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Some Important Characteristics of Fuzzy Number Mappings

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Abstract

This paper presents the Characteristics of convex fuzzy number mappings defined over fuzzy number space through illustrative examples. Also we framed some important results based on the characteristics of fuzzy number mappings.

Keywords : Convex fuzzy number mappings, triangular fuzzy numbers, trapezoidal fuzzy numbers,Gaussian fuzzy numbers

1.Introduction

A fuzzy number is an ordinary number whose precise value is somewhat uncertain.Fuzzy numbers are used in Statistics,Computer Programming, Engineering and Experimental Science. Any fuzzy number can be thought of as a function whose domain is a specified set. In many respects, fuzzy numbers depict the physical world more realistically than single valued numbers[2-5].A fuzzy number should be normalized and convex, condition for normalized implies that maximum membership value is 1. Generally a fuzzy number represents a real number interval whose boundary is fuzzy and the fuzzy interval is represented by two end points.

3.Preliminaries

A Fuzzy number μ is defined as $\mu : R \rightarrow [0,1]$ which is normal ,fuzzy convex, upper semi-continuous with bounded support .

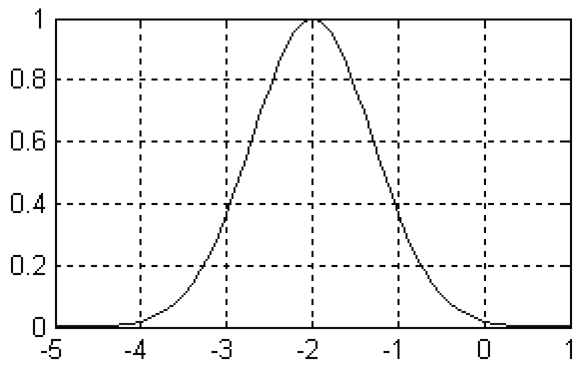

Fig.1 Fuzzy number representation

Now is called a fuzzy number space.

Obviously, a Fuzzy set is a fuzzy number if and only if is a closed and bounded interval for each] and ,null set

Also, ,]

Moreover

A mapping is said to be a *Fuzzy Number Mapping.*

Also ( ],

μ E and] and

Also itself a fuzzy number (fig.2)

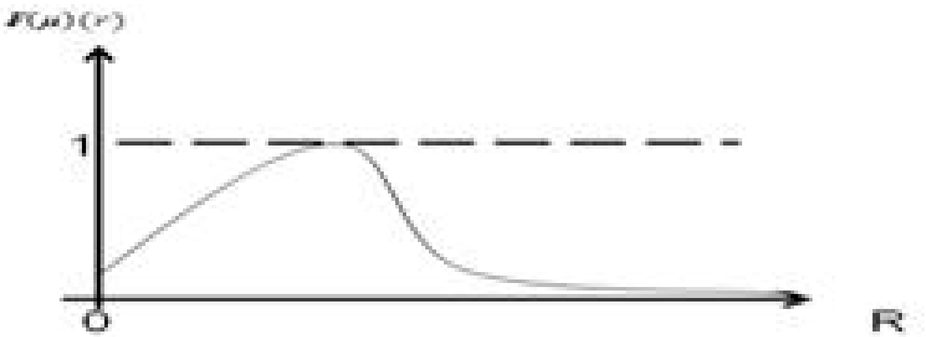

fig.2

A Convex Fuzzy Number Mapping is defined as

tμ+(1-t)) where

3. Main Results

Here the results are based on the partitioning of the membership values of the fuzzy number interval.

R→ Real Numbers *μ → Membership Grade*

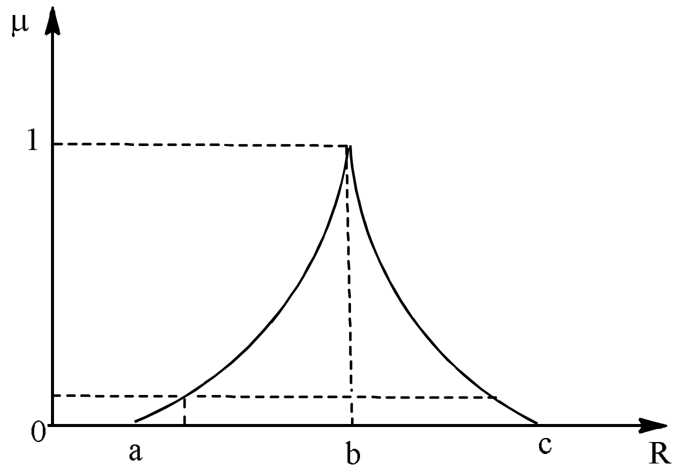

Fig 3

The membership functions of fuzzy number interval gives two parts, left part values and right part values - left part membership values 'from 0 to 1' which in *monotonically increasing* (Fig.4) while the right part membership values 'from 1 to 0' which is *monotonically decreasing* (Fig.5).Since this type of fuzzy number interval having maximum membership grade '1' at the mid value of the interval, for proving the results for whole interval, it is enough to prove for the left part.

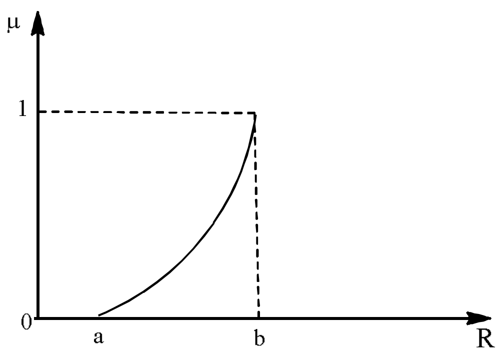

Fig. 4 Left part

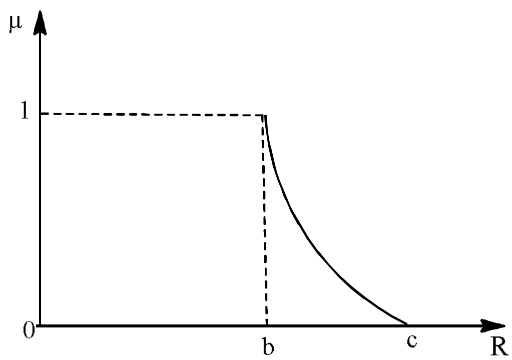

Fig. 5 Right part

Consider a triangular fuzzy number $\mu = [a, c]$, take r = 0

Clearly triangular fuzzy number is a triangular fuzzy number interval $\mu = [a, c]$, with median about b (as in fig.3)

Consider the left part $[a, b]$ (figure .4)

Let ψ ={μ_0(r), μ_1(r),…., μ_n(r)} be a possible partition of the membership values of [a,b]. Clearly μ_0(r) = 0 and μ_n(r) = 1.Similarly it is possible to partition the membership values of right part $[b, c]$

Thus for proving the results of membership values on the whole interval, it in enough to prove one part (left or right).

The same is applicable for Gaussian and Trapezoidal fuzzy numbers.

Illustration I

Consider a fuzzy function $F(\mu)$ (x) = $e^{\mu(x)-1}$ for all $\mu \in E$, $x \in R$ Assume that this convex fuzzy number function F maps a triangular fuzzy number interval [a, c] where $\mu(x)$ is the triangular membership function

$$\mu(x) = \begin{cases} \frac{x-\mathrm{a}}{\mathrm{b}-\mathrm{a}}, & \mathrm{a} \le x \le \mathrm{b} \\ \frac{\mathrm{c}-x}{\mathrm{c}-\mathrm{b}}, & \mathrm{b} \le x \le \mathrm{c} \end{cases}$$

Example: Consider a triangular fuzzy number interval [2, 6]

$$\mu(x) = \begin{cases} \frac{x-63}{67-63}, & 63 \le x \le 67 \\ \frac{71-x}{71-67}, & 67 \le x \le 71 \end{cases}$$

Let ψ ={$\mu_0(r), \mu_1(r), \mu_2(r)$} be a possible partition of the membership values of left part [2,4]. Corresponding partition of membership values of $[F(\mu)]^r$ is given in the table below:

x	$\mu(x)$ or $\mu_i(r)$	F(μ_i) (r)
2	0	0.38
3	0.5	0.61
4	1	1
5	0.5	0.61
6	0	0.38

Example: Consider a Gaussian fuzzy number interval [10,70] with

$\mu(x) = e^{\frac{-(x-m)^2}{2\sigma^2}}$, $10 \le x \le 70$ where mean m= 40 and standard deviation $\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{\mathrm{n}}} = \sqrt{400}$

x	$\mu(x)$ or $\mu_i(r)$	F(μ_i) (r)
10	0.33	0.51
20	0.61	0.68
30	0.58	0.89
40	1	1
50	0.88	0.89
60	0.61	0.68
70	0.33	0.51

Example: Consider a trapezoidal fuzzy number interval [40,80] with a partition {40,45,50,55,60,65,70,75,80}

$$\mu(x) = \begin{cases} \frac{x-40}{55-40}, & 40 \le x \le 55 \\ 1 & 55 \le x \le 65 \\ \frac{80-x}{80-65}, & 65 \le x \le 80 \end{cases}$$

$\boldsymbol{x}$	**μ** $(\boldsymbol{x})$ or $\boldsymbol{\mu}_i(\boldsymbol{r})$	F($\boldsymbol{\mu}_i$) $(\boldsymbol{r})$
40	0	0.37
45	0.33	0.51
50	0.67	0.72
55	1	1
60	1	1
65	1	1
70	0.67	0.72
75	0.33	0.51
80	0	0.37

From the above illustrations, it is found that $F(\mu)(x) = e^{\mu(x)-1}$, for all $\mu \in E$, $x \in R$ is a fuzzy number membership function.

$F(\mu)(x) = 1$, $x \in R$ (exactly one $x \in R$ other than trapezoidal Fuzzy numbers)

Obviously, F(μ) is convex

$F(\mu)$is piecewise continuous

Clearly $F(\mu)(x) = e^{\mu(x)-1}$ maps fuzzy numbers to fuzzy numbers

Result : $F: E \to E$ defined by F(μ)(x) = $e^{\mu(x)-1}$, $x \in R$,$\forall\, \mu \in E$ is a bijection.

Proof

$F(\mu)(x) = e^{\mu(x)-1}$

For μ , $\lambda \in E$ and $x \in R$, $F(\mu)(x) = F(\lambda)(x)$

$\Rightarrow\ e^{\mu(x)-1} = e^{\lambda(x)-1} \Rightarrow \mu(x) = \lambda(x)$, for all x

Then F is one –one (Injective)

For every $\lambda \in E$, there exists a $\mu \in E$ such that $F(\mu)(x) = \lambda(x)$, $x \in R$

Then clearly F is on to (subjective)

Thus F is a bijection.

Illustration II.

Consider a fuzzy function on E defined by

$$F(\mu)(x) = \begin{cases} \mu(x) - \mu(a), & a \le x \le b \\ 1 & x = b \\ \mu(c) - \mu(a), & b \le x \le c \end{cases}$$

Where $\mu(x)$ is a triangular fuzzy number [a, c].
Let [a, c] = [812, 824] ,b = 818

x	**μ**(x)	F (**μ**) (x)
812	0	0
814	0.333	0.333
816	0.667	0.667
818	1	1
820	0.667	0.667
822	0.333	0.333
824	0	0

Clearly this is a fuzzy number mapping which maps all triangular fuzzy numbers to itself.

Assume that E has only triangular fuzzy numbers. Then this fuzzy function F is an *identity Fuzzy number* mapping on E.

Define the same function for Gaussian fuzzy numbers and Trapezoidal fuzzy numbers.

This mapping for Gaussian fuzzy numbers is illustrated by the following table

x	**μ**(x)	F (**μ**) (x)
480	0.37	0
481	0.78	0.41
482	1	1
483	0.78	0.41
485	0.37	0

From the table it is easy to verify that F is a fuzzy number mapping on E ,which maps Gaussian fuzzy numbers to fuzzy numbers(need not be Gaussian).

The mapping F for Trapezoidal fuzzy numbers is illustrated by the following table

x	**μ**(x)	F (**μ**) (x)
100	0	0
15	0.33	0.33
200	1	1
250	1	1
300	1	1
350	0.33	0.33
400	0	0

Here F maps trapezoidal fuzzy numbers to itself .

Result: Let F: E→ E be a convex fuzzy number mapping defined as

$$F(\mu)(x) = \begin{cases} \mu(x) - \mu(a), & a \le x \le b \\ 1 & x = b \\ \mu(c) - \mu(a), & b \le x \le c \end{cases}$$

where μ = [a, c] be the fuzzy number interval about mean 'b'. Then F is a bijection.

Proof.

For $a \le x \le b$

$F(\mu)(x) = F(\lambda)(x)$, for $\mu, \lambda \in E$

$\mu(x) - \mu(a) = \lambda(x) - \lambda(a)$

Case 1 : μ is a triangular fuzzy number or trapezoidal fuzzy number.

Here $\mu(a) = \lambda(a) = 0$ Thus $\mu(x) = \lambda(x)$

F is then one- one

Case 2: μ is a Gaussian fuzzy number.

$$\mu(x) - e^{\frac{-(a-b)^2}{2\sigma^2}} = \lambda(x) - e^{\frac{-(a-b)^2}{2\sigma^2}}$$

Thus $\mu(x) = \lambda(x)$ and hence F is one – one.

Similarly F is one-one for *for* $b \le x \le c$

Thus F is an injective fuzzy number mapping.

It is obvious that $F(\mu)(x) = \begin{cases} \mu(x) - \mu(a), & a \le x \le b \\ 1 & x = b \\ \mu(c) - \mu(a), & b \le x \le c \end{cases}$

is a surjective (onto) fuzzy number mapping and hence F is a bijection.

From the above explanations and illustrations, it is possible to generate following concepts

F is an inventible fuzzy number mapping.

The inverse function of a bijective function is also bijective. Thus $F^{-1}$is also a bijection.

F is an endomorphic function on E

F is a permutation (a bijective function from a set to itself) on E.

References

[1] D. Dubois, H. Prade,” Fuzzy Sets and Systems:Theory and Application” Academic Press,New York, 1980.

[2] Gui-Xiang Wang, You-Ming Li,Wei Li, “ The Boundedness and Continuity of Convex Fuzzy number Mappings” Fourth international conference on Machine learning,Cybernetics,Guangzlou 18-21,August 2005

[3] Haddi Nasseri, ” Fuzzy Numbers : Positive and Non negative” International Mathematical Forum, 2008, No.3 , 1777-1780

[4] S.Q.Jiang,H.Yan ,”Fuzzy Bounded Sets and Totally Fuzzy Bounded Sets in I-Topological Vector spaces” Iranian Journal of Fuzzy systemsVol 6,no.3(2009) pp.73-90

[5] S.K.Mishra,S.Y.Wang,K.k.Lai, “Pseudolinear Fuzzymappings” ,European Journal of Operations Research” (2006)

[6] Shang Gao and Zaijue Zhang ,”Multiplication Operation on Fuzzy Numbers” Journal of Software , Vol.4,June2009 pp.331-338

[7] Timothy.J.Ross ,”Fuzzy Logic with Engineering Applications” : McGraw-Hill Inc.,Newyork 1995

[8] Wang Guixiang,and Wu Congxin, “Directional derivatives and sub differential of convex mappings and application in convex fuzzy pragramming’, Fuzzy Sets and Systems,Vol.138,pp.559-591,2003

[9] Witold Kosinski,” On Fuzzy Number Calculus”Int.J.Appl.Math.Comput.Sci., 2006,Vol.16,No.1,51-57