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One Modulo Three Harmonic Mean Labeling of Graphs

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Abstract

In this paper, we introduce a new labeling called one modulo three harmonic mean labeling. A graph G is said to be one modulo three harmonic mean graph if there is a function φ from the vertex set of G to $\{1, 3, 4, \ldots, 3q-2, 3q\}$ with φ is one-one and φ induces a bijection φ* from the edge set of G to {1, 4, …, 3q - 2}, where φ*(e = uv) = $\left\lceil \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rceil$ or $\left\lfloor \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rfloor$ and the function φ is called as one modulo three harmonic mean labeling of G. Further, we prove that some graphs are one modulo three mean graphs

Key words: One modulo three harmonic mean labeling, one modulo three harmonic mean graphs.

Introduction and definitions

All graphs considered here are simple, finite, connected and undirected. The symbols V(G) and E(G) denote the vertex set and the edge set of a graph G. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Several types of graph labeling and a detailed survey is available in [1]. For standard terminology and notations we follow harary[2].

V.swaminathan and C. Sekar introduce the concept of one modulo three graceful labeling in [3]. S.Somasundaram and R.Ponraj introduced mean labeling of graphs in

[4]. S.Somasundaram and S.S Sandhya introduced the concept of harmonic mean labeling in [5] and studied their behavior in [6], [7] and [8]. In this paper ,we introduce a new type of labeling known as one modulo three harmonic mean labeling and investigate one modulo three harmonic mean graphs.

We will provide brief summary of definitions and other information which are necessary for the present investigation.

Definition 1.1: The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1(which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertices in the i^{th} copy of G_2.

Definition1.2: The graph $P_n \odot K_1$ is called Comb.

Definition 1.3: The graph $C_n \odot K_1$ is called crown.

2. One modulo three harmonic mean labeling

Definition 2.1: A graph G is said to be one modulo three harmonic mean graph if there is a function φ from the vertex set of G to $\{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ with φ is one-one and φ induces a bijection φ* from the edge set of G to {1, 4, …, 3q - 2}, where φ*(e = uv) = $\left\lceil \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rceil$ or $\left\lfloor \frac{2\varphi(u)\varphi(v)}{\varphi(u)+\varphi(v)} \right\rfloor$ and the function φ is called as one modulo three harmonic mean labeling of G.

Remark 2.2: If G is a one modulo three harmonic mean graph, then 1 must be a label of one of the vertices of G, since an edge must get the label 1.

Theorem 2.3: If G is a one modulo three harmonic mean graph, then there is atleast one vertex of degree one.

Proof.

Suppose G is a one modulo three harmonic mean graph. Then by remark 2.2, 1 must be the label of one of the vertices of G. If the vertex u get the label 1, then any edge incident with u get the label 1 or 2, since $1 \leq \frac{2m}{m+1} < m$. But 2 is not in the set $\{1, 3, 4, \ldots, 3q-2, 3q\}$. Therefore, this vertex must have degree one. Hence the theorem.

Theorem 2.4: Any k-regular graph, k>1, is not one modulo three harmonic mean graph.

Proof.

Let G be a k-regular graph. Suppose G is one modulo three harmonic mean graph. Then by theorem 2.3, there is atleast one vertex of degree one. Which is a contradiction to k>1. Hence the theorem.

Corollary 2.5: Any cycle C_n is not a one modulo three harmonic mean graph.

Proof.
The proof follows from the theorem 2.4.

Theorem 2.6: Any path P_n is a one modulo three harmonic mean graph.

Proof.
Let P_n be the path $u_1u_2\ldots u_n$. Define a function φ: V(G) → $\{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by $\varphi(u_1) = 1$, $\varphi(u_i) = 3(i-1)$, $2 \leq i \leq n$. Then φ induces a bijection φ*: E(G) → {1, 4, …, 3q - 2},where $\varphi^*(u_iu_{i+1}) = 3i-2, 1 \leq i \leq n$. Therefore, φ is a one modulo three harmonic mean labeling. Hence P_n is a one modulo three harmonic mean graph.

Example 2.7: One modulo three harmonic mean labeling of P_8 is shown in figure 1.

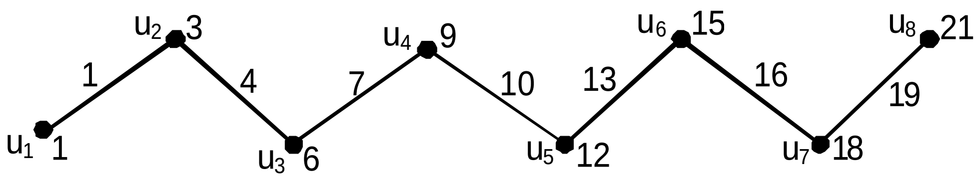

Figure 1

Theorem 2.8:A comb $P_n \odot K_1$ is a one modulo three harmonic mean graph.

Proof.
Let $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n$ be the vertices of $P_n \odot K_1$. Here we consider two cases.

Case (1): n is odd.
Define a function f: $V(P_n \odot K_1) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 3$
$\varphi(u_i) = 6i - 3$, for all $i = 2, 4, \ldots, n-1$
$\varphi(u_i) = 6(i-1)$, for all $i = 3, 5, \ldots, n$
$\varphi(v_1) = 1$
$\varphi(v_i) = 6(i-1)$, for all $i = 2, 4, \ldots, n-1$
$\varphi(v_i) = 6i - 3$, for all $i = 3, 5, \ldots, n$.

Then φ induces a bijective function $\varphi^*: E(P_n \odot K_1) \rightarrow \{1, 4, 7, \ldots, 3q-2\}$,where
$\varphi^*(u_iu_{i+1}) = 6i-2,\ 1 \leq i \leq n-1$
$\varphi^*(u_iv_i) = 6i-5,\ 1 \leq i \leq n$

Thus the edges get the distinct labels 1, 4, …, 3q – 2. In this case φ is a one modulo three harmonic mean labeling.

Case (2): n is even.

Define a function $\varphi : V(P_n \odot K_1) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 3$

$\varphi(u_i) = 6i - 3$, for all $i = 2, 4, \ldots, n$

$\varphi(u_i) = 6(i - 1)$, for all $i = 3, 5, \ldots, n - 1$

$\varphi(u_i) = 3i, 1 \leq i \leq n$

$\varphi(v_1) = 1$

$\varphi(v_i) = 6(i - 1)$, for all $i = 2, 4, \ldots, n$

$\varphi(v_i) = 6i - 3$, for all $i = 3, 5, \ldots, n - 1$.

Then φ induces a bijective function $\varphi^*: E(P_n \odot K_1) \rightarrow \{1, 4, 7, \ldots, 3q - 2\}$,where

$\varphi^*(u_iu_{i+1}) = 6i - 2, 1 \leq i \leq n - 1$

$\varphi^*(u_iv_i) = 6i - 5, 1 \leq i \leq n$

Thus the edges get the distinct labels $1, 4, \ldots, 3q - 2$. In this case φ is a one modulo three harmonic mean labeling. From case (1) and case (2) we conclude that $P_n \odot K_1$ is a one modulo three harmonic mean graph.

Example 2.9: One modulo three harmonic mean labeling of $P_6 \odot K_1$ and $P_7 \odot K_1$ is shown in figure 2 and figure 3 respectively.

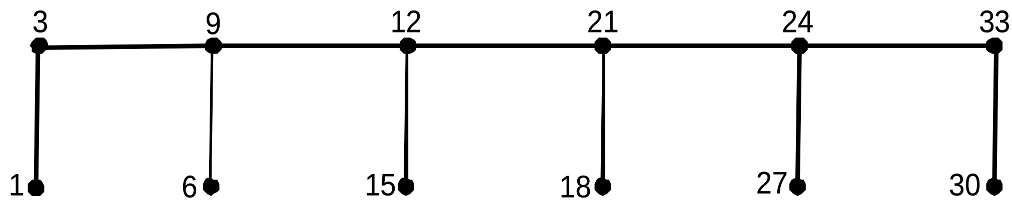

Figure 2

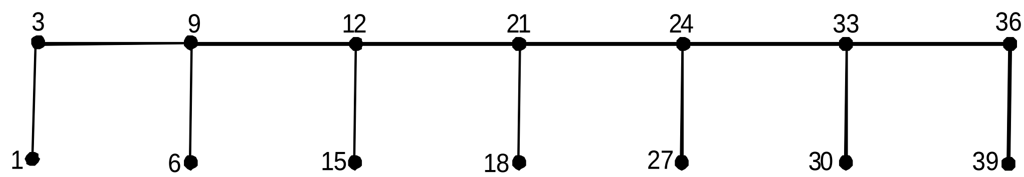

Figure 3

We have $P_2 = K_2$, which is one modulo three harmonic mean graph. Next we have the following theorem.

Theorem 2.10: If $n > 2$, K_n is not a one modulo three harmonic mean graph.

Proof.
Suppose K_n, $n > 2$ is a one modulo three harmonic mean graph. Then by theorem 1, there exists atleast one vertex of degree one. This is the contradiction to the fact that all the vertices are of degree greater than one. Hence the theorem.

Theorem 2.11: $K_{1,n}$ is one modulo three harmonic mean graph if and only if $n \leq 6$.

Proof.
$K_{1,1}$ is same as P_2 and $K_{1,2}$ is P_3. Hence by theorem 2.3 $K_{1,1}$ and $K_{1,2}$ are one modulo three harmonic mean graphs. One modulo three harmonic mean labeling for $K_{1,3}$, $K_{1,4}$, $K_{1,5}$ and $K_{1,6}$ are displayed below.

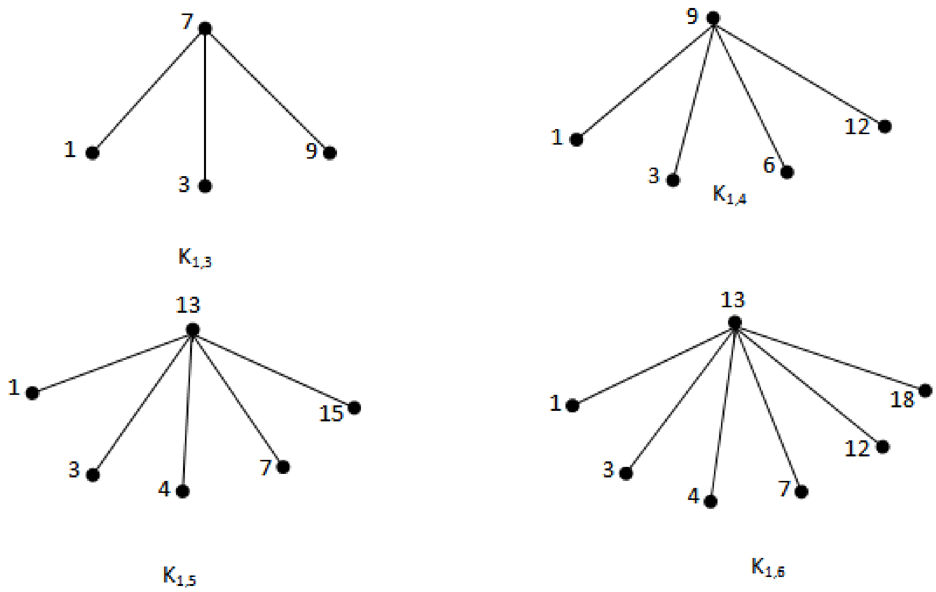

Figure 4

Assume $n > 6$ and suppose $K_{1,n}$ has one modulo three harmonic mean labeling. Let $\varphi(u)$ be the central vertex of $K_{1,n}$. Here we consider two cases.

Case(1): $2 \leq \varphi(u) \leq 13$. Then clearly there is no edge with label $3q - 2, q > 6$, since the highest edge label is $\frac{2.13.3q}{13+3q}$. Hence in this case $K_{1,n}$ is not a one modulo three harmonic mean graph.

Case (2) :$\varphi(u) \geq 15$. Then there is no edge with label 4 as shown in the following figure 5.

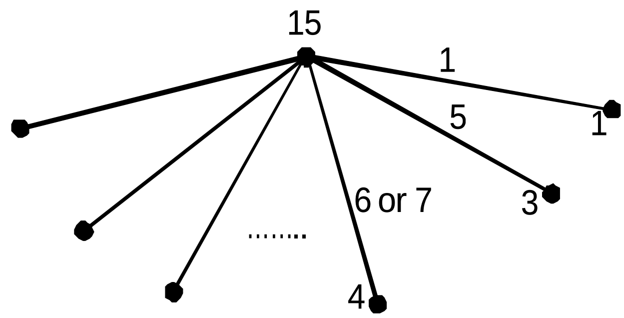

Figure 5

Hence $K_{1,n}$ is not a one modulo three harmonic mean graph for n > 6.

Theorem 2.12: $P_n \odot \overline{K}_2$ is a one modulo three Harmonic mean graph.

Proof.

Let u_i, u_i^j, $1 \leq i \leq n$, $1 \leq j \leq 2$ be the vertices of $P_n \odot \overline{K}_2$. Define a function φ: $V(P_n \odot \overline{K}_2) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 4$

$\varphi(u_i) = 9i - 5$, $2 \leq i \leq n$

$\varphi(u_1^1) = 1$

$\varphi(u_i^1) = 9(i-1)$, $2 \leq i \leq n$

$\varphi(u_i^2) = 9i - 6$, $1 \leq i \leq n$.

Then φ induces a bijective function $\varphi^*: E(P_n \odot \overline{K}_2) \rightarrow \{1, 4, \ldots, 3q-2\}$, where

$\varphi^*(u_iu_{i+1}) = 9i - 2$, $1 \leq i \leq n-1$

$\varphi^*(u_iu_i^1) = 9i - 8$, $1 \leq i \leq n$

$\varphi^*(u_iu_i^2) = 9i - 5$, $1 \leq i \leq n$.

Therefore, φ is a one modulo three harmonic mean labeling of $P_n \odot \overline{K}_2$. Hence $P_n \odot \overline{K}_2$ is one modulo three harmonic mean graph.

Example 2.13: One modulo three harmonic mean labeling of $P_5 \odot \overline{K}_2$ is shown in figure 6.

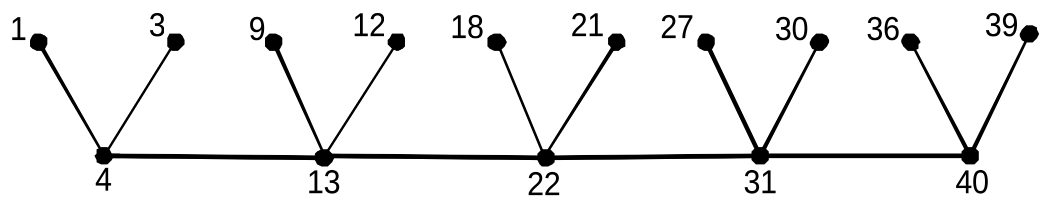

Figure 6

Theorem 2.14: A graph obtained by attaching a path of length two at each vertex of P_n is one modulo three harmonic mean graph.

Proof.
Let u_i be the vertices of P_n and v_i, w_i, be the vertices path of length two, $1 \le i \le n$. Let G be a graph obtained by attaching a path of length two at each vertex of P_n. Define a function φ: $V(G) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_i) = 9i - 5$, $1 \le i \le 2$
$\varphi(u_i) = 9i - 6$, $3 \le i \le n$
$\varphi(v_i) = 9i - 6$, $1 \le i \le 2$
$\varphi(v_i) = 9i - 5$, $3 \le i \le n$
$\varphi(w_1) = 1$
$\varphi(w_2) = 9$
$\varphi(w_i) = 9(i-2) - 2$, $3 \le i \le n$.

Then φ induces a bijective function φ^*: $E(G) \to \{1, 4, \ldots, 3q-2\}$, where
$\varphi^*(u_iu_{i+1}) = 9i - 2$, $1 \le i \le n-1$
$\varphi^*(u_iv_i) = 9i - 5$, $1 \le i \le n$
$\varphi^*(u_iw_i) = 9i - 8$, $1 \le i \le n$.

Therefore, φ is one modulo three harmonic mean labeling of G. Hence G is one modulo three harmonic mean graph.

Example 2.15: One modulo three harmonic mean labeling of G, When n=7 is shown in figure 7.

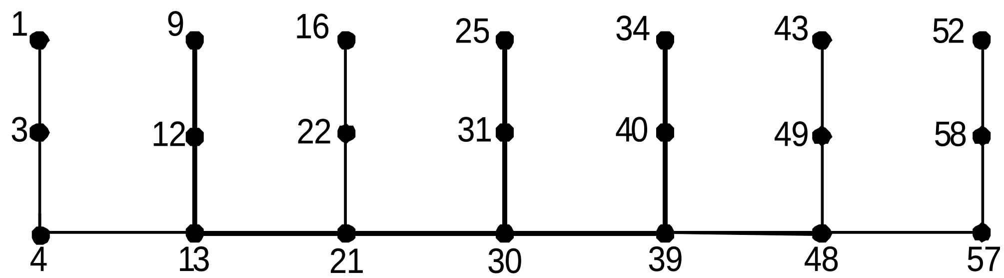

Figure 7

Theorem 2.16: $C_n \odot K_1$ is one modulo three Harmonic mean graph.

Proof.
Let u_i, v_i, $1 \le i \le n$, be the vertices of $C_n \odot K_1$. Define a function φ: $V(C_n \odot K_1) \to \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

Case(1): if n is odd,

$\varphi(u_1) = 7$
$\varphi(u_i) = 6i$, $i = 3, 5, \ldots, n$
$\varphi(u_2) = 15$,
$\varphi(u_i) = 6i - 3$, $i = 4, 6, \ldots, n - 1$
$\varphi(v_1) = 1$
$\varphi(v_3) = 4$
$\varphi(v_i) = 6i - 3$, $i = 5, 7, \ldots, n$
$\varphi(v_i) = 6i$, $i = 4, 6, \ldots, n - 1$

Case(2): if n is even,

$\varphi(u_1) = 7$,
$\varphi(u_i) = 6i$, $i = 3, 5, \ldots, n - 1$
$\varphi(u_2) = 15$
$\varphi(u_i) = 6i - 3$, $i = 4, 6, \ldots, n$
$\varphi(v_1) = 1$
$\varphi(v_3) = 4$
$\varphi(v_i) = 6i - 3$, $i = 5, 7, \ldots, n - 1$
$\varphi(v_2) = 3$
$\varphi(v_i) = 6i$, $i = 4, 6, \ldots, n$.

Then φ induces a bijective function $\varphi^*: E(C_n \odot K_1) \to \{1, 4, \ldots, 3q - 2\}$, where
$\varphi^*(u_iu_{i+1}) = 6i + 4$, $1 \le i \le 2$
$\varphi^*(u_iu_{i+1}) = 6i + 1$, $3 \le i \le n - 1$
$\varphi^*(u_iv_i) = 3i - 2$, $1 \le i \le 3$
$\varphi^*(u_iv_i) = 6i - 2$, $4 \le i \le n$.

Therefore, φ is one modulo three harmonic mean labeling of $C_n \odot K_1$. Hence $C_n \odot K_1$ is one modulo three harmonic mean graph.

Example 2.17: One modulo three harmonic mean labeling of $C_7 \odot K_1$ is given in figure 9.

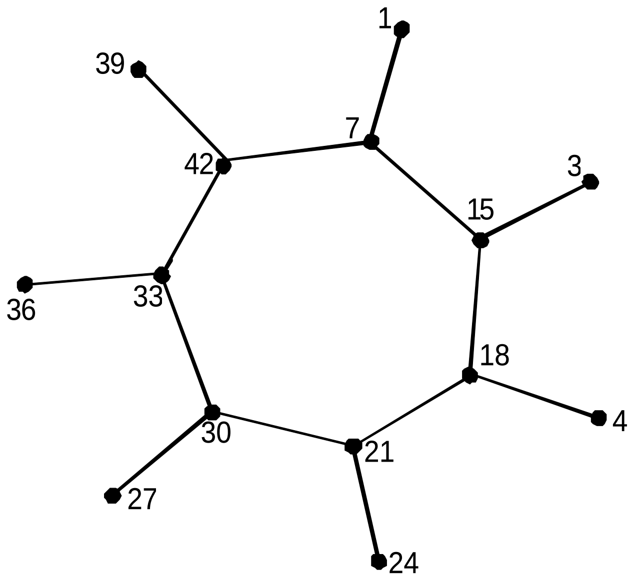

Figure 9

Theorem 2.18: $C_n \odot \overline{K}_2$ is one modulo three Harmonic mean graph.

Proof.

Let u_i, u_i^j, $1 \le i \le n$, $1 \le j \le 2$ be the vertices of $C_n \odot \overline{K}_2$. Define a function φ: $V(C_n \odot \overline{K}_2) \rightarrow \{1, 3, 4, 6, \ldots, 3q-2, 3q\}$ by

$\varphi(u_1) = 7$

$\varphi(u_i) = 9i - 3$, $2 \le i \le n$

$\varphi(u_1^1) = 1$

$\varphi(u_2^1) = 4$

$\varphi(u_i^1) = 9i - 6$, $3 \le i \le n$

$\varphi(u_1^2) = 3$

$\varphi(u_i^2) = 9i$, $2 \le i \le n$.

Then φ induces a bijective function $\varphi^*: E(C_n \odot \overline{K}_2) \rightarrow \{1, 4, \ldots, 3q-2\}$, where

$\varphi^*(u_i u_{i+1}) = 9i + 1$, $1 \le i \le n-1$

$\varphi^*(u_1 u_1^1) = 1$,

$\varphi^*(u_2 u_2^1) = 7$, $1 \le i \le n-1$

$\varphi^*(u_i u_i^1) = 9i - 5$, $3 \le i \le n$

$\varphi^*(u_1 u_1^2) = 4$

$\varphi^*(u_i u_i^2) = 9i - 2$, $2 \le i \le n$.

Therefore, φ is one modulo three harmonic mean labeling of $C_n \odot \overline{K}_2$. Hence $C_n \odot \overline{K}_2$ is one modulo three harmonic mean graph.

Example 2.19: One modulo three harmonic mean labeling of $C_5 \odot \overline{K}_2$ is shown in figure 10.

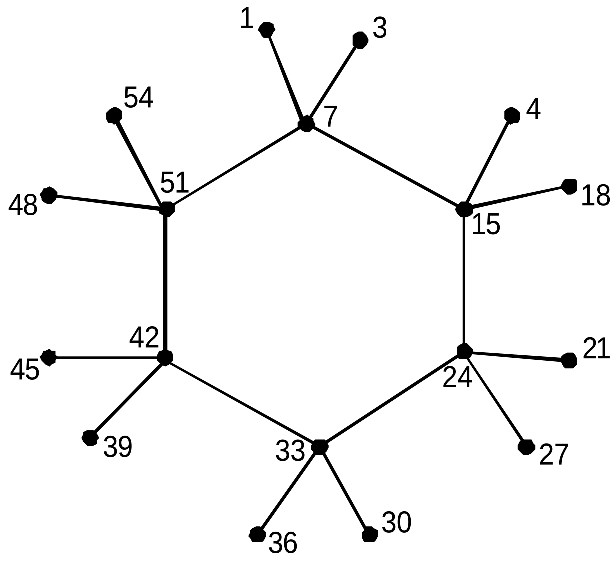

Figure 10

Theorem 2.20: A graph obtained by attaching a path of length two at each vertex of C_n admits One Modulo three Harmonic mean labeling.

Proof.

Let G be the graph obtained by attaching a path of length two at each vertex of C_n. Let u_i, v_i, w_i, $1 \le i \le n$ be the vertices of G. Define a function φ: $V(G) \to \{1, 3, 4, 6, \ldots, 3q - 2, 3q\}$ by

$\varphi(u_1) = 7$
$\varphi(u_i) = 9i - 3, 2 \le i \le n$
$\varphi(v_1) = 4$
$\varphi(v_i) = 9i - 2, 2 \le i \le n$
$\varphi(w_1) = 1$
$\varphi(w_2) = 4$
$\varphi(w_i) = 9i - 8, 3 \le i \le n.$

Then φ induces a bijective function φ^*: $E(G) \to \{1, 4, \ldots, 3q - 2\}$, where
$\varphi^*(u_iu_{i+1}) = 9i + 1, 1 \le i \le n - 1$
$\varphi^*(u_1v_1) = 4$
$\varphi^*(u_iv_i) = 9i - 2, 2 \le i \le n$
$\varphi^*(u_1w_1) = 1$
$\varphi^*(u_2w_2) = 7$
$\varphi^*(u_iw_i) = 9i - 5, 3 \le i \le n.$

Therefore, φ is one modulo three harmonic mean labeling of G. Hence G is one modulo three harmonic mean graph.

Example 2.21: One modulo three harmonic mean labeling of G, n=6 is shown in figure 11.

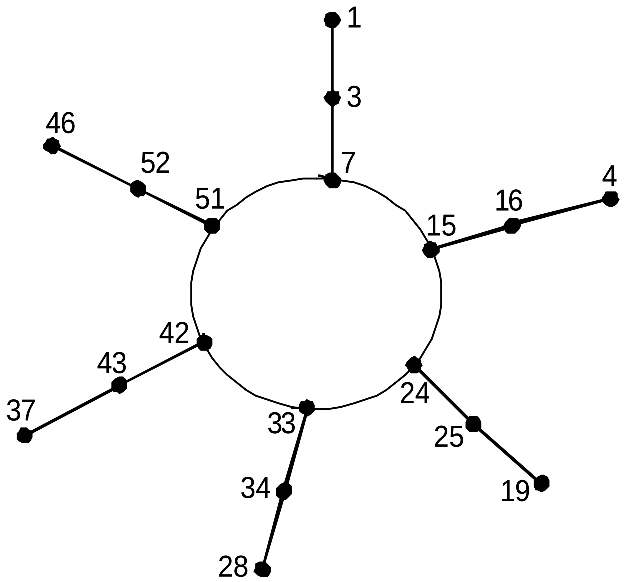

Figure 11

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