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Analysis of Cost-Service Trade-off in Inventory Management through Evolutionary Multi-objective Optimization

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Abstract

Inventory management involves trade-offs between conflicting objectives such as cost minimization and service level maximization. The trade-off analysis of cycle stock investment and workload, so called the exchange curve, possibly dates back to several decades ago. These analyses seldom formulated inventory trade-offs as a multi-objective optimization problem and their solution procedures were all based on single objective optimization. To our best knowledge, there do exist some studies that propose non-classical approach to multi-objective inventory management. However, some of the objectives in earlier studies are not conflicted each other such that the multi-objective models were not properly justified. In this paper, a Inventory Management Model without redundancy is discussed first. Then a solution procedure based on evolutionary multi-objective optimization is introduced to effectively solve the fixed order model. The results show that the intrinsic multi-objective approach can find efficient policies of order size and safety factor simultaneously without estimating shortage cost or service level. Moreover, a fitted exchange curve of cost and service is useful in determining the best customer service possible for the given investment in inventory management. The cost-service trade-off can be observed in a single run of an iterative computation, so that it is more appropriate for the practice of inventory management.

Keywords: Inventory Management; Exchange Curve; Multi-objective optimization; Multi-Objective Particle Swarm Optimization (MOPSO)

1. Introduction

Most business decisions involve more than one conflicting objectives. For example, inventory management has to operate in an efficient way while maintaining appropriate customer service. This uncomfortable fact has been ignored for some time, although some modifications of single objective models have been employed to reach a compromise solution. Due to the advances in information technology, analysis adhering to the essence of multi-objective optimization is increasingly taking on recently.

Multi-objective inventory management is seldom found in literatures, although trade-off analysis of cycle stock and workload (i.e. number of annual setups) possibly dates back to several decades ago (Brown, 1961). The oldest inventory model, economic order quantity (EOQ), implicitly aggregated two conflicting objectives into a single one by introducing the inventory carrying rate. Since Brown (1961), practitioners argued that there is no correct value for the inventory-carrying rate. Instead, it is a management policy that can be changed from time to time to meet the volatile environment. Brown (1967) later introduced the concept of exchange curve to delineate the trade-off between workload and cycle stock. Starr and Miller (1962) also determined trade-off between the number of annual setups and average investment by the method of Lagrangian relaxation.

For items with probabilistic demand, all cost components including setup, carrying, and shortage costs could be considered as management policies. Controlling shortage is often treated as another objective, in addition to the efforts of carrying inventory, by many inventory consultants (Silver et al., 1998). Gardner and Dannenbring (1979) specifically introduced customer service maximization as another objective, along with minimizing workload and inventory investment. They extended the exchange curve to a response surface in three–dimensional space. Unfortunately, there is no direct solution for any variable in their model, and so the problem is solved by successive approximation. Alscher and Schneider (1982) used the same model in Gardner and Dannebring (1979) and proposed another method instead of Lagrangian multiplier to solve the tri-objective probabilistic model. Literatures mentioned above belong to a branch of inventory management studies that usually refers to the exchange curve analysis. They did not explicitly formulate inventory trade-off as a multi-objective optimization problem. Moreover, their solution procedures are all based on single objective optimization that either unifies several objectives into a single one or treats all objectives except one as constraints. Thus, the transformed problems can be solved by classical optimization methods. However, only a single trade-off solution can be obtained in each run of most classical methods, consequently the process to generate exchange curve is lengthy and frustrated. Last but not the least, this approach contradicts our intuition that single-objective optimization is a degenerate case of multi-objective optimization (Deb, 2001).

Bookbinder and Chen (1992) proposed a multi-criteria (or multi-objective) approach for analyzing multi-echelon inventory and distribution systems. They described their approach as multiple criteria decision making (MCDM) generalizations of earlier studies (Brown, 1961; Brown, 1967; Star and Miller, 1962; Gardner and Dannenbring, 1979). Their formulations are intrinsically multi-objective;

however, the solution procedure still follows the single objective optimization. Puerto et al. (2002) commented on the paper by Bookbinder and Chen (1992) that their solution procedure does not determine the Pareto-optimal set properly. Besides giving a procedure producing the set, they also stated that the models reduces to a bi-criteria nonlinear mixed integer programming problem that is well known to be the hardest kind of problem in multi-objective optimization for which no general tools have been yet developed.

Agrell (1995) presented a multi-criteria framework for probabilistic inventory systems with backordering. Three criteria including expected cost, expected number of shortage occasions and expected number of demand not covered from stock are minimized in this work. The goal is to find efficient order size and safety stock (defined in Section 3 later) to strike a balance among conflicting objectives. Therefore, the trade-off among objectives should be assessed before a management policy has been made. Agrell (1995) implemented a commonly used method of multi-objective optimization, named Step method (STEM), in Microsoft Excel. Users can use the decision support system to interactively find one or several efficient solutions of order size and safety stock. Nevertheless, a sequence of single objective optimization problems still needs to be solved as in the case of the Lagrangian relaxation approach.

For the non-classical approach to multi-objective inventory management, Tsou (2008) presented a two-stage decision framework based on multi-objective particle swarm optimization (MOPSO) and technique for order preference by similarity to ideal solution (TOPSIS). MOPSO is used to generate the efficient policies of the multi-objective inventory management problem in Agrell (1995). Then, a compromise solution was selected by TOPSIS according to subjective preferences of decision makers. Tsou and Kao (2008) also developed a metaheuristic based on Electromagnetism-like Mechanism (EM) to approximate the efficient front without using any prior or interactive preference. They showed that the metaheuristic could find similar trade-off solutions in cost and shortage as the interactive procedure STEM did. Tsou (2009) further showed that evolutionary Pareto approach could generate trade-off solutions potentially ignored by the well-known simultaneous method. Nevertheless, we recently notice that the trade-off solutions of above studies actually laid on an exchange curve, *instead of* forming a trade-off surface in the 3D objective space. It apparently indicates that some of the objectives, including minimization of expected annual cost, expected annual number of stock out occasions, and the expected annual number of items stocked out, are not conflicted each other. Among which, the last two objectives are redundant because they are related to the same concept of shortage but different measures. The use of a tri-objective model, consequently, was not properly justified in above studies. .

The purposes of this paper are twofold. Firstly, we present a fixed order inventory model without redundancy under lost sales. Secondly, a solution procedure based on evolutionary multi-objective optimization is used to address the inventory management problem. It is well known that applying evolutionary multi-objective optimization methods to solve business and industrial problems, such as portfolio selection and facility location problem, is getting popular recently (Chiam et al., 2008, Rabbani et al., 2010, Bhattacharya and Bandyopodhyay, 2010). In this paper, we combine the MOPSO with local search mechanism to solve the problem, which was suggested by Tsou et al. (2006) and has been shown that it is competitive with other well-known evolutionary multi-objective optimization algorithm, such as the Strength Pareto Evolutionary Algorithm (SPEA) (Zitzler and Thiele, 1999). The rest of this paper is organized as follows. Section 2 describes the bi-objective (r,Q) system under lost sales. An algorithm based on MOPSO is described in Section 3. Computational experiments are carried out in Section 4. Finally, conclusions are presented accordingly.

2. Inventory System with Fixed Order under Lost Sales

A common mechanism to managing items with probabilistic demand is the fixed order system (r,Q). It is widely used in business because it is easy to understand and implement. Managers will place an order of size Q when inventory position drops to or lower than the reorder point r. Reorder point is equal to expected lead time demand (μ_L) plus safety stock (SS), which is the safety factor (k) times a standard deviation of lead time demand (σ_L). Therefore, the decision is boiled down to determine management policy described by k and Q in order to meet the business target. Agrell (1995) presented a tri-objective model concerning cost and shortage to plan for these management policies. As mentioned earlier, trade-off solutions based on Agrell's model actually formed an exchange curve, because there exists redundancy among objectives. One of the shortage objectives is simply dropped out to have a non-redundant model under lost sales shown as follows.

$$\underset{k,Q}{\text{Minimize}}\, C(k,Q)=\frac{AD}{Q}+hc\ \ \frac{Q}{2}+k\sigma_L+\frac{D\sigma_L}{Q}\left(\varphi(k)-k(1-\Phi(k))\right) \quad \text{And} \tag{1}$$

$$\underset{k,Q}{\text{minimize}}\ \ S(k,Q)=\frac{D}{Q}\int_k^\infty \varphi(z)dz\,, \tag{2}$$

Subject to

$$\sqrt{\frac{2AD}{hc}}\le Q\le D \quad \text{and} \tag{3}$$

$$0\le k\le\frac{D}{\sigma_L}\,. \tag{4}$$

Where

Q is the lot size,

D is the average annual demand,

A is the ordering/setup cost,

c is the unit item cost,

h is the inventory carrying rate,

L is the lead time,

D_L is the lead time demand.

It is normally distributed with mean μ_L and standard deviation σ_L, $\varphi(x)$ and $\Phi(x)$ are the probability distribution and cumulative distribution functions of the standard normal random variables, respectively, and r is the order point, which equals to the expected lead time demand plus the safety stock. That is $r = \mu_L + k\sigma_L$.

Objective (1) is to minimize the expected cost per year for setup and holding inventory under lost sales, and objective (2) is to minimize the expected number of shortage occasions per year. The derivations of these objectives can be found in a classical inventory management book (Silver et al., 1998). Generally speaking, the function in (1) is an approximation of the inventory cost when the quantity stocked out is very small.

Inequality (3) ensures that the order size should be no less than EOQ quantity and no more than the average annual demand. This means that the order size for a probabilistic inventory system should be greater than or equal to that of deterministic case in order to counteract uncertainty. Inequality (4) guarantees that the safety stock will not be greater than the average annual demand and must be non-negative.

One of the important trades-off under probabilistic demand is whether to hold more inventories or to face possible stock outs. Both circumstances incur some sort of cost, for example, holding and ordering cost have been widely monitored by many companies across industries and countries for a long time. But, shortage cost is not easy to determine till now. Gardner and Dannenbring (1979) found that most practitioners have not adopted shortage cost, since there is no basis for its measurement in accounting methodology. Alscher and Schneider (1982) also mentioned that most practitioners do not make use of shortages but preferring to use measures of customer service. Due to these practices in inventory management, expected total relevant cost and customer service measured by the expected number of stock out occasions are chosen as inventory trade-off.

3. A Pareto approach based on MOPSO

For the minimization problem described above, an inventory decision $\vec{\mathbf{x}}_1 = (k_1, Q_1)$ is said to strongly dominate $\vec{\mathbf{x}}_2 = (k_2, Q_2)$ (denoted by $\vec{\mathbf{x}}_1 \prec\prec \vec{\mathbf{x}}_2$) if and only if $C(\vec{\mathbf{x}}_1) < C(\vec{\mathbf{x}}_2)$ and $S(\vec{\mathbf{x}}_1) < S(\vec{\mathbf{x}}_2)$. That is, solution $\vec{\mathbf{x}}_1$ is strictly better than solution $\vec{\mathbf{x}}_2$ in both the cost and shortage criteria. Less stringently, a decision vector $\vec{\mathbf{x}}_1$ dominate $\vec{\mathbf{x}}_2$ (denoted by $\vec{\mathbf{x}}_1 \prec \vec{\mathbf{x}}_2$) if and only if $C(\vec{\mathbf{x}}_1) \le C(\vec{\mathbf{x}}_2)$ and $S(\vec{\mathbf{x}}_1) \le S(\vec{\mathbf{x}}_2)$ and at least one of above inequality is strictly held. Vector $\vec{\mathbf{x}}_1$ weakly dominate another vector $\vec{\mathbf{x}}_2$, notated as $\vec{\mathbf{x}}_1 \preceq \vec{\mathbf{x}}_2$ if and only if $C(\vec{\mathbf{x}}_1) \le C(\vec{\mathbf{x}}_2)$ and $S(\vec{\mathbf{x}}_1) \le S(\vec{\mathbf{x}}_2)$.

In general, the search process does not interest solutions dominated by other solutions. It is also the case that no feasible solution could dominate all other feasible solutions; otherwise there would be no need to conduct multi-objective optimization. Therefore, we need to find solutions that are not dominated by any other solutions. Such solutions are called *efficient* or *Pareto-optimal* solutions. Their counterparts in the objective space form the efficient or Pareto-optimal front that is the trade-off among objectives. Efficiency results from the conflicts among objectives, which are inherent in multi-objective optimization and can be formally defined as follows. A solution $\dot{\mathbf{x}}_t$ is called a efficient solution of the bi-objective inventory problem if there does not exist any solution $\dot{\mathbf{x}}$, $(\dot{\mathbf{x}} \neq \dot{\mathbf{x}}_t)$ so that $C(\dot{\mathbf{x}}) \leq C(\dot{\mathbf{x}}_t)$ and $S(\dot{\mathbf{x}}) \leq S(\dot{\mathbf{x}}_t)$ and at least one of above inequality is strictly held. The condition of efficient solution is rather strict and many multi-objective optimization algorithms cannot guarantee to generate efficient solutions but only weakly efficient solutions. A solution $\dot{\mathbf{x}}_t$ is called a weakly efficient solution of the bi-objective inventory problem if there does not exist any solution $\dot{\mathbf{x}}$, $(\dot{\mathbf{x}} \neq \dot{\mathbf{x}}_t)$ so that $C(\dot{\mathbf{x}}) < C(\dot{\mathbf{x}}_t)$ and $S(\dot{\mathbf{x}}) < S(\dot{\mathbf{x}}_t)$.

The geometric interpretation of efficiency can be shown as in Figure 1 (Liu et al., 2003). The weakly efficient solutions include $\dot{\mathbf{x}}_1$, $\dot{\mathbf{x}}_2$, $\dot{\mathbf{x}}_3$, $\dot{\mathbf{x}}_5$, $\dot{\mathbf{x}}_7$, $\dot{\mathbf{x}}_8$ and $\dot{\mathbf{x}}_9$, among which $\dot{\mathbf{x}}_2$ and $\dot{\mathbf{x}}_8$ are not efficient and they are weakly dominated by $\dot{\mathbf{x}}_1$ and $\dot{\mathbf{x}}_9$, respectively. Finally, solutions $\dot{\mathbf{x}}_4$ and $\dot{\mathbf{x}}_6$ are strongly dominated by $\dot{\mathbf{x}}_5$ and $\dot{\mathbf{x}}_7$, respectively.

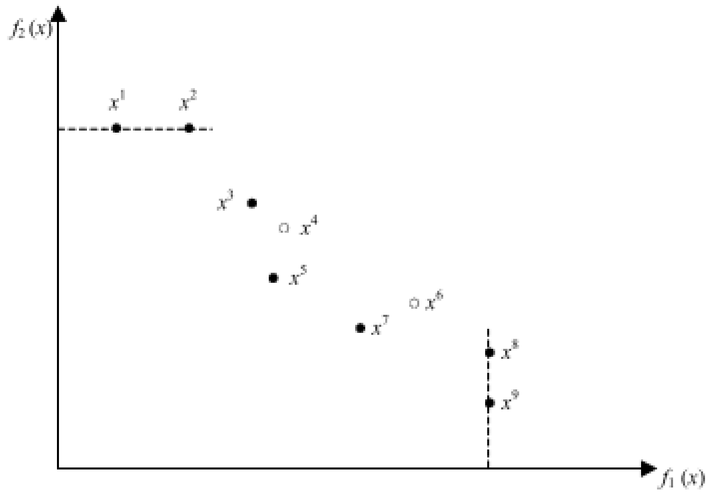

Figure 1. Efficient and weakly efficient solutions.

The MOPSO algorithm is based on the concept of efficiency described earlier (Coello-Coello and Lechuga, 2002). Particle swarm optimization (PSO), which the MOPSO derived from, is a population-based search algorithm for optimization problems. Because PSO has excellent exploration capability, incorporation of local search into MOPSO is essential when the exploitative capability should be emphasized; especially there are many efficient solutions for multi-objective problems. The local search mechanism not only can fully exploit a potential good solution, but also may prevent MOPSO from premature convergence. Hence combining local search and MOPSO will hopefully improve the chance of finding better trade-offs in inventory cost and customer service. This kind of synergy between exploitation and exploration is intended to enhance the proximity of efficient set to the true Pareto-optimal front. The pseudo code of the proposed approach is shown in Table 1.

Table 1. The pseudo code of proposed approach.

```
Randomly initialize the position and velocity of each particle
Evaluate objective vector and initialize individual bests
Generate efficient archive
For t = 1 to T (maximum number of iterations)
  For i = 1 to N (number of particles)
    Randomly select a global best from the archive
    Fly the particle according to individual best and global best
End for
    Evaluate objective vector and update individual best
    Update efficient archive
Apply local search on efficient archive and update the archive
End for
```

4. Computational Results

Inventory data from pharmaceutical industry: $D=$ 3,412, σ_L =53.354, $A=$ 80, $c=$ 27.5, and $h=0.26$ are solved by MOPSO with local search. Although the hybrid MOPSO is slower than the MOPSO, it surpasses the MOPSO in light of the number of efficient policies, the accuracy of the efficient set, and the spread of the solutions. Table 2 contains thirty efficient (k,Q) policies generated from the MOPSO with local search. Plenty of solutions offered by a single run of the multi-objective approach can help decision makers develop management policies under a changing environment.

The trade-off between expected cost and service level is shown in Figure 2. To outline a policy for inventory management, a regression curve is fitted for the cost and service trade-off of the pharmaceutical data. The non-parametric curve on the plot is drawn by a local regression smoother. It works by fitting a least squares line in the neighborhood of each solution, placing greater weight on points closer to the focal solution. This curve demonstrates how capital investment in inventory management can be traded for customer service. The coordinates shown on the top-right corner are the seventeenth and twenty-sixth solutions in Table 2. Points below the curve are infeasible and decisions located above the curve are suboptimal. They can be improved by moving back to the curve.

Table 2. Efficient solutions produced by the MOPSO with local search.

Sol. No.	*Q*	*k*	*C*	*S*	*Service Level*
1	294.05	1.55	2725	0.7044	0.9394
2	304.15	1.55	2726	0.6810	0.9394
3	300.25	1.98	2795	0.2706	0.9761
4	298.58	1.47	2722	0.8106	0.9292
5	300.25	1.98	2795	0.2706	0.9761
6	303.58	1.95	2789	0.2873	0.9744
7	291.75	1.69	2738	0.5332	0.9545
8	310.97	1.49	2724	0.7490	0.9319
9	323.67	2.43	2945	0.0778	0.9925
10	312.76	1.68	2741	0.5080	0.9535
11	303.58	2.01	2804	0.2491	0.9778
12	276.49	2.14	2835	0.1984	0.9838
13	862.19	4.44	5093	0.0000	1.0000
14	300.7	1.75	2748	0.4552	0.9599
15	300.25	1.98	2795	0.2706	0.9761
16	285.12	2.28	2876	0.1336	0.9887
17	419.32	2.85	3242	0.0165	0.9978
18	329.07	1.81	2774	0.3648	0.9649
19	321.22	2.25	2884	0.1284	0.9878
20	290.47	1.63	2732	0.6068	0.9484
21	284.06	2.25	2866	0.1452	0.9878
22	310.26	2.2	2861	0.1516	0.9861
23	313.06	1.89	2780	0.3202	0.9706
24	287.39	1.86	2766	0.3734	0.9686
25	300.25	1.98	2795	0.2706	0.9761
26	302.76	3.68	3389	0.0009	0.9999
27	295.07	1.84	2763	0.3805	0.9671
28	261.33	2.74	3034	0.0379	0.9969
29	300.25	1.98	2795	0.2706	0.9761
30	322.64	1.84	2776	0.3480	0.9671

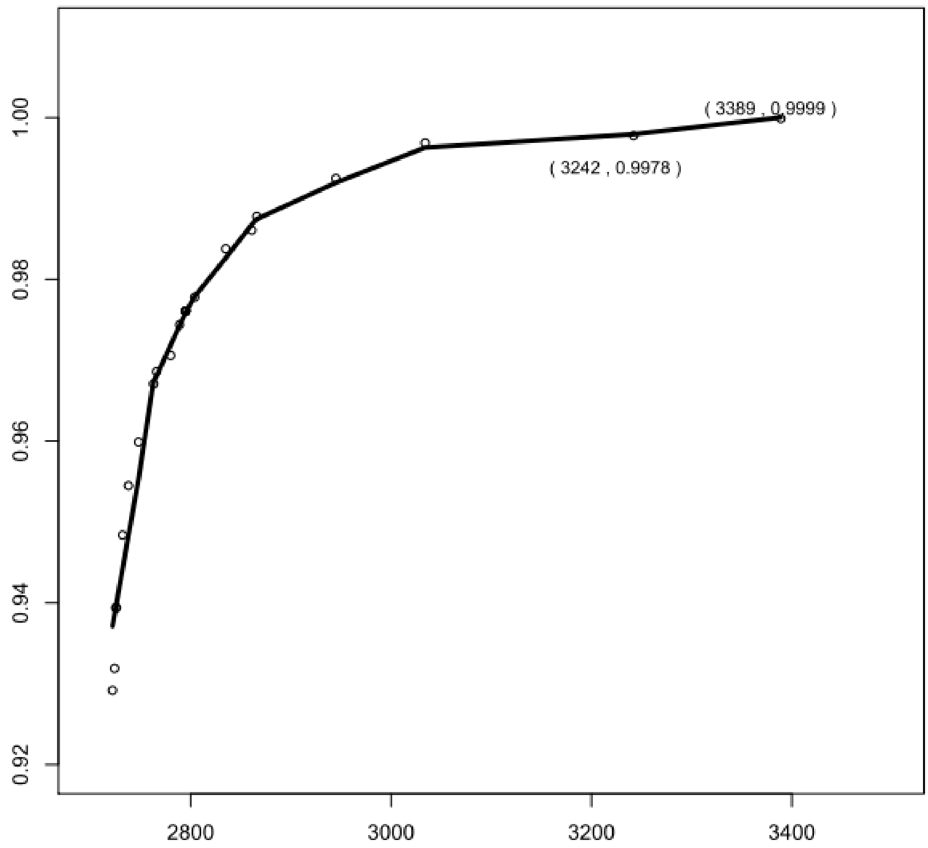

Figure 2. Exchange curve for the trade-off between expected cost (*x*-axis) and service level (*y*-axis).

5. Conclusions

Inventory management is usually formulated as single objective optimization models. To build such models, managers have to estimate shortage cost in order to aggregate conflicting and incommensurate objectives into a single one. Not only the difficulty in estimation has been generally acknowledged, but also the trade-off in inventory management targets should be explicitly put in a multi-objective formulation. Nevertheless, an appropriate model (i.e. without redundant objectives) when the trade-off multi-objectively formulated is hardly found in literatures. For a probabilistic inventory system, to strike a balance between operating the management system least costly or taking risk on possible shortage is an important trade-off to be addressed. Traditional approach, well known as the exchange curve, usually explores the trade-off of cost and service from a single objective formulation. And it was mostly solved by successive approximation based on Lagrange method that is lengthy and frustrating.

This paper presented a model without redundancy in the objectives, which represents a fixed order system under lost sales. A solver based on MOPSO was utilized to find the inventory management policies. It employed a local search to fully exploit a potentially good solution. Such hybridization could strengthen the possibilities of particles in MOPSO for flying towards the Pareto-optimal front and generate a well-distributed trade-off set.

The way of multi-objective analysis has been shown that it can find the whole picture of efficient order size and safety factor, not only simultaneously but also in a single run. A lot of solutions generated by the multi-objective approach facilitate decision makers develop management policies under a changing environment. Besides that, an estimated exchange curve is useful in determining the possibly best customer service under given investment in inventory management. Therefore, it indeed answers the need of practical inventory management. Finally, extending the analysis to the case of supply chain would be valuable because inventory management is a critical issue under such business environment.

References

[1] Alscher J, Schneider H (1982). Resolving a multi-item inventory problem with unknown costs. Engineering Costs and Production Economics 6: 9-15.

[2] Agrell PJ (1995). A multi-criteria framework for inventory control. Int. J. Prod. Econ. 41: 59-70

[3] Bhattacharya R, Bandyopodhyay S (2010). Solving conflicting bi-objective facility location problem by NSGA II evolutionary algorithm. Int. J. Adv. Manuf. Technol. In press. (DOI 10.1007/S00170-010-2262-6)

[4] Bookbinder JH, Chen Vincent YX (1992). Multi criteria trade-offs in a warehouse/retailer system. J. Opl. Res. Soc. 43: 707-720.

[5] Brown RG (1961). Use of the carrying charge to control cycle stocks. APICS Q. Bull. 2: 29-46.

[6] Brown RG (1967). Decision Rules for Inventory Management. Holt, Rinehart and Winston, New York.

[7] Chiam SC, Tan KC, Al Mamum A (2008). Evolutionary multi objective portfolio optimization in practical context. Int. J. of Autom. and Comput. 5(1): 67-80.

[8] Coello-Coello CA, Lechuga MS (2002). MOPSO: a proposal for multiple objective particle swarm optimization. *Proceedings of the 2002 IEEE Congress on Evolutionary Computation*, 12-17.

[9] Deb K (2001). Multi-Objective Optimization using Evolutionary Algorithms. John Wiley & Sons: New York.

[10] Gardner ES, Dannenbring DG (1979). Using optimal policy surfaces to analyze aggregate inventory tradeoffs. Manage. Sci. 25: 709-720.

[11] Liu GP, Yang JB, Whidborne JF (2003). Multi objective Optimization and Control. Research Studies Press, Hertfordshire.

[12] Puerto J, Gutiérrez J, Sicilia J (2002). Bi-criteria trade-off analysis in a two-echelon inventory/distribution system. J. Oper. Res. Soc. 53: 468-469.

[13] Rabbani M, Bajestani MA, Khoshkhou GB (2010). A multi-objective particle swarm optimization for project selection problem. Expert Syst. Appl. 37(1): 315-321.

[14] Sliver EA, Pyke DF, Peterson R (1998). Inventory Management and Production Planning and Scheduling. John Wiley & Sons, New York.

[15] Star MK, Miller DW (1962). Inventory Control: Theory and Practice. Prentice-Hall, New Jersey.

[16] Tsou C-S, Fang H-H, Chang, H-H, Kao C-H (2006). An improved particle swarm Pareto optimizer with local search and clustering. *Sixth International Conference on Simulated Evolution And Learning (SEAL 2006),* volume 4247 of *Lecture Notes in Computer Science*, 15-18 October 2006, Hefei, China, pp. 400-407, Spinger-Verlag.

[17] Tsou C-S (2008). Multi-objective inventory planning using MOPSO and TOPSIS. Expert Syst. Appl. 35(1-2): 136-142.

[18] Tsou C-S, Kao C-H (2008). Multi-objective inventory control using electromagnetism-like meta-heuristic. Int. J. Prod. Res. 46(14): 3859-3874.

[19] Tsou C-S (2009). Evolutionary Pareto optimizers for continuous review stochastic inventory systems. Eur. J. Oper. Res. 195(2): 364-371.

[20] Zitzler E, Thiele L (1999). Multi-objective evolutionary algorithms: a comparative case study and the strength Pareto approach. IEEE Trans. Evol. Comput. 3(4): 257-271.