo-viscous fluids, heat flux bi-vector, Steady flow, couple, drag force, Nussult number, Stresses.

INTRODUCTION:

A Thermo-viscous fluid (i.e. a viscous fluid in a thermal state) is characterized by two sets of constitutive equations, one for the stress and the other for the heat. The stress tensor S and the heat flux bi-vector h are polynomial functions of the kinematic tensors, the deformation rate tensor d and the thermal bi-gradient vector b; where

$$S = S(d,b;\rho,\theta),\ h = h(d,b;\rho,\theta),\ d = (u_{i,j} + u_{j,i})/2$$

$$b \equiv \| b_{ij} \| \equiv \| \in_{ijk} \theta_{,k} \|,\ h \equiv \| h_{ij} \| \equiv \| \in_{ijk} q_{,k} \|$$

where u_i is the velocity in the i^{th} - direction, θ the temperature, ρ the density and $q_{,k}$ heat flux in the k^{th} - direction. These equations can be seen in agreement with the principles of determinism, material objectivity and equipresence. Further, these functions are hemitropic polynomial functions of tensors d_{km} and b_{km}.

The thermo-viscous fluids are classified by the combined degrees N&P of d and b respectively in the constitutive relations.

ZERO ORDER THEORY:

In this case $N = P = 0$. Therefore all the constitutive coefficients with the exception of α_1 are equal to zero. The constitutive equations reduce to

$$S = \alpha_1 I \tag{1}$$

$$h = 0 \tag{2}$$

where α_1 is independent of d and b and is simply a function of ρ and θ.The materials characterized by these equations are nothing but so called ideal (non-Viscous) fluids with α_1 (<0) representing the hydrostatic pressure.

FIRST ORDER THEORY

In this case, $\text{Max}|N + P| = 1$. Therefore the constitutive equations for this case become

$$S = \alpha_1 I + \alpha_3 d \tag{3}$$

$$h = \beta_1 b \tag{4}$$

where the constitutive coefficients α_1, α_3 and β_1 are scalar polynomials in trd with coefficients which are functions of ρ and θ.

The equations (3) may be recognized as the constitutive equations of the first order simple fluid of Noll's type or that of the classical Newtonian –viscous fluids. The equation (4) is the same as the linear law of heat conduction i.e. Fourier law. For this reason the fluids characterized by these equations (3) and (4) have been referred to as classical Newtonian-viscous and Fourier heat conducting fluids in the foregoing chapters.

It may be noticed that from the theory of zeroth and first order theories, the constitutive relations are decoupled in deformation rate tensor d and thermal bi-gradient vector b and therefore both these theories do not truly describe the interaction between the viscous and thermal characteristics of the medium. It is therefore necessary to consider higher order theories which exhibit such interactions.

SECOND-ORDER THERMO-VISCOUS FLUIDS

In this case, we have Max$|N+P|=2$.The second order thermo-viscous fluids are characterized by the constitutive equations

$$S=\alpha_1 I+\alpha_3 d+\alpha_5 d^2+\alpha_6 b^2+\alpha_8(db-bd) \tag{5}$$

$$h=\beta_1 b+\beta_3(db+bd) \tag{6}$$

Where the constitutive coefficients $\alpha_i^{'s}$ and $\beta_i^{'s}$ are scalar polynomials in the traces of respective arguments.

$\alpha_1=-p^*$: p^* is the fluid pressure

$\alpha_3=2\mu$: μ is the coefficient of classical (Newtonian) viscosity

$\alpha_5=4\mu_c$: μ_c is the coefficient of (Reiner-Rivlin) cross-viscosity

α_6 : Thermo-stress coefficient

$2\alpha_8$: coefficient of Kho-Eringen thermo-stress-viscous

$-\beta_1=k$: $k=$ (Fourier) thermal conductivity coefficient.

β_3 : strain thermal conductivity coefficient.

This is the simplest model for a thermo-viscous fluid which could contain the interaction between mechanical and thermal phenomena in both constitutive equations.

FORMU(LATION AND SOLUTION OF THE PROBLEM

Introducing the spherical polar coordinate system (r,θ,ϕ) with $\theta=0$ coinciding with the axis of rotation in which the common centre is situated. Further r is the radial distance from the centre. The fluid flow is represented by the velocity $(0,0,w,(r,\theta))$ and the temperature distribution $\eta(r,\theta)$. This choice of velocity evidently satisfies the continuity equation. r = b

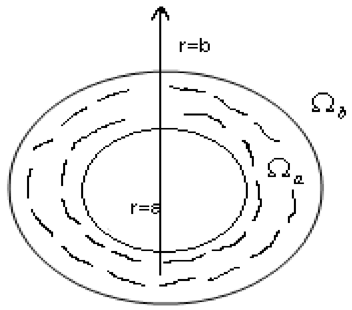

Fig 1. Flow configuration

The rate of deformation tensor d:

$$d = \frac{1}{2}\begin{bmatrix} 0 & 0 & \frac{\partial w}{\partial r} - \frac{w}{r} \\ 0 & 0 & \frac{1}{r}\left(\frac{\partial w}{\partial \theta} - w\cot\theta\right) \\ \frac{\partial w}{\partial r} - \frac{w}{r} & \frac{1}{r}\left(\frac{\partial w}{\partial \theta} - w\cot\theta\right) & 0 \end{bmatrix} \tag{7}$$

Thermal bi-gradient matrix:

$$b = \begin{bmatrix} 0 & 0 & -\frac{1}{r}\frac{\partial \eta}{\partial \theta} \\ 0 & 0 & \frac{\partial \eta}{\partial r} \\ \frac{1}{r}\frac{\partial \eta}{\partial \theta} & -\frac{\partial \eta}{\partial r} & 0 \end{bmatrix} \tag{8}$$

The thermo-viscous fluid is characterized by the following stress- tensor S and heat-flux vector h and these are given by The stress tensor

$$S = -pI + 2\mu d + 4\mu_c d^2 + \alpha_6 b^2 + 2\alpha_8 (db - bd) \tag{9}$$

$$
= \begin{bmatrix}
\begin{array}{l} -p+\mu_c\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)^2+\alpha_6\left(\frac{1}{r}\frac{\partial \eta}{\partial \theta}\right)^2 \\ +2\alpha_8\frac{1}{r}\frac{\partial \eta}{\partial \theta}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right) \end{array} &
\begin{array}{l} \mu_c\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right) \\ -\alpha_6\frac{1}{r}\frac{\partial \eta}{\partial \theta}\frac{\partial \eta}{\partial r} \\ +\alpha_8\left\{-\frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)+\frac{1}{r}\frac{\partial \eta}{\partial \theta}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)\right\} \end{array} &
\mu\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right) \\
\begin{array}{l} \mu_c\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right) \\ -\alpha_6\frac{1}{r}\frac{\partial \eta}{\partial \theta}\frac{\partial \eta}{\partial r} \\ +\alpha_8\left\{-\frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)\right. \\ \left.+\frac{1}{r}\frac{\partial \eta}{\partial \theta}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)\right\} \end{array} &
\begin{array}{l} -p+\mu_c\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)^2 \\ +\alpha_6\left(\frac{\partial \eta}{\partial r}\right)^2 \\ -2\alpha_8\frac{\partial \eta}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right) \end{array} &
\mu\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right) \\
\mu\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right) &
\mu\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right) &
\begin{array}{l} -p+\mu_c\left\{\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)^2+\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)^2\right\} \\ +\alpha_6\left\{\left(\frac{\partial \eta}{\partial r}\right)^2+\left(\frac{1}{r}\frac{\partial \eta}{\partial \theta}\right)^2\right\} \\ +2\alpha_8\left\{-\frac{\partial \eta}{\partial \theta}\frac{1}{r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)+\frac{\partial \eta}{\partial r}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)\right\} \end{array}
\end{bmatrix}
$$

Heat flux vector

$$h=-kb+2\beta_3(db+bd) \tag{10}$$

$$
h=\begin{bmatrix}
0 & \beta_3\left\{-\frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)-\frac{1}{r}\frac{\partial \eta}{\partial \theta}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)\right\} & \frac{k}{r}\frac{\partial \eta}{\partial \theta} \\
\beta_3\left\{\frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)+\frac{1}{r}\frac{\partial \eta}{\partial \theta}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}-\frac{w\cot\theta}{r}\right)\right\} & 0 & -k\frac{\partial \eta}{\partial r} \\
-\frac{k}{r}\frac{\partial \eta}{\partial \theta} & k\frac{\partial \eta}{\partial r} & 0
\end{bmatrix}
$$

THE EQUATIONS OF MOTION:

In the radial direction:

$$
\begin{aligned}
\frac{\partial p}{\partial r} &= -\frac{\rho w^2}{r}+\mu_c\left[\frac{1}{r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)^2+\frac{\partial}{\partial r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)^2\right] \\
&+\alpha_6\left[\frac{1}{r}\left(\frac{1}{r}\frac{\partial \eta}{\partial \theta}\right)^2-\frac{2}{r}\left(\frac{\partial \eta}{\partial r}\right)^2+\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \eta}{\partial \theta}\right)^2-\frac{\cot\theta}{r^2}\frac{\partial \eta}{\partial \theta}\frac{\partial \eta}{\partial r}-\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial \eta}{\partial \theta}\frac{\partial \eta}{\partial r}\right)\right]+ \\
&\alpha_8\left[\frac{6}{r^2}\frac{\partial \eta}{\partial \theta}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)+2\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \eta}{\partial \theta}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)\right)-\frac{\cot\theta}{r}\frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)-\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}-\frac{w}{r}\right)\right)\right]
\end{aligned}
\tag{11}
$$

In the transverse direction

$$\rho F_\theta = \frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\alpha_6 \frac{1}{r}\frac{\partial \eta}{\partial \theta}\frac{\partial \eta}{\partial r} + \alpha_8 \frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}\frac{1}{r}\right)\right)\right)$$

$$-\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\left(-p + \alpha_6\left(\frac{\partial \eta}{\partial r}\right)^2\right)\sin\theta\right)$$

$$+\frac{1}{6}\left(\frac{\alpha_6}{r}\frac{\partial \eta}{\partial \theta}\frac{\partial \eta}{\partial r} + \alpha_8\frac{\partial \eta}{\partial r}\left(\frac{\partial w}{\partial r}\frac{1}{r}\right)\right) + \frac{\cot\theta}{r}\left[-p + \mu_c\left(\frac{\partial w}{\partial r}\frac{w}{r}\right)^2\right.$$

$$\left. + \alpha_6\left(\left(\frac{\partial \eta}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial \eta}{\partial r}\right)^2\right)\right] + 2\alpha_8\left(\frac{\partial \eta}{\partial \theta}\right)\frac{1}{r}\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right) + \rho\left(\frac{w^2\cot\theta}{r}\right) \tag{12}$$

In ϕ direction:

$$\rho F_\phi = -\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\mu\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right)\right) + \frac{\mu}{r}\left(\frac{\partial w}{\partial r} - \frac{w}{r}\right)\right]$$

$$+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\mu}{r}\left(\frac{\partial w}{\partial \theta} - w\cot\theta\right)\right) + \frac{2\mu}{r^2}\cot\theta\left(\frac{\partial w}{\partial \theta} - w\cot\theta\right) \tag{13}$$

Equation of motion in ϕ direction reduced to

$$\nabla^2 w = \frac{w}{r^2}\operatorname{cosec}^2\theta \tag{14}$$

The energy equation reduces to

$$\nabla^2\eta = 0 \quad \textit{where} \quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial}{\partial \theta} \tag{15}$$

Consider the slow steady flow of a second order thermo-viscous fluid between two spheres of radii $a,\ b(b > a)$. When the spheres are rotating with constant angular velocities $\Omega_{a,}\Omega_b$ respectively and rotating about a common diameter and kept at temperatures η_a, η_b respectively

The boundary conditions

$$\left.\begin{aligned} &w(a,\theta) = a\Omega_a \sin\theta \\ &w(a,\theta) = b\Omega_b \sin\theta \\ &\eta(a,\theta) = \eta_a,\ \eta(b,\theta) = \eta_b \end{aligned}\right\} \tag{16}$$

Introducing the non-dimensional quantities.

$$\left.\begin{aligned} &r = aR,\ w(r,\theta) = a\Omega_a W(R,\theta) \\ &\frac{b}{a} = \sigma,\ \gamma = \frac{\Omega_b}{\Omega_a},\ \frac{\eta-\eta_a}{\eta_b-\eta_a} = T(R,\theta) \end{aligned}\right\} \tag{17}$$

The boundary conditions (16) reduce to

$$\left.\begin{aligned} &W(1,\theta) = \sin\theta \\ &W(\sigma,\theta) = \sigma\gamma\sin\theta \\ &T(1,\theta) = 0, T(\sigma,\theta) = 1 \end{aligned}\right\} \tag{18}$$

Now the equations in the Non-dimensional form
i.e. the momentum and energy equations (14) and (15) with(17) reduce to

$$\nabla^2 W - \frac{W}{R^2}\operatorname{cosec}^2\theta = 0 \tag{19}$$

and

$$\nabla^2 T = 0 \tag{20}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial R^2} + \frac{2}{R}\frac{\partial}{\partial R} + \frac{1}{R^2}\frac{\partial^2}{\partial\theta^2} + \frac{\cot\theta}{R^2}\frac{\partial}{\partial\theta}$$

Let $W(R,\theta) = F(R)\sin\theta$ (21)

Substituting (21) in (19), we get

$$\left(F^{11} + \frac{2F^1}{R} - \frac{2F}{R^2}\right)\sin\theta = 0 \tag{22}$$

$$\because \sin\theta \neq 0,\ F^{11} + \frac{2F^1}{R} - \frac{+2F}{R^2} = 0 \tag{23}$$

Hence

$$F(R) = AR + \frac{B}{R^2} \tag{24}$$

$$W(R,\theta) = \left(AR + \frac{B}{R^2}\right)\sin\theta \tag{25}$$

Using the boundary conditions, the equation (25) reduces to the form

$$W(R,\theta) = \left(\left(\frac{1-\gamma\sigma^3}{1-\sigma^3}\right)R + \left(\frac{(\gamma-1)\sigma^3}{1-\sigma^3}\right)\frac{1}{R^2}\right)\sin\theta \tag{26}$$

Let

$$T(R,\theta) = G(R) + H(R)\sin^2\theta \tag{27}$$

substituting (27) in (20), we get

$$G^{11}+\frac{2G^{1}}{R}+\frac{4H}{R^{2}}+\sin^{2}\theta\left(H^{11}+\frac{2H^{1}}{R}-\frac{4H}{R^{2}}-\frac{2H}{R^{2}}\right)=0 \tag{28}$$

consider

$$G^{11}+\frac{2G^{1}}{R}+\frac{4H}{R^{2}}=0, \tag{29}$$

$$H^{11}+\frac{2H^{1}}{R}-\frac{6H}{R^{2}}=0 \tag{30}$$

The boundary conditions for G *and* H are

$$\left.\begin{array}{l}G(1)=0,\ G(\sigma)=1\\ H(1)=0,\ H(\sigma)=0\end{array}\right\} \tag{31}$$

using the boundary conditions (31) we get

$$G(R)=\frac{\sigma}{\sigma-1}\left(1-\frac{1}{R}\right)\ \ and\ \ H(R)=0 \tag{32}$$

Hence the temperature distribution (the solution of (27))

$$T(R)=G(R)=\left(1-\frac{1}{R}\right)\frac{\sigma}{\sigma-1} \tag{33}$$

The pressure distribution is obtained from the momentum equation in radial direction (11).

i.e. $$p^{*}(p-p_{0})=\left(-\rho a\Omega_{a}^{\ 2}\sin^{2}\theta\right)\left(\frac{A^{2}R^{2}}{2}-\frac{B^{2}}{4R^{4}}-\frac{2AB}{R}\right)$$
$$+\frac{15\mu_{c}B^{2}\Omega_{a}^{\ 2}\sin^{2}\theta}{2R^{6}}+\frac{\alpha_{6}\sigma^{2}(\eta_{b}-\eta_{a})^{2}}{2a^{2}(\sigma-1)^{2}R^{4}}$$
$$-\alpha_{8}\left[\frac{6}{5}\frac{B\Omega_{a}\sigma}{aR^{5}(\sigma-1)}\left(\eta_{b}-\eta_{a}\right)\cos\theta\right] \tag{34}$$

where

$$A=\frac{1-\gamma\sigma^{3}}{1-\sigma^{3}},\ \ B=\frac{(\gamma-1)\sigma^{3}}{1-\sigma^{3}}$$

Stress components are obtained as follows

$$S_{rr}=-P+\frac{9\mu_{c}B^{2}\Omega_{a}^{\ 2}\sin^{2}\theta}{R^{6}} \tag{35}$$

$$S_{\theta\theta} = -p + \alpha_6 \left(\frac{\eta_b - \eta_a}{a} \right)^2 \left(\frac{\sigma}{\sigma - 1} \right)^2 \frac{1}{R^4} \tag{36}$$

$$S_{\phi\phi} = -P + \frac{9\mu_c B^2 \Omega_a{}^2 \sin^2\theta}{R^6} + \alpha_6 \left(\frac{\sigma}{\sigma - 1} \right)^2 \left(\frac{\eta_b - \eta_a}{a} \right)^2 \frac{1}{R^4} \tag{37}$$

$$S_{r\theta} = \frac{3\alpha_8 (\eta_b - \eta_a) \Omega_a B \sigma \sin\theta}{a R^5 (\sigma - 1)} \tag{38}$$

$$S_{\theta\phi} = 0 \tag{39}$$

$$S_{r\phi} = \frac{-3\mu B \Omega_a \sin\theta}{R^3} \tag{40}$$

The force component in the transverse direction

$$\rho F_\theta = -\frac{3}{2} \rho a \Omega_a{}^2 \sin 2\theta \left(A^2 R - \frac{2AB}{R^2} \right) + \frac{39}{2} \frac{\mu_c B^2 \Omega_a{}^2 \sin 2\theta}{a R^7} + \frac{42}{5} \frac{\alpha_8 B \Omega_a (\eta_b - \eta_a) \sigma \sin\theta}{a^2 (\sigma - 1) R^6} \tag{41}$$

The drag force on the inner sphere

$$= \frac{2\pi}{\rho} \alpha_3{}^2 \int_0^\pi r^2 \left\{ S_{rr} |_{R=1} \cos\theta \sin\theta - S_{r\theta} |_{R=1} \sin^2\theta \right\} d\theta$$

$$= \frac{-128\pi}{5} \mu^2 \frac{\alpha_8 B (\Omega_a b (\eta_b - \eta_a)}{\rho (\sigma - 1)} \tag{42}$$

The drag force on the outer sphere

$$= \frac{2\pi}{\rho} \alpha_3{}^2 \int_0^\pi r^2 \left\{ S_{rr} |_{R=\sigma} \sin\theta \cos\theta - S_{r\theta} |_{R=\sigma} \sin^2\theta \right\} d\theta$$

$$= \frac{-128\pi}{5} \mu^2 \frac{a \alpha_8 B \Omega_a (\eta_b - \eta_a)}{(\sigma - 1) \rho \sigma^2} \tag{43}$$

The viscous couple on the inner sphere

$$= \int_0^\pi 2\pi r^3 S_{r\phi} |_{R=1} \sin^2\theta d\theta$$

$$= \frac{-8\pi\mu a^3 \Omega_a \sigma^3 (\gamma - 1)}{(1 - \sigma^3)} \tag{44}$$

The viscous couple on the outer sphere

$$= \int_0^{\pi} 2\pi r^3 S_{r\phi} |_{R=\sigma} \quad \sin^2 \theta d\theta$$

$$= \frac{-8\pi\mu a^3 \Omega_a \sigma^3 (\gamma - 1)}{(1-\sigma^3)} \quad (45)$$

= the viscous couple on the inner sphere.

Nussult number on the inner sphere

$$= \frac{\partial \eta}{\partial r} |_{r=a}$$

$$= \frac{(\eta_b - \eta_a)}{a}\left(\frac{\sigma}{\sigma - 1}\right) \quad (46)$$

Nussult number on the outer sphere

$$= \frac{\partial \eta}{\partial r} |_{r=b}$$

$$= \frac{(\eta_b - \eta_a)}{b(\sigma - 1)} \quad (47)$$

The dissipation of energy is given by $\sum d_{ij} S_{ij}$

$$= \frac{9B^2 \mu \Omega_a{}^2 \sin^2 \theta}{R^6} \quad (48)$$

References

[1] Koh, S.L., and On the foundations of Eringen, A.C non-linear thermo viscoelasticity, Int.J.Engg.Sci1, pp 199 – 229 (1963)

[2] Nageswara Rao.P and Steady flow of a Pattabhi Ramacharyulu.N.Ch thermoviscous fluid over an infinite plate. (1979) Proc.Ind.Acad.Sci., Vol 88 A, Part III, No.2, P-157 -167

[3] Nageswara Rao and Steady flow of a Pattabhiramacharyulu thermoviscous fluids through straight tubes J.I.I Sc., Vol.61, No.6 pp 89-102 (1979)

[4] Nageswara Rao and Steady slow motion Pattabhiramacharyulu through a circular tube Ind.J.P and APP.maths. (1980)

[5] Nageswara Rao .P some problems in thermo-viscous fluid dynamics. Ph.D.thesis submitted to K.U., Warangarl (1980)