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Cordial Labeling in Context of Barycentric Subdivision of Special Graphs

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Abstract

In this paper we discuss cordial labeling in context of barycentric subdivision of shell graph, complete bipartite graph $K_{n,n}$ and wheel graph.

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1. Introduction

We begin with simple, finite, undirected graph $G = (V, E)$. In this paper P_n denotes path with n vertices and C_n denotes cycle with n vertices. For all other terminology and notations we follow Harary [8].

Definition 1.1. A *shell* S_n is the graph obtained by taking $n - 3$ concurrent chords in a cycle C_n. The vertex at which all the chords are concurrent is called the *apex* vertex.

Definition 1.2. A graph $G = (V, E)$ is said to be *bipartite graph* if the vertex set can be partitioned into two subsets V_1 and V_2 such that for every edge $e_i = v_i v_j \in E$, $v_i \in V_1$ and $v_j \in V_2$.

Definition 1.3. A *complete bipartite graph* is a simple bipartite graph such that two vertices are adjacent if and only if they are in different partite sets. If partite sets are having m and n vertices then the related complete bipartite graph is denoted by $K_{m,n}$.

Definition 1.4. Let G and H be two graphs such that $V(G)\bigcap V(H) = \phi$. Then join of G and H is denoted by $G+H$. It is the graph with $V(G+H) = V(G)\bigcup V(H)$, $E(G+H) = E(G)\bigcup E(H)\bigcup J$, where $J = \{uv|u \in V(G), v \in V(H)\}$.

Definition 1.5. A *wheel* W_n is join of the graphs C_n and K_1. i.e. $W_n = C_n + K_1$. Here vertices corresponding to C_n are called *rim vertices* and C_n is called *rim* of W_n while the vertex corresponding to K_1 is called *apex* vertex.

Definition 1.6. If the vertices of the graph are assigned values subject to certain conditions is known as *graph labeling*.

A dynamic survey of graph labeling is published and updated every year by Gallian [5]. The reference cited here is the updated survey of 2012.

Definition 1.7. A function $f : V(G) \to \{0,1\}$ is called a *binary vertex labeling* of a graph G and $f(v)$ is called *label of the vertex* v of G under f.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0,1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$, $v_f(1)$ denote the number of vertices of G with labels $0,1$ respectively under f and let $e_f(0)$, $e_f(1)$ denote the number of edges of G with labels $0,1$ respectively under f^*.

Definition 1.8. A binary vertex labeling of a graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *cordial* if it admits cordial labeling.

The concept of cordial graphs was introduced by Cahit [3]. Cahit [4] proved that complete bipartite graphs $K_{m,n}$ are cordial for all m and n and wheel graph W_n is cordial if and only if $n \equiv 3(mod 4)$. Vaidya et al.[11], proved that star of petersen graph, the graph obtained by joining two copies of petersen graph by a path of arbitrary length and the graph obtained by joining two copies of wheel graph by a path of arbitrary length are cordial graphs. Andar et al. [1], [2] proved that helms, closed helms, flowers, multiple shells are cordial.

In this paper we prove that barycentric subdivision of shell graph, complete bipartite graph, wheel graph are cordial graphs.

2. Main Results

Theorem 2.1. The barycentric subdivision of shell graph S_n is cordial for all n.

Proof, Let G be the barycentric subdivision of shell graph S_n. Let $v_1, v_2, \ldots, v_{2n-1}$ be external vertices of G and $v_1^{'}, v_2^{'}, v_3^{'}, \ldots, v_{n-3}^{'}$ be the internal vertices in G. Here the vertices $v_2, v_4, \ldots v_{2n-2}$ and $v_1^{'}, v_2^{'}, v_3^{'}, \ldots, v_{n-3}^{'}$ are formed by barycentric subdivision of shell graph S_n, where $v_i^{'}$ is the vertex adjacent to apex vertex v_0 and the vertex $v_{2(i+1)}, i = 1, 2, 3, \ldots, n-3$. We define labeling function $f : V(G) \rightarrow \{0, 1\}$ as follows.

Case 1: $n = 4$
$f(v_0) = 0,$
$f(v_i) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_i^{'}) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n-3$
Case 2: $n \equiv 0(mod4)$
$f(v_0) = 0,$
$f(v_i) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_1^{'}) = 1,$
$f(v_i^{'}) = 0;$ if $i \equiv 1, 2(mod4)$
$= 1;$ if $i \equiv 0, 3(mod4), 2 \leq i \leq n-3$
Case 3: $n \equiv 1(mod4)$
$f(v_i) = 0;$ if $i \equiv 1, 2(mod4)$
$= 1;$ if $i \equiv 0, 3(mod4), 1 \leq i \leq n$
$f(v_i^{'}) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n-3$
Case 4: $n \equiv 2(mod4)$
$f(v_0) = 1,$
$f(v_i) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_i^{'}) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n-3$
Case 5: $n \equiv 3(mod4)$
$f(v_0) = 0,$
$f(v_i) = 0;$ if $i \equiv 2, 3(mod4)$
$= 1;$ if $i \equiv 0, 1(mod4), 1 \leq i \leq n$
$f(v_1^{'}) = 1,$
$f(v_i^{'}) = 0;$ if $i \equiv 1, 2(mod4)$
$= 1;$ if $i \equiv 0, 3(mod4), 2 \leq i \leq n-3.$ ■

The labeling pattern defined above satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in *Table 1*. Hence the graph under consideration is cordial graph.
Let $n = 4a + b$, where $n \in N$.

Illustration 2.1 Cordial labeling of the graph obtained by barycentric subdivision of

Table 1: Table for Theorem 2.1

b	vertex conditions	edge conditions
0	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$

shell graph S_5 is shown in *Figure 1* as an illustration for the *Theorem 2.1*.

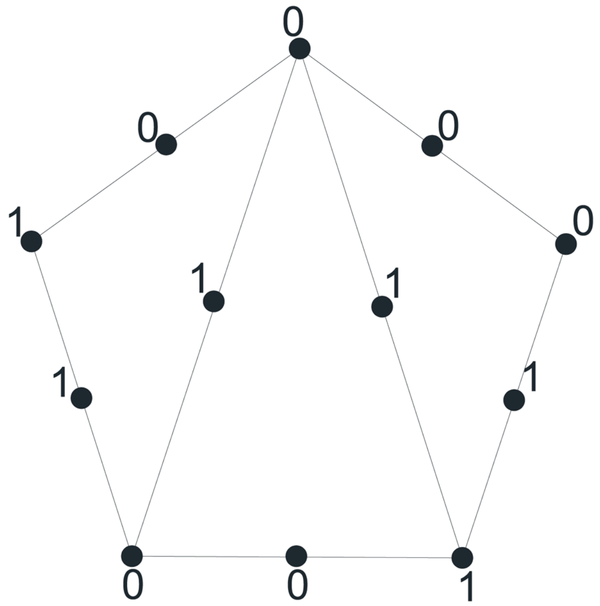

Figure 1: Cordial labeling of barycentric subdivision of shell graph S_5

Theorem 2.2. The barycentric subdivision of complete bipartite graph $K_{n,n}$ is cordial for all n.

Proof. Let G be the barycentric subdivision of complete bipartite graph $K_{n,n}$. Let $V = V_1 \bigcup V_2$ be the bipartition of vertex set V complete bipartite graph $K_{n,n}$. Let $\{v_i | i = 1, 2, \dots, n\}$ denote the vertices of V_1 and let $\{v_j | j = 1, 2, \dots, n\}$ denote the vertices of V_2.

Let $\{v_{ij}/i = 1, 2, \dots, n, j = 1, 2, \dots, n\}$ be the vertices formed by barycentric subdivision of $K_{n,n}$ where v_{ij} is the vertex adjacent to v_i and v_j, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, n$. We define labeling function $f : V(G) \to \{0, 1\}$ as follows.

Case 1: $n \equiv 0(mod4)$
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$

$= 1$; if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4), 1 \leq j \leq n$
For $i \equiv 0, 1, 2, 3(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4), 1 \leq j \leq n$
Case 2: $n \equiv 1(mod4)$
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4), 1 \leq j \leq n$
For $i \equiv 1(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 1 \leq j \leq n$
For $i \equiv 0, 2(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4), 1 \leq j \leq n$
For $i \equiv 3(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4), 1 \leq j \leq n$
Case 3: $n \equiv 2(mod4)$
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4), 1 \leq j \leq n$
For $i \equiv 1, 3(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 1 \leq j \leq n$
For $i \equiv 0, 2(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4), 1 \leq j \leq n$
Case 4: $n \equiv 3(mod4)$
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4), 1 \leq j \leq n$
For $i \equiv 0(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $j \equiv 1, 2(mod4), 1 \leq j \leq n$
For $i \equiv 1(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4), 1 \leq j \leq n$
For $i \equiv 2(mod4)$

$f(v_{ij}) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4), 1 \leq j \leq n$
For $i \equiv 3(mod4)$
$f(v_{ij}) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4), 1 \leq j \leq n$. ■

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case as shown in *Table 2*. Hence the graph under consideration is cordial graph.

Let $n = 4a + b, k = 4c + d$, where $n, k \in N$.

Table 2: Table for Theorem 2.2

b	vertex conditions	edge conditions
0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$

Illustration 2.2 Cordial labeling for the graph obtained by barycentric subdivision of graph $K_{3,3}$ is shown in *Figure 2* as an illustration for the *Theorem 2.2*. It is the case related to $n \equiv 3(mod4)$.

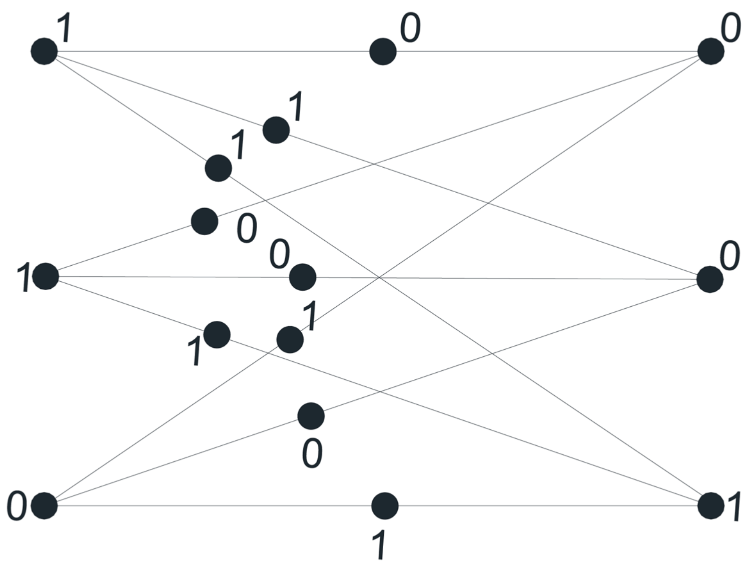

Figure 2: Cordial labeling of barycentric subdivision of complete bipartite graph $K_{3,3}$

Theorem 2.3. The barycentric subdivision of wheel graph W_n is cordial for all n.

Proof. Let G be the barycentric subdivision of wheel W_n. Let $v_1, v_2, \ldots, v_{2n}$ be rim vertices of G. Let $v_1^{'}, v_2^{'}, \ldots, v_n^{'}$ be internal vertices of G. Let v_0 be the apex vertex of G. To define labeling function $f : V \rightarrow \{0, 1\}$ we consider the following cases.

Case 1: $n \equiv 0(mod4)$
$f(v_0) = 0,$
$f(v_i) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
$f(v_i^{'}) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n$
Case 2: $n \equiv 1(mod4)$
$f(v_0) = 1, f(v_2) = 0,$
$f(v_i) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 1 \leq i \leq n, i \neq 2$
$f(v_i^{'}) = 0;$ if $i \equiv 1, 2(mod4)$
$= 1;$ if $i \equiv 0, 3(mod4), 1 \leq i \leq n$
Case 3: $n \equiv 2(mod4)$
$f(v_0) = 1, f(v_1) = 0$
$f(v_i) = 0;$ if $i \equiv 0, 3(mod4)$
$= 1;$ if $i \equiv 1, 2(mod4), 2 \leq i \leq n$
$f(v_i^{'}) = 0;$ if $i \equiv 2, 3(mod4)$
$= 1;$ if $i \equiv 0, 1(mod4), 1 \leq i \leq n$
Case 4: $n \equiv 3(mod4)$
$f(v_0) = 0, f(v_1^{'}) = 1, f(v_2^{'}) = 0$
$f(v_i) = 0;$ if $i \equiv 2, 3(mod4)$
$= 1;$ if $i \equiv 0, 1(mod4), 1 \leq i \leq n$
$f(v_i^{'}) = 0;$ if $i \equiv 0, 1(mod4)$
$= 1;$ if $i \equiv 2, 3(mod4), 3 \leq i \leq n.$ ■

The labeling pattern defined in above cases satisfies the conditions of cordial labeling which is shown in *Table 3*. Hence the graph under consideration is cordial graph.

Let $n = 4a + b, k = 4c + d$, where $n, k \in N$.

Table 3: Table for Theorem 2.3

b	vertex conditions	edge conditions
0,2	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
1,3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$

Illustration 2.3 Cordial labeling for the graph obtained by barycentric subdivision of W_6 is shown in *Figure 3* as an illustration for *Theorem 2.3*. It is the case related to $n \equiv 2(mod4)$.

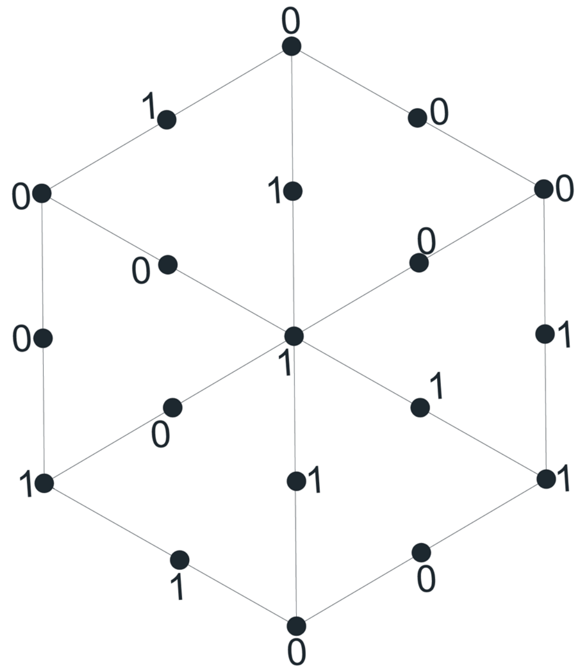

Figure 3: Cordial labeling of barycentric subdivision of wheel graph W_6

3. Conclusion

The cordial labeling for wheel graph, complete bipartite graph and shell graph is already discussed in [2] and [4]. We derived results on cordial labeling of barycentric subdivision of these graphs.

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