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On Soft Preopen Sets in Soft Topological Spaces

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Abstract

This paper aims at introducing a soft topology via soft preopen sets. We define soft preopen sets and prove some of its properties. Also we state and prove the condition for collection of soft preopen sets to be a soft topology.

Keywords: soft preopen sets, soft preclosed sets, soft dense sets, soft preclosure, soft preinterior, soft point, soft submaximal.

Introduction

Soft set was introduced by Molodtsov [14] in the year 1999. Soft topology was introduced by Muhammad Shabir et al [15] in 2011. Now many researchers are working on various properties and types of soft topological spaces. Cagman et al [16] introduced soft limit points, soft Hausdroff space etc,. Won Keun [20] investigated soft regular spaces and some properties of them. Sabir Hussain et al [18] defined and discussed properties of soft interior, soft exterior and soft boundary. Samanta et al [8] introduced mappings in soft topological spaces. Mahanta et al [10] made a study of soft topology via soft semi-open sets, which is a motivation and base for this paper. Kannan [9] studied on soft g closed sets. Zorlutuna et al [22] provided the relationship between soft topology and fuzzy topology. Pazzar et al [4] defined L_ fuzzy soft topology. Nazmul et al [17] studied properties soft neighbourhood systems. Arokia Lancy et al [3] introduced soft $g\beta$ and soft $gs\beta$ closed sets. Andrijevic [1,2,5,6] and Mashhour [11,12] gave many results on pre open sets in general topology. This paper aims at developing a soft topology via soft preopen sets.

Preliminaries

Definition 1 ([14]). Let U be an initial universal set and E be the set of parameters. Let $P(U)$ denote the power set of U and let $A \subseteq E$. A pair (F,A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

The collection of soft sets (F,A) over a universe U and the parameter set A is a family of soft sets denoted by $SS(U)_A$.

Definition 2 ([15]). Let τ be a collection of soft sets over a universe U with a fixed set A of parameters, then $\tau \subseteq SS(U)_A$ is called a soft topology on U with a fixed set A if,
(i) ϕ_A, U_A belong to τ.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (U,A,τ) is called a soft topological space over U. The members of τ are called soft open sets in U and complements of them are called soft closed sets in U.

Soft operations are denoted by usual set theoretical operations with '~' symbol above. Soft interior and soft closure are denoted by $\widetilde{s}int$ and $\widetilde{s}cl$ respectively.

Theorem 1 ([15]). Arbitrary union of soft open sets is soft open and finite intersection of soft closed sets is soft closed.

Definition 3 ([22]). Let (U,A,τ) be a soft topological space and let (G,A) be a soft set. Then
(i) The soft closure of (G,A) is the soft set
$\widetilde{s}cl(G,A) = \widetilde{\cap}\{(S,A): (S,A)$ is soft closed and $(G,A) \widetilde{\subseteq} (S,A)\}$

(ii) The soft interior of (G,A) is the soft set
$\widetilde{s}int(G,A) = \widetilde{\cup}\{(S,A): (S,A)$ is soft open and $(S,A) \widetilde{\subseteq} (G,A)\}$

Remark 1 ([22]). $\widetilde{s}cl(G,A)$ is the smallest soft closed set containing (G,A) and $\widetilde{s}int(G,A)$ is the largest soft open set contained in (G,A).

Theorem 2 ([22]). *Let* (U,τ,A) *be a soft topological space and let* (F,A) *and* (G,A) *be soft sets over* U. *Then,*
(i) (F,A) *is soft closed if and only if* $(F,A) = \widetilde{s}cl(F,A)$
(ii) (G,A) *is soft open if and only if* $(G,A) = \widetilde{s}int(G,A)$

Remark 2 ([22]). If $\{(G,A)_\alpha \mid \alpha \in I\}$ is a collection of soft sets, then
$\widetilde{\cup}\, \widetilde{s}int(G,A)_\alpha \ \widetilde{\subseteq}\ \widetilde{s}int(\widetilde{\cup}\,(G,A)_\alpha)$
$\widetilde{\cup}\, \widetilde{s}cl(G,A)_\alpha \ \widetilde{\subseteq}\ \widetilde{s}cl(\widetilde{\cup}\,(G,A)_\alpha)$

Remark 3 ([7]). If (F,A) and (K,A) are any two soft sets in (U,A,τ) then
$U_A - ((F,A)\,\widetilde{\cap}\,(K,A)) = (U_A - (F,A))\,\widetilde{\cup}\,(U_A - (K,A))$.

Theorem 3 ([6]).
(i) For every soft open set (G, A) in a soft topological space (U, τ, A) and every soft set (K, A) we have $\tilde{s}cl(K, A) \tilde{\cap} (G, A) \tilde{\subseteq} \tilde{s}cl((K, A) \tilde{\cap} (G, A))$
(ii) For every soft closed set (F, A) in a soft topological space (U, τ, A) and every soft set (K, A) we have $\tilde{s}int((K, A) \tilde{\cup} (F, A)) \tilde{\subseteq} \tilde{s}int(K, A) \tilde{\cup} (F, A)$

Soft preopen sets
Definition 4. In a soft topological space (U, A, τ), a soft set
(i) (G, A) is said to be soft preopen set if $(G, A) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(G, A))$
(ii) (F, A) is said to be soft preclosed set if $(F, A) \tilde{\supseteq} \tilde{s}cl(\tilde{s}int(F, A))$

A soft preclosed set is nothing but the complement of a soft preopen set.

Definition 5. A subset (D, A) of a soft topological space (U, A, τ) is said to be soft dense if $\tilde{s}cl(D, A) = U_A$.

The collection of all soft dense sets in a soft topological space (U, A, τ) is denoted as $D(U, A, \tau)$.

Definition 6. (U, A, τ) is said to be soft submaximal if each soft dense subset is soft open.

Definition 7. In a soft topological space (U, A, τ), a soft set (G, A) is said to be soft regular open if $(G, A) = \tilde{s}int(\tilde{s}cl(G, A))$.

Remark 4 . $\tilde{s}int(\tilde{s}cl(K, A))$ is always soft regular open for any soft set (K, A).

Example 1. Let $U = \{a, b\}, A = \{e_1, e_2\}$ Define

$(F, A)_1 = \{(e_1, \phi), (e_2, \phi)\}$, $(F, A)_2 = \{(e_1, \phi), (e_2, \{a\})\}$,
$(F, A)_3 = \{(e_1, \phi), (e_2, \{b\})\}$, $(F, A)_4 = \{(e_1, \phi), (e_2, \{a, b\})\}$,
$(F, A)_5 = \{(e_1, \{a\}), (e_2, \phi)\}$, $(F, A)_6 = \{(e_1, \{a\}), (e_2, \{a\})\}$,
$(F, A)_7 = \{(e_1, \{a\}), (e_2, \{b\})\}$, $(F, A)_8 = \{(e_1, \{a\}), (e_2, \{a, b\})\}$,
$(F, A)_9 = \{(e_1, \{b\}), (e_2, \phi)\}$, $(F, A)_{10} = \{(e_1, \{b\}), (e_2, \{a\})\}$,
$(F, A)_{11} = \{(e_1, \{b\}), (e_2, \{b\})\}$, $(F, A)_{12} = \{(e_1, \{b\}), (e_2, \{a, b\})\}$,
$(F, A)_{13} = \{(e_1, \{a, b\}), (e_2, \phi)\}$, $(F, A)_{14} = \{(e_1, \{a, b\}), (e_2, \{a\})\}$,
$(F, A)_{15} = \{(e_1, \{a, b\}), (e_2, \{b\})\}$, $(F, A)_{16} = \{(e_1, \{a, b\}), (e_2, \{a, b\})\}$

are all soft sets on universal set U under the parameter set A.

$\tau = \{(F, A)_1, (F, A)_5, (F, A)_7, (F, A)_8, (F, A)_{16}\}$ is a soft topology over U.

Soft open sets are $(F, A)_1, (F, A)_5, (F, A)_7, (F, A)_8, (F, A)_{16}$

Soft closed sets are $(F, A)_1, (F, A)_9, (F, A)_{10}, (F, A)_{12}, (F, A)_{16}$

Soft preopen sets are $(F, A)_1, (F, A)_5, (F, A)_6, (F, A)_7, (F, A)_8, (F, A)_{13}, (F, A)_{14},$

$(F,A)_{15}, (F,A)_{16}$

Soft preclosed sets are $(F,A)_1, (F,A)_2, (F,A)_3, (F,A)_4, (F,A)_9, (F,A)_{10}, (F,A)_{11}, (F,A)_{12}, (F,A)_{16}$

Soft dense sets are $(F,A)_5, (F,A)_6, (F,A)_7, (F,A)_8, (F,A)_{13}, (F,A)_{14}, (F,A)_{15}, (F,A)_{16}$.

Remark 5. The Cardinality of $SS(U)_A$ is given by $n(SS(U)_A) = 2^{n(U).n(A)}$.

Example 2. In Example 1, $n(SS(U)_A) = 2^{3\times 2} = 2^4 = 16$.

And, if $U = \{a,b,c\}$, $A = \{e_1, e_2\}$ then $n(SS(U)_A) = 2^{3\times 2} = 2^6 = 64$.

Also if $U = \{a,b\}$, $A = \{e_1, e_2, e_3\}$ then $n(SS(U)_A) = 2^{2\times 3} = 2^6 = 64$.

Remark 6. (ϕ, A) and (U, A) are always soft preopen and soft preclosed set.

Remark 7. Every soft open (soft closed) set is a soft preopen (soft preclosed) set.

Converse of this Remark need not be true.

Example 3. In Example 2 $(F,A)_6$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$ are soft preopen but not soft open.

Example 4. Let $U = \{a,b,c\}$, $A = \{e_1, e_2\}$.

Let F, G, H, I, J, K, L be the mappings from A to $P(U)$ defined by,

$(F,A) = \{(e_1,\{a,b\}),(e_2,\{a,b\})\}$, $(G,A) = \{(e_1,\{b\}),(e_2,\{a,c\})\}$

$(H,A) = \{(e_1,\{b,c\}),(e_2,\{a\})\}$, $(I,A) = \{(e_1,\{b\}),(e_2,\{a\})\}$

$(J,A) = \{(e_1,\{a,b\}),(e_2,\{a,b,c\})\}$, $(K,A) = \{(e_1,\{a,b,c\}),(e_2,\{a,b\})\}$

$(L,A) = \{(e_1,\{b,c\}),(e_2,\{a,c\})\}$ are soft sets over U and consider

$\tau = \{\tilde{\phi}, \tilde{U}, (F,A),(G,A),(H,A),(I,A),(J,A),(K,A),(L,A)\}$ a soft topology over U.

Then $(M,A) = \{(e_1,\{a,b,c\}),(e_2,\{a,c\})\}$ is soft preopen set but not soft open.

Theorem 4. *Arbitrary union of soft preopen sets is a soft preopen set.*

Proof. Let $\{(G,A)_\alpha \mid \alpha \in I\}$ be a collection of soft preopen sets of a soft topological space (U, A, τ). Then, for each α, $(G,A)_\alpha \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(G,A)_\alpha)$

Claim: $\tilde{\cup}\,(G,A)_\alpha \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(\tilde{\cup}\,(G,A)_\alpha))$

By definition, $\tilde{\cup}\,(G,A)_\alpha \tilde{\subseteq} \tilde{\cup}\,\tilde{s}int(\tilde{s}cl(G,A)_\alpha)$

$\tilde{\subseteq} \tilde{s}int(\tilde{\cup}\,\tilde{s}cl(G,A)_\alpha)$ (by Remark 2 (i))

$\tilde{\subseteq} \tilde{s}int(\tilde{s}cl(\tilde{\cup}\,(G,A)_\alpha))$ (by Remark 2 (ii))

Therefore, $\tilde{\cup}\,(G,A)_\alpha \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(\tilde{\cup}\,(G,A)_\alpha))$

Remark 8. Arbitrary intersection of soft preclosed sets is soft preclosed.

Remark 9. Finite intersection of soft preopen sets need not be a soft preopen set.

Example 5. In Example 4, $(N, A) = \{(e_1, \{b\}), (e_2, \{b, c\})\}$ and $(O, A) = \{(e_1, \{a, c\}), (e_2, \{a, c\})\}$ are soft preopen sets.

But, $(N, A) \tilde{\cap} (O, A) = \{(e_1, \phi), (e_2, \{c\})\}$ is not soft preopen.

Theorem 5. *If* (G, A) *is soft preopen set such that* $(H, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}cl(H, A)$, *then* (H, A) *is also a soft preopen set.*

Proof. $(H, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}cl(G, A) \tilde{\subseteq} \tilde{s}cl(H, A)$ and (G, A) is soft preopen, $(G, A) \tilde{\subseteq} \tilde{s}int\, \tilde{s}cl(G, A)$.

$\tilde{s}cl(G, A) \tilde{\subseteq} \tilde{s}cl(H, A) \Longrightarrow \tilde{s}int\, \tilde{s}cl(G, A) \tilde{\subseteq} \tilde{s}int\, \tilde{s}cl(H, A)$ and $(H, A) \tilde{\subseteq} (G, A) \tilde{\subseteq} \tilde{s}int\, \tilde{s}cl(G, A) \tilde{\subseteq} \tilde{s}int\, \tilde{s}cl(H, A)$

$\Longrightarrow (H, A) \tilde{\subseteq} \tilde{s}int\, \tilde{s}cl(H, A) \Longrightarrow (H, A)$ is a soft preopen set.

We shall denote the family of all soft preopen sets (soft preclosed sets) of a soft topological space (U, A, τ) by $POSS(U)_A$ $(PCSS(U)_A)$.

Definition 8. Let (U, A, τ) be a soft topological space and (G, A) be a soft set over U.

(i) The soft preclosure of (G, A) is a soft set,

$\tilde{s}pcl(G, A) = \tilde{\cap} \{(S, A) : (G, A) \tilde{\subseteq} (S, A)$ and $(S, A) \in PCSS(U)_A\}$

(ii) The soft preinterior of (G, A) is a soft set,

$\tilde{s}pint(G, A) = \tilde{\cup} \{(S, A) : (S, A) \tilde{\subseteq} (G, A)$ and $(S, A) \in POSS(U)_A\}$

Note that, $\tilde{s}pcl(G, A)$ is the smallest soft preclosed set containing (G, A) and $\tilde{s}pint(G, A)$ is the largest soft preopen set contained in (G, A).

Theorem 6. *Let* (U, A, τ) *be a soft topological space and* (G, A) *be a soft set over* U. *Then,*

(i) $(G, A) \in PCSS(U)_A$ *if and only if* $(G, A) = \tilde{s}pcl(G, A)$

(ii) $(G, A) \in POSS(U)_A$ *if and only if* $(G, A) = \tilde{s}pint(G, A)$

(iii) $\tilde{s}pcl(\phi, A) = (\phi, A)$ *and* $\tilde{s}pcl(U, A) = (U, A)$

(vi) $\tilde{s}pint(\phi, A) = (\phi, A)$ *and* $\tilde{s}pint(U, A) = (U, A)$

(v) $\tilde{s}pcl(\tilde{s}pcl(G, A)) = \tilde{s}pcl(G, A)$

(vi) $\tilde{s}pint(\tilde{s}pint(G, A)) = \tilde{s}pint(G, A)$

(vii) $(\tilde{s}pcl(G, A))^c = \tilde{s}pint(G^c, A)$

(viii) $(\tilde{s}pint(G, A))^c = \tilde{s}pcl(G^c, A)$

Proof. Let (G, A) be a soft set over U.

(i) Let (G, A) be a soft preclosed set.

Then it is the smallest soft preclosed set containing itself, by Remark 1, and hence $(G, A) = \tilde{s}pcl(G, A)$.

Conversely, let $(G, A) = \tilde{s}pcl(G, A)$ $\tilde{s}pcl(G, A)$ being the intersection of soft preclosed sets is soft preclosed so $\tilde{s}pcl(G, A) \in PCSS(U)_A$ implies that $(G, A) \in$

$PCSS(U)_A$

(ii) Let (G,A) be a soft preopen set.

Then it is the largest soft preopen set contained in (G,A), by Remark 1, and hence $(G,A) = \tilde{s}pint(G,A)$

Conversely, let $(G,A) = \tilde{s}pint(G,A)$ $\tilde{s}pint(G,A)$ being the union of soft preopen sets is soft preopen so $\tilde{s}pint(G,A) \in POSS(U)_A$ implies $(G,A) \in POSS(U)_A$

(iii) Since (ϕ,A) and (U,A) are soft preclosed sets, so by (i)

$\tilde{s}pcl(\phi,A) = (\phi,A)$ and $\tilde{s}pcl(U,A) = (U,A)$

(iv) Similar to (iii).

(v) Since $\tilde{s}pcl(G,A) \in PCSS(U)_A$

By (i), $(G,A) \in PCSS(U)_A$ iff $(G,A) = \tilde{s}pcl(G,A)$

Therefore, $\tilde{s}pcl(\tilde{s}pcl(G,A)) = \tilde{s}pcl(G,A)$

(vi) Similar to (v).

(vii)
$$\begin{aligned}(\tilde{s}pcl(G,A))^c &= (\tilde{\cap}\{(S,A) : (G,A) \tilde{\subseteq} (S,A) \;\&\; (S,A) \in PCSS(U)_A\})^c \\ &= \tilde{\cup}\{(S,A)^c : (S,A)^c \tilde{\subseteq} (G,A)^c \;\&\; (S,A)^c \in POSS(U)_A\} \\ &= \tilde{\cup}\{(S^c,A) : (S^c,A) \tilde{\subseteq} (G^c,A) \;\&\; (S^c,A) \in POSS(U)_A\} \\ &= \tilde{s}pint\,(G^c,A)\end{aligned}$$

(viii)
$$\begin{aligned}(\tilde{s}pint(G,A))^c &= (\tilde{\cup}\{(S,A) : (S,A) \tilde{\subseteq} (G,A) \;\&\; (S,A) \in POSS(U)_A\})^c \\ &= \tilde{\cap}\{(S,A)^c : (G,A)^c \tilde{\subseteq} (S,A)^c \;\&\; (S,A)^c \in PCSS(U)_A\} \\ &= \tilde{\cap}\{(S^c,A) : (G^c,A) \tilde{\subseteq} (S^c,A) \;\&\; (S^c,A) \in PCSS(U)_A\} \\ &= \tilde{s}pcl\,(G^c,A)\end{aligned}$$

Theorem 7. *Let* (U,A,τ) *be a soft topological space and* (G,A) *and* (K,A) *be two soft sets over* U*. Then,*

(i) $(G,A) \tilde{\subseteq} (K,A) \Rightarrow \tilde{s}pint(G,A) \tilde{\subseteq} \tilde{s}pint(K,A)$

(ii) $(G,A) \tilde{\subseteq} (K,A) \Rightarrow \tilde{s}pcl(G,A) \tilde{\subseteq} \tilde{s}pcl(K,A)$

(iii) $\tilde{s}pcl((G,A) \tilde{\cup} (K,A)) = \tilde{s}pcl(G,A) \tilde{\cup} \tilde{s}pcl(K,A)$

(iv) $\tilde{s}pint((G,A) \tilde{\cap} (K,A)) = \tilde{s}pint(G,A) \tilde{\cap} \tilde{s}pint(K,A)$

(v) $\tilde{s}pcl((G,A) \tilde{\cap} (K,A)) \tilde{\subset} \tilde{s}pcl(G,A) \tilde{\cap} \tilde{s}pcl(K,A)$

(vi) $\tilde{s}pint((G,A) \tilde{\cup} (K,A)) \tilde{\supset} \tilde{s}pint(G,A) \tilde{\cup} \tilde{s}pint(K,A)$

Proof. *(i)* By definition,

$\tilde{s}pint(G,A) = \tilde{\cup}\{(S,A) : (S,A) \tilde{\subseteq} (G,A) \;\&\; (S,A) \in POSS(U)_A\}$

& $\tilde{s}pint(K,A) = \tilde{\cup}\{(T,A) : (T,A) \tilde{\subseteq} (K,A) \;\&\; (T,A) \in POSS(U)_A\}$

Now, $\tilde{s}pint(G,A) \tilde{\subseteq} (G,A) \tilde{\subseteq} (K,A) \Rightarrow \tilde{s}pint(G,A) \tilde{\subseteq} (K,A)$

Since $\tilde{s}pint(K,A)$ is the largest soft preopen set contained in (K,A)

Therefore, $\tilde{s}pint(G,A) \tilde{\subseteq} \tilde{s}pint(K,A)$

(ii) By definition,

$\widetilde{s}pcl(G,A) = \widetilde{\cap}\{(S,A) : (G,A) \widetilde{\subseteq} (S,A) \,\&\, (S,A) \in PCSS(U)_A\}$

& $\widetilde{s}pint(K,A) = \widetilde{\cap}\{(T,A) : (K,A) \widetilde{\subseteq} (T,A) \,\&\, (T,A) \in PCSS(U)_A\}$

since $(G,A) \widetilde{\subseteq} \widetilde{s}pcl(G,A)$ and $(K,A) \widetilde{\subseteq} \widetilde{s}pcl(K,A)$

$\Rightarrow (G,A) \widetilde{\subseteq} (K,A) \widetilde{\subseteq} \widetilde{s}pcl(K,A) \Rightarrow (G,A) \widetilde{\subseteq} \widetilde{s}pcl(K,A)$

But $\widetilde{s}pcl(G,A)$ is the smallest soft preclosed set containing (G,A)

Therefore, $\widetilde{s}pcl(G,A) \widetilde{\subseteq} \widetilde{s}pcl(K,A)$.

(iii) We have, $(G,A) \widetilde{\subset} (G,A) \widetilde{\cup} (K,A)$ and $(K,A) \widetilde{\subset} (G,A) \widetilde{\cup} (K,A)$

By (ii), $(G,A) \widetilde{\subseteq} (K,A) \Rightarrow \widetilde{s}pcl(G,A) \widetilde{\subseteq} \widetilde{s}pcl(K,A)$,

$\widetilde{s}pcl(G,A) \widetilde{\subset} \widetilde{s}pcl((G,A) \widetilde{\cup} (K,A))$ and

$\widetilde{s}pcl(K,A) \widetilde{\subset} \widetilde{s}pcl\big((G,A) \widetilde{\cup} (K,A)\big)$

$\Rightarrow \widetilde{s}pcl(G,A) \widetilde{\cup} \widetilde{s}pcl(K,A) \widetilde{\subset} \widetilde{s}pcl((G,A) \widetilde{\cup} (K,A))$ . . . (1)

Now, $\widetilde{s}pcl(G,A), \widetilde{s}pcl(K,A) \in PCSS(U)_A$

$\Rightarrow \widetilde{s}pcl(G,A) \widetilde{\cup} \widetilde{s}pcl(K,A) \in PCSS(U)_A$

Then $(G,A) \widetilde{\subset} \widetilde{s}pcl(G,A)$ and $(K,A) \widetilde{\subset} \widetilde{s}pcl(K,A)$

$\Rightarrow (G,A) \widetilde{\cup} (K,A) \widetilde{\subset} \widetilde{s}pcl(G,A) \widetilde{\cup} \widetilde{s}pcl(K,A)$

i.e., $\widetilde{s}pcl(G,A) \widetilde{\cup} \widetilde{s}pcl(K,A)$ is a soft preclosed set containing $(G,A) \widetilde{\cup} (K,A)$. But, $\widetilde{s}pcl((G,A) \widetilde{\cup} (K,A))$ is the smallest soft preclosed set containing $(G,A) \widetilde{\cup} (K,A)$.

Hence $\widetilde{s}pcl\big((G,A) \widetilde{\cup} (K,A)\big) \widetilde{\subset} \widetilde{s}pcl(G,A) \widetilde{\cup} \widetilde{s}pcl(K,A)$ . . . (2)

From (1) & (2), $\widetilde{s}pcl((G,A) \widetilde{\cup} (K,A)) = \widetilde{s}pcl(G,A) \widetilde{\cup} \widetilde{s}pcl(K,A)$.

(iv) We have, $(G,A) \widetilde{\cap} (K,A) \widetilde{\subset} (G,A)$ and $(G,A) \widetilde{\cap} (K,A) \widetilde{\subset} (K,A)$

By (i), $(G,A) \widetilde{\subseteq} (K,A) \Rightarrow \widetilde{s}pint(G,A) \widetilde{\subseteq} \widetilde{s}pint(K,A)$,

$\widetilde{s}pint((G,A) \widetilde{\cap} (K,A)) \widetilde{\subset} \widetilde{s}pint(G,A)$ and

$\widetilde{s}pint\big((G,A) \widetilde{\cap} (K,A)\big) \widetilde{\subset} \widetilde{s}pint(K,A)$

$\Rightarrow \widetilde{s}pint((G,A) \widetilde{\cap} (K,A)) \widetilde{\subset} \widetilde{s}pint(G,A) \widetilde{\cap} \widetilde{s}pint(K,A)$ . . . (3)

Now, $\widetilde{s}pint(G,A), \widetilde{s}pint(K,A) \in POSS(U)_A$

$\Rightarrow \widetilde{s}pint(G,A) \widetilde{\cap} \widetilde{s}pint(K,A) \in POSS(U)_A$

Then $\widetilde{s}pint(G,A) \widetilde{\subset} (G,A)$ and $\widetilde{s}pint(K,A) \widetilde{\subset} (K,A)$

$\Rightarrow \widetilde{s}pint(G,A) \widetilde{\cap} \widetilde{s}pint(K,A) \widetilde{\subset} (G,A) \widetilde{\cap} (K,A)$

i.e., $\widetilde{s}pint(G,A) \widetilde{\cap} \widetilde{s}pint(K,A)$ is a soft preopen set contained in $(G,A) \widetilde{\cap} (K,A)$. But, $\widetilde{s}pint((G,A) \widetilde{\cap} (K,A))$ is the largest soft preopen set contained in $(G,A) \widetilde{\cap} (K,A)$.

Hence $\widetilde{s}pint(G,A) \widetilde{\cap} \widetilde{s}pint(K,A) \widetilde{\subset} \widetilde{s}pint\big((G,A) \widetilde{\cap} (K,A)\big)$ . . . (4)

From (3) & (4),

$\widetilde{s}pint((G,A) \widetilde{\cap} (K,A)) = \widetilde{s}pint(G,A) \widetilde{\cap} \widetilde{s}pint(K,A)$.

(v) We have, $(G,A) \widetilde{\cap} (K,A) \widetilde{\subset} (G,A)$ and $(G,A) \widetilde{\cap} (K,A) \widetilde{\subset} (K,A)$

$\Rightarrow \tilde{s}pcl((G,A)\ \tilde{\cap}\ (K,A)) \tilde{\subset} \tilde{s}pcl(G,A)$ and
$\tilde{s}pcl((G,A)\ \tilde{\cap}\ (K,A)) \tilde{\subset} \tilde{s}pcl(K,A)$
$\Rightarrow\ \tilde{s}pcl((G,A)\ \tilde{\cap}\ (K,A)) \tilde{\subset}\ \tilde{s}pcl(G,A)\ \tilde{\cap}\ \tilde{s}pcl(K,A)$

(vi) Similar to (v)

Theorem 8. Let (K,A) be a soft subset of a soft topological space (U,τ,A). Then (i) $\tilde{s}pcl(K,A) = (K,A)\ \tilde{\cup}\ \tilde{s}cl(\tilde{s}int(K,A))$; (ii) $\tilde{s}pint(K,A) = (K,A)\ \tilde{\cap}\ \tilde{s}int(\tilde{s}cl(K,A))$

Proof. (i) $\tilde{s}cl(\tilde{s}int((K,A)\ \tilde{\cup}\ \tilde{s}cl(\tilde{s}int(K,A)))) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(K,A)\ \tilde{\cup}\ \tilde{s}cl(\tilde{s}int(K,A))) = \tilde{s}cl(\tilde{s}int(K,A)) \tilde{\subseteq} (K,A)\ \tilde{\cup}\ \tilde{s}cl(\tilde{s}int(K,A))$ by Theorem 3. Hence $(K,A)\ \tilde{\cup}\ \tilde{s}cl(\tilde{s}int(K,A))$ is soft preclosed and thus $\tilde{s}pcl(K,A) \tilde{\subseteq} (K,A)\ \tilde{\cup}\ \tilde{s}cl(\tilde{s}int(K,A))$. On the other hand, since $\tilde{s}pcl(K,A)$ is soft preclosed, we have $\tilde{s}cl(\tilde{s}int(K,A)) \tilde{\subseteq} \tilde{s}cl(\tilde{s}int(\tilde{s}pcl(K,A)) \tilde{\subseteq} \tilde{s}pcl(K,A)$ and hence $(K,A)\ \tilde{\cup}\ \tilde{s}cl(\tilde{s}int(K,A)) \tilde{\subseteq} \tilde{s}pcl(K,A)$.

(ii) is a consequence of (i).

Theorem 9. *Let* (U,A,τ) *be a soft topological space. Then for any* $(F,A) \tilde{\subseteq} U_A$, $\tilde{s}pint(F,A) = U_A - \tilde{s}pcl(\tilde{U} - (F,A))$.

Proof. $U_A - \tilde{s}pint(F,A) = U_A - ((F,A)\ \tilde{\cap}\ \tilde{s}int(\tilde{s}cl(F,A))) = (U_A - (F,A))\ \tilde{\cup}\ (U_A - \tilde{s}int(\tilde{s}cl(F,A))) = (U_A - (F,A))\ \tilde{\cup}\ \tilde{s}cl(U_A - \tilde{s}cl(F,A)) =$
$(U_A - (F,A))\ \tilde{\cup}\ \tilde{s}cl\ \tilde{s}int(U_A - (F,A)) = \tilde{s}pcl(U_A - (F,A))$, by Remark 3 and Theorem 8. Therefore, $\tilde{s}pint(F,A) = U_A - \tilde{s}pcl(U_A - (F,A))$.

Definition 9 ([22]). The soft set $(F,A) \in SS(U)_A$ is called a soft point in U_A, denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e') = \phi$ for all $e' \in A - \{e\}$.

The soft point e_F is said to be in the soft set (G,A), denoted by $e_F \tilde{\in} (G,A)$, if for the element $e \in A$ and $F(e) \subseteq G(e)$.

Theorem 10. *Let* $(D,A) \tilde{\in} D(U,A,\tau)$, *with* $\tilde{s}int(D,A) \neq \phi_A$. *Then* $\tilde{s}pcl(D,A) = U_A$.

Proof. If possible suppose that, there is an $e_F \tilde{\in} U_A$ but $e_F \tilde{\notin} \tilde{s}pcl(D,A)$. So $e_F \tilde{\in} (U_A - \tilde{s}pcl(D,A))$, implies that $e_F \tilde{\in} \tilde{s}pint(U_A - (D,A)) \tilde{\subset} \tilde{s}cl(U_A - (D,A))$ [by Theorem 9]. So $e_F \tilde{\in} (U_A - \tilde{s}int(D,A))$. Hence $e_F\ \tilde{\notin}\ \tilde{s}int(D,A)$. So there is no soft open set $(H,A)_{e_F}$ (containing e_F) such that $(H,A)_{e_F} \tilde{\subset} (D,A)$ and so $(H,A)_{e_F}\ \tilde{\cap}\ (D,A) = \phi_A$, a contradiction that (D,A) is a soft dense set. Hence $\tilde{s}pcl(D,A) = U_A$.

Theorem 11. If (O,A) is soft open and (G,A) is soft preopen, then $(G,A)\ \tilde{\cap}\ (O,A)$ is soft preopen.

Proof. $(G,A) \tilde{\cap} (O,A) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(G,A)) \tilde{\cap} (O,A) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(G,A) \tilde{\cap} (O,A))$ $\tilde{\subseteq} \tilde{s}int(\tilde{s}cl((G,A) \tilde{\cap} (O,A)))$ by Theorem 3.

Theorem 12. *Every soft dense set is a soft preopen set.*

Proof. Proof is obvious.

Theorem 13. *Let* (U,A,τ) *be a soft topological space and* $(D,A) \tilde{\subset} \tilde{U}$. If $(D,A)\tilde{\cap}(G,A) \neq \phi_A$ *for every* $(G,A) \in \tau$, *then* (D,A) *is dense in* (U,A,τ).

Proof. Since $(D,A)\tilde{\cap}(G,A) \neq \phi_A$, $(D,A) \tilde{\nsubseteq} (G,A)^c$ for any (G,A) which intersects (D,A). $\tilde{s}cl(D,A) = \tilde{U}$ therefore, (D,A) is soft dense in (U,A,τ).

Theorem 14. *Let* (U,A,τ) *be a soft topological space and* $(D,A) \tilde{\subset} U_A$. *If* $(D,A)\tilde{\cap}(G,A) \neq \phi_A$ *for every* $(G,A) \in POSS(U)_A$, *then* (D,A) *is dense in* (U,A,τ).

Proof. Since $\tau \tilde{\subset} POSS(U)_A$ and collection soft closed sets is a subset of $PCSS(U)_A$, the theorem holds.

Theorem 15. *For any subset of a soft topological space* (U,A,τ) *the following are equivalent:* $(F,A) \in POSS(U)_A$

There is a soft regular open set $(G,A) \tilde{\subseteq} U_A$ *such that* $(F,A) \tilde{\subseteq} (G,A)$ *and* $\tilde{s}cl(F,A) = \tilde{s}cl(G,A)$. (F,A) *is the intersection of a soft regular open set and a soft dense set.* (F,A) *is the intersection of a soft open set and a soft dense set.*

Proof. (i) $\Longrightarrow$ (ii): Let $(F,A) \in POSS(U)_A$, i.e. $(F,A) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(F,A))$. Let $(G,A) = \tilde{s}int(\tilde{s}cl(F,A))$. Then (G,A) is soft regular open with $(F,A) \tilde{\subseteq} (G,A)$ and $\tilde{s}cl(F,A) = \tilde{s}cl(G,A)$ by Remark 4 and $\tilde{s}cl(G,A) = \tilde{s}cl(\tilde{s}int(\tilde{s}cl(F,A))) = \tilde{s}cl(F,A)$.
(ii) $\Longrightarrow$ (iii): Suppose (ii) holds. Let $(D,A) = (F,A) \tilde{\cup} (U_A - (G,A))$. Then $\tilde{s}cl(D,A) = \tilde{s}cl((F,A) \tilde{\cup} (G,A)^c)) = \tilde{s}cl(F,A) \tilde{\cup} \tilde{s}cl(G,A)^c) =$
$\tilde{s}cl(G,A) \tilde{\cup} \tilde{s}cl(G,A)^c) = U_A$. Hence (D,A) is soft dense and $(F,A) = (G,A) \tilde{\cap} (D,A)$.
(Iii) $\Longrightarrow$ (iv): This is trivial.
(iv) $\Longrightarrow$ (i): Suppose $(F,A) = (G,A) \tilde{\cap} (D,A)$ with (G,A) soft open and (D,A) soft dense. Then $\tilde{s}cl(F,A) = \tilde{s}cl(G,A)$. $(F,A) \tilde{\subseteq} (G,A) \tilde{\subseteq} \tilde{s}cl(G,A) = \tilde{s}cl(F,A)$. Also $(F,A) \tilde{\subseteq} \tilde{s}int(G,A) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(G,A)) = \tilde{s}int(\tilde{s}cl(F,A))$, since (G,A) is soft open. So $(F,A) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(F,A))$.

Remark 10. From the above theorem we see that in a soft topological space (U,A,τ), $(F,A) \in POSS(U)_A$ if and only if (F,A) is the intersection of a soft open set and a soft dense set.

Theorem 16. *Every soft preopen set is soft open if and only if* (U,A,τ) *is submaximal.*

Proof. By Theorem 12, every soft dense set is soft preopen and by Remark 10, any soft preopen set is the soft intersection of a soft open set and a soft dense set. Also by Theorem 15, soft intersection of soft open and soft preopen is soft preopen. Hence the proof.

Theorem 17. *For a soft topological space* (U, A, τ), the collection $POSS(U)_A$ *is a soft topology if and only if the intersection of any two soft dense sets is soft preopen.*

Proof. By Remark 10, any soft preopen set is soft intersection of soft open and soft dense sets. So intersection of two soft preopen sets involve intersection of two soft dense sets and a soft open set. Hence if the intersection of two soft dense sets is preopen then by Theorem 15 the proof follows.

Conclusion

In this paper, soft preopen sets are defined and few properties are studied. Some results relating soft dense sets and soft preopen sets are established. Also the condition for a collection of soft preopen sets to be a soft topology is given.

Competing interests

The authors declare that that they have no competing interests.

Authors' contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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