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Conformal Randers Change of a Finsler Space with (α, β) Metric of Douglas Type

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Abstract

A change of Finsler metric $L(\alpha, \beta) \rightarrow \bar{L}\ (\alpha, \beta) = e^{\sigma(x)}\ \{ L(\alpha, \beta) + \beta \}$ where σ is a function of position x^i only, α is Riemannian fundamental function and β a differentiable one-form is called conformal Randers change. This change is generalization of conformal change as well as Randers change. The purpose of the present paper is devoted to studying for Finsler space $\bar{F}^n = (M^n, \bar{L})$ which is obtained by Conformal Randers change of Finsler spaces $F^n = (M^n, L)$ of Douglas type, remains to be Douglas type and vice versa.

Key words: Douglas space, conformal change, Randers change, (α, β) metric

Mathematics subject classification: 2000: 53B 20, 53B 28, 53B 40, 53B 18.

Introduction

G.Randers ([6], [7],) in the year 1941 introduce a special metric ds = $\sqrt{a_{ij}(x)y^i y^j} + b_i(x)y^i$ in a view point of General theory of Relativity. Since then many Physicist had developed the General theory of Relativity. By this time Finsler spaces has already be coined. This metric was first recognized by as a kind of Finsler metric in 1957 by R.S.Ingarden and M.Matsumoto ([1], [4], [6],) produced the (α, β) – metric by generalizing Randers Metric [2]. The theory of Finsler space with (α, β) - metric has been developed into faithful branch of Finsler Geometry. From stand point of Finsler Geometry itself Randers metric is very interesting because its form of simple and properties of Finsler spaces equipped this metric can be looked as Riemannian spaces equipped with the metric L(α, β) = α + β. The conformal theory of Finsler metrics based on the theory of Finsler spaces by M. Matsomoto, M. Hashiguchi ([3], [7]) in 1976 studied the conformal change of a Finsler metric namely

$\bar{L}(x, y) = e^{\sigma(x)} L(x, y)$. The concept of Douglas space ([1], [8], [9], [10], [11]) has been introducing by M. Matsumoto and S. Bacso as a generalization of Berward spaces from stand point of view of geodesic equation. Finsler space is said to be of Douglas space if $D_{ij} = G_i y_j - G_j y_i$ are homogeneous polynomial in y^i of degree – 3. It has been shown by M. Matsomoto in papers ([1], [7], [8], [9], [10]) that $F^n = (M^n, L)$ is a Douglas type iff the Douglas tensor $D^h_{ijk} = G^h_{ijk} - \frac{1}{n-1}\left(G_{ijk} y^h + \delta^h_i G_{jk} + \delta^h_j G_{ik} + \delta^h_k G_{ij}\right)$ vanishes identically, where G^h_{ijk} is hv – curvature tensor of Berward connection $B\Gamma$. The Conformal Randers change can be consider as generalization of conformal as well as Randers change because writing $\beta = 0$ it reduces to Conformal change and when $\sigma(x) = 0$ it reduces to Randers change.It is compositions of Randers change and conformal change. In present paper we shall workout the condition under which a change of Finsler metric $L(\alpha, \beta) \rightarrow \bar{L}(\alpha, \beta) = e^{\sigma(x)} \{ L(\alpha, \beta) + \beta \}$ is that Conformal Randers change of Finsler spaces of Douglas type remains to be Douglas type.

2. Preliminaries.

Let $\alpha(x, y) = \sqrt{a_{ij}(x) y^i y^j}$ be Riemannian metric and $\beta(x, y) = b_i(x) y^i$ be a differentiable one-form in an n-dimensional differentiable manifold M^n. If a fundamental metric function $L(\alpha, \beta)$ is positively homogeneous of degree one in α and β in M^n, then $F^n = (M^n, L(\alpha, \beta))$ is called a Finsler space with (α, β)-metric [5]. The space $R^n = (M^n, \alpha)$ is called a Riemannian space associated with F^n [5], Christoffel symbols of R^n are indicated by $\gamma_j{}^i{}_k$, and covariant differentiation with respect to $\gamma_j{}^i{}_k(x)$ by ∇. We shall use the symbols as follows:

(2.1) $$r_{ij} = \frac{1}{2}(\nabla_j b_i + \nabla_i b_j),\quad s_{ij} = \frac{1}{2}(\nabla_j b_i - \nabla_i b_j),\quad s^i{}_j = a^{ir} s_{rj},\quad s_j = b_r s^r{}_j$$

It is to be noted that $s_{ij} = \frac{1}{2}(\partial_j b_i - \partial_i b_j)$. Throughout the paper the symbols ∂_j and $\dot{\partial}_j$ stand for $\frac{\partial}{\partial x^j}$ and $\frac{\partial}{\partial y^j}$ respectively. We are concerned with the Berwald connection $B\Gamma = (G^i_{jk}, G^i_j)$ which is given by $2G^i(x, y) = g^{ij}(y^r \dot{\partial}_j \partial_r F - \partial_j F)$, where $F = L^2/2$, $G^i_j = \dot{\partial}_j G^i$ and $G^i_{jk} = \dot{\partial}_k G^i_j$.

The Finsler space F^n is said to be of Douglas type (Douglas space) [1] if $D^{ij} = G^i(x, y) y^j - G^j(x, y) y^i$ are homogeneous polynomial in y^i of degree three. we shall denote the "homogeneous polynomials in y^i of degree r" by hp(r).

For a Finsler space F^n with (α, β)-metric ([3], [5]), we have

(2.2) $$2G^i = \gamma^i_{00} + 2B^i,$$

where

(2.3) $$B^i = \frac{E}{\alpha} y^i + \frac{\alpha L_\beta}{L_\alpha} s^i_0 - \frac{\alpha L_{\alpha\alpha}}{L_\alpha} C^* \left(\frac{y^i}{\alpha} - \frac{\alpha}{\beta} b^i \right),$$

$E = \dfrac{\beta L_\beta}{L} C^*$, $C^* = \dfrac{\alpha\beta(r_{00} L_\alpha - 2\alpha s_0 L_\beta)}{2(\beta^2 L_\alpha + \alpha\gamma^2 L_{\alpha\alpha})}$, $b^i = a^{ij} b_j$,

$\gamma^2 = b^2 \alpha^2 - \beta^2$, $b^2 = a^{ij} b_i b_j$

and the subscript α and β in L denote the partial differentiation with respect to α and β respectively. Since $\gamma^i_{00} = \gamma^i_{jk}(x) y^j y^k$ is homogenous polynomial in (y^i) of degree two, we have

Proposition (2.1) [7]. A Finsler space with (α, β)-metric is a Douglas space if and only if $B^{ij} = B^i y^j - B^j y^i$ are hp(3). Equation (2.3) gives

(2.4) $$B^{ij} = \frac{\alpha L_\beta}{L_\alpha}(s^i_0 y^j - s^j_0 y^i) + \frac{\alpha^2 L_{\alpha\alpha}}{\beta L_\alpha} C^* (b^i y^j - b^j y^i)$$

LEMMA -2.1 [8] If $\alpha^2 \equiv 0$ (mod - β) that is $a_{ij}(x)\, y^i y^j$ contains $b_i(x) y^i$ as a factor then the dimension is two and $b^2 = 0$.In this case, we have $\delta = d_i(x) y^i$ satisfying $\alpha^2 = \beta\delta$ and $d_i(x) b^i = 2$.

3 - Conformal Randers change of Finsler spaces with (α, β)-metric of Douglas type

Let $F^n = (M^n, L)$ and $\bar{F}^n = (M^n, \bar{L}(\alpha, \beta) = e^\sigma [L(\alpha, \beta) + \beta])$ be two Finsler spaces on the same underlying manifold M^n. If we have a function $\sigma(x)$ in each coordinate neighborhoods of M^n such that

$\bar{L}(\alpha, \beta) = e^\sigma [L(\alpha, \beta) + \beta]$ then F^n is called conformably Randers to $\bar{F}^n$, and change $L \rightarrow \bar{L}$ of metric is called conformal Randers change of (α, β) metric. As to (α, β) metric $\bar{L}(\alpha, \beta) = e^\sigma [L(\alpha, \beta) + \beta] = L(\bar{\alpha}, \bar{\beta})$, by homogeneity. Therefore, a conformal Randers change of (α, β) metric is expressed as $(\alpha, \beta) \rightarrow (\bar{\alpha}, \bar{\beta})$ where $\bar{\alpha} = e^\sigma \alpha$, $\bar{\beta} = e^\sigma \beta$,. Therefore, we have $\bar{y}^i = y^i$, $\bar{y}_i = e^{2\sigma} y_i$, $\bar{a}_{ij} = e^{2\sigma} a_{ij}$, $\bar{b}_i = e^\sigma b_i$, $\bar{a}^{ij} = e^{-2\sigma} a^{ij}$, $\bar{b}^i = e^{-\sigma} b^i$ and $\bar{b}^2 = b^2$.

Proposition (3.1): In a Finsler spaces with (α, β) – metric the length b of b_i with respect to the Riemannian α is invariant under conformal Randers change.

The conformal Randers change $(\alpha, \beta) \rightarrow (\bar{\alpha}, \bar{\beta})$ gives rise to the conformal change of R^n: $\alpha \rightarrow \bar{\alpha} = e^\sigma \alpha$ and hence we get the conformal Randers change of Christoffel symbols γ^i_{jk} are same as conformal change of Christoffel symbols γ^i_{jk}. So it follows [1] as

(3.3) $$\bar{\gamma}^i_{jk} = \gamma^i_{jk} + \delta^i_j \sigma_k + \delta^i_k \sigma_j - \sigma^i a_{jk}$$

where $\sigma_j = \partial_j \sigma$ and $\sigma^i = a^{ij}\sigma_j$.

From (3.2) and (3.3) we have the following conformal Randers change

(3.4)(a) $$\bar{\nabla}_j \bar{b}_i = e^{\sigma}(\nabla_j b_i - b_j \sigma_i + \rho a_{ij})$$

(b) $$\bar{r}_{ij} = e^{\sigma}[r_{ij} - \frac{1}{2}(b_i \sigma_j + b_j \sigma_i) + \rho a_{ij}]$$

(c) $$\bar{s}_{ij} = e^{\sigma}[s_{ij} + \frac{1}{2}(b_i \sigma_j - b_j \sigma_i)]$$

(d) $$\bar{s}^i_j = e^{-\sigma}[s^i_j + \frac{1}{2}(b^i \sigma_j - b_j \sigma^i)]$$

(e) $$\bar{s}_j = s_j + \frac{1}{2}(b^2 \sigma_i - \rho b_i). \text{ where } \rho = b_r \sigma^r,$$

From (3.3) and (3.4) we can easily obtain the following:

(3.5)(a) $$\bar{\gamma}_0{}^i{}_0 = \gamma_0{}^i{}_0 + 2\sigma_0 y^i - \alpha^2 \sigma^i$$

(b) $$\bar{r}_{00} = e^{\sigma}(r_{00} + \rho\, \alpha^2 - \sigma_0 \beta)$$

(c) $$\bar{s}^i_0 = e^{-\sigma}[s^i_0 + \frac{1}{2}(b^i \sigma_0 - \beta \sigma^i)]$$

(d) $$\bar{s}_0 = s_0 + \frac{1}{2}(b^2 \sigma_0 - \rho \beta)$$

To find the conformal Randers change of B^{ij} given in (2.3), we first find the conformal Randers change of C* given in (2.3). Since $L(\alpha, \beta) = e^{\sigma}[L(\alpha, \beta) + \beta]$, we have

(3.6) $$\bar{L}_{\bar{\alpha}} = L_{\alpha}, \quad \bar{L}_{\bar{\alpha}\bar{\alpha}} = e^{-\sigma} L_{\alpha\alpha}, \quad \bar{L}_{\bar{\beta}} = L_{\beta}, \quad \bar{\gamma}^2 = e^{2\sigma} \gamma^2.$$

From (2.3), (3.4) and (3.5), we have

(3.7) $$\bar{C}^* = e^{\sigma}(C^* + D^*),$$

where

(3.8) $$D^* = \frac{\alpha\beta[(\rho\alpha^2 - \sigma_0\beta) - e^{\sigma}\{2s_0 + (b^2\sigma_0 - \rho\beta)(L_{\beta}e^{-\sigma} + 1)\}]}{2(\beta^2 L_{\alpha} + \gamma^2 \alpha L_{\alpha\alpha})}$$

Hence conformal Randers change of B^{ij} is written in the form

(3.9) $$\bar{B}^{ij} = B^{ij} + C^{ij},$$

where

(3.10) $$2C^{ij} = \frac{1}{2L_{\alpha}}[2\alpha e^{\sigma}(s^i{}_0 y^j - s^j{}_0 y^i) + (L_{\beta} + e^{\sigma})\alpha\{\sigma_0(b^i y^j - b^j y^i) - \beta(\sigma^i y^j - \sigma^j y^i)\}] + \frac{D^* \alpha^2 L_{\alpha\alpha}(b^i y^j - b^j y^i)}{\beta L_{\alpha}}$$

Theorem (3.1). A Douglas space with (α, β) –metric transformed to a Douglas space with (α, β) –metric under Conformal Randers change if and only If C^{ij} defined in equation (3.10) is hp(3).

4 - Conformal Randers change of Finsler spaces with some (α, β)-metric

For a Randers metric we have $L = \alpha+\beta$ so that $L_\alpha = 1$, $L_\beta = 1$ and $L_{\alpha\alpha} = 0$.Then we have

(4.1) $$2C^{ij} = \alpha[2e^\sigma(s^i{}_0 y^j - s^j{}_0 y^i) + (1+e^\sigma)\{\sigma_0(b^i y^j - b^j y^i) - \beta(\sigma^i y^j - \sigma^j y^i)\}].$$

We know that [6] Finsler spaces with Randers metric is Douglas space iff $s_{ij} = 0$. Under this condition equation (4.1) becomes $2C^{ij} = \alpha(1+e^\sigma)\{\sigma_0(b^i y^j - b^j y^i) - \beta(\sigma^i y^j - \sigma^j y^i)\}$. Since α is irrational function in y^i, from above it follows that C^{ij} is hp -3 iff

(4.2) $$\sigma_0(b^i y^j - b^j y^i) - \beta(\sigma^i y^j - \sigma^j y^i) = 0$$

The equation (4.2) may be written as

(4.3) $$(\sigma_k\delta^j_h + \sigma_h\delta^j_k)b^i - (b_k\delta^j_h + b_h\delta^j_k)\sigma^i - (\sigma_k\delta^i_h + \sigma_h\delta^i_k)b^j$$
$$+ (b_k\delta^i_h + b_h\delta^i_k)\sigma^j = 0$$

Contracting (4.3) by j and h we get $b_i\sigma_j - b_i\sigma_i = 0$ which gives $\sigma_i = \frac{\rho}{b^2}b_i$. Conversely if $S_{ij} = 0$ and $\sigma_i = \frac{\rho}{b^2}b_i$ then(4.1) gives $C^{ij} = 0$.Hence equation (3.9) gives $\bar{B}^{ij} = B^{ij}$. Thus we have

Theorem - (4.1): The Douglas space with Randers metric transformed to a Douglas space under Conformal Randers change if and only If $S_{ij} = 0$ and $\sigma_i = \frac{\rho}{b^2}b_i$, where $\rho = b_r\sigma^r$.

For a Kropina metric, we have $L = \frac{\alpha^2}{\beta}$, so that $L_\alpha = \frac{2\alpha}{\beta}$, $L_{\alpha\alpha} = \frac{2}{\beta}$, $L_\beta = -\frac{\alpha^2}{\beta^2}$. Hence the value of D^* given by (3.7) reduces to

(4.4) $$D^* = \frac{1}{4b^2\alpha^2}[\beta^2(\rho\alpha^2 - \sigma_0\beta) - \alpha e^\sigma\{2s_0\beta^2 + (b^2\sigma_0 - \rho\beta)(\beta^2 - \alpha^2 e^{-\sigma})\}]$$

Therefore the value of C^{ij} given by (2.10) reduces to

(4.5) $$C^{ij} = \frac{1}{2}e^\sigma\beta(s^i{}_0 y^j - s^j{}_0 y^i) + (\frac{e^\sigma\beta}{4} - \frac{\alpha^2}{4\beta})\{\sigma_0(b^i y^j - b^j y^i) - \beta(\sigma^i y^j - \sigma^j y^i)\}$$
$$-\ (b^i y^j - b^j y^i)[(\frac{\beta^2\rho}{2b^2} - \frac{\sigma_0\beta^3}{2b^2\alpha^2}) - \frac{e^\sigma\beta^2 s_0}{b^2\alpha} +\ (b^2\sigma_0 + \rho\beta)$$
$$(\frac{\beta^2\rho}{2b^2} - \frac{\sigma_0\beta^3}{2b^2\alpha^2})].$$

Since $\frac{1}{2}e^\sigma\beta(s^i{}_0 y^j - s^j{}_0 y^i) + \frac{e^\sigma\beta}{4}\{\sigma_0(b^i y^j - b^j y^i) - \beta(\sigma^i y^j - \sigma^j y^i)\} - (b^i y^j - b^j y^i)[\frac{\beta^2\rho}{2b^2} - \frac{\alpha}{2b^2}(b^2\sigma_0 + \rho\beta)$] are hp(3).These terms may be neglected in our discussion and we treat only of

$$H^{ij} = -\frac{\alpha^2}{4\beta}\sigma_0(b^i y^j - b^j y^i) - (b^i y^j - b^j y^i)[\frac{e^\sigma\beta^2}{2b^2\alpha}(b^2\sigma_0 + \sigma\beta) - \frac{\sigma_0\beta^3}{2b^2\alpha^2} - \frac{e^\sigma\beta^2 s_0}{b^2\alpha}]$$

Above equation may be written as

(4.6) $4b^2\alpha^2\beta H^{ij} = -(b^i y^j - b^j y^i)[\alpha^4 b^2\sigma_0 - 2\sigma_0\beta^4 - 4e^\sigma s_0\alpha\beta^3 + 2e^\sigma b^2\sigma_0\beta^3 - 2\rho e^\sigma\alpha\beta^4]$

Equating rational and irrational terms we get

(4.7) $4b^2\alpha^2\beta H^{ij} = -(b^i y^j - b^j y^i)(\alpha^4 b^2\sigma_0 - 2\sigma_0\beta^4)$ and

(4.8) $2e^\sigma\beta^3(b^i y^j - b^j y^i)(2s_0 + b^2\sigma_0 + \rho\beta) = 0.$

Take $n > 2$, $\alpha^2 \neq 0$ (mod - β) [8]. If $(b^i y^j - b^j y^i) = 0$, by transvection of $b_i y_j$ we get $b^2\alpha^2 - \beta^2 = 0$ which arise contradiction. So

(4.9) $(2s_0 + b^2\sigma_0 + \rho\beta) = 0$

Also from equation (4.7)

$b^2\alpha^2[4\beta H^{ij} + (b^i y^j - b^j y^i)\alpha^2\sigma_0] + 2\sigma_0\beta^4(b^i y^j - b^j y^i) = 0$

which implies $4\beta H^{ij} + (b^i y^j - b^j y^i)\alpha^2\sigma_0 = 0$ and $\sigma_0 = 0$

Therefore from equation (4.9) $2s_0 = -\rho\beta$.

Thus we have Theorem 4.2 – A Finsler spaces $\bar{F}^n$ (n > 2) which is obtained by conformal Randers change of a Kropina space F^n with $b^2 \neq 0$ is of Douglas type if and only if $\sigma_0 = 0$ and $2s_0 + \rho\beta = 0$, where $\rho = b_r\sigma^r$,.

For a Finsler spaces with metric

(4.10) $L = \alpha + \frac{\beta^2}{\alpha}$.

Under Randers change it become

(4.11) $L^* = \alpha + \beta + \frac{\beta^2}{\alpha}$.

The (α, β) –metric (4.11) is called an Approximate Matsumoto metric.

Lemma (4.1) [10] – A Finsler spaces with an Approximate Matsumoto metric is a Douglas spaces if and only if $\alpha^2 \neq 0$ (mod - β), $b^2 \neq 1$, $\Delta_j b_i = k\{(1+2b^2)a_{ij} - 3b_i b_j\}$ where $k = \frac{h}{b^2-1}$, $h(x)$ is scalar function, that is b_i is gradient vector. $\alpha^2 \equiv 0$ (mod - β): $n = 2$, $\Delta_j b_i = \frac{1}{2}\{v_i(d_j + 3b_j) + v_j(d_i + 3b_i)\}$ where $v_0 = v_i(x)y^i$.

Also conformal change of an Approximate Matsumoto metric is approximate Matsumoto metric. Take

(4.12) $A_{ij} = \Delta_j b_i - k\{(1+2b^2)a_{ij} - 3b_i b_j\} = 0$

Assume $(F^n, \bar{L} = e^\sigma(\alpha + \beta + \frac{\beta^2}{\alpha}))$ is Douglas Spaces.Then $\bar{A}_{ij} = 0$ This can be expressed as

(4.13) $e^{\sigma}(A_{ij} + \boldsymbol{\rho a}_{ij} - \boldsymbol{\sigma}_i b_j) = 0$

In view of equation (4.10), the equation (4.13) become $\boldsymbol{\rho a}_{ij} = \boldsymbol{\sigma}_i b_j$, contracting by y^j gives

(4.14) $\boldsymbol{\rho y}_i = \sigma_i \beta$

Again if n = 2, $\alpha^2 \equiv 0 \pmod{-\beta}$ assume

(4.15) $w_{ij} = \Delta_j b_i - \frac{1}{2}\{v_i(d_j + 3b_j) + v_j(d_i + 3b_i)\} = 0$

The $\bar{w}_{ij} = 0$ implies

(4.16) $e^{\sigma}(W_{ij} + \boldsymbol{\rho a}_{ij} - \boldsymbol{\sigma}_i b_j) = 0$, we note that $\bar{v}_i = e^{\sigma} v_i$.

In view of (4.14), the equation (4.16) becomes $(\rho a_{ij} - \sigma_i b_j) = 0$. After contacting it by y^j gives $\rho Y_i = \sigma_i \beta$. Thus in both cases we see that

Theorem (4.3): A Finsler space $\bar{F}^n$ (n > 2) which is obtained by conformal Randers change of $F^n = (M^n, L = \alpha + \frac{\beta^2}{\alpha})$ with $b^2 \neq 0$ is of Douglas type, remains to be douglas type if and only if $\rho Y_i = \sigma_i \beta$ where $\rho = b_r \sigma^r$.

Lemma [11] (4.2): Let F^n be a Dauglas space with (α, β) metric $L = (c_1\alpha + c_2\beta + \frac{\alpha^2}{\beta})$ for which $b^2 \neq 0$ and *if* $\alpha^2 \neq 0 \pmod{-\beta}$, then there exists a scalar function u(x) and a tensor function $V_{ij}(x)$ such that $\nabla_j b_i = (r_{ij} + S_{ij})$ is given by $S_{ij} = \frac{1}{b^2}(b_i S_j - b_j S_i) - \frac{1}{(n-1)} V_{ij}$ $r_{ij} = \frac{c_2}{2c_1}(b_i S_j + b_j S_i) - 4a_{ij}$.

For a Finsler spaces with metric $L = (\alpha + \frac{\alpha^2}{\beta})$

Under Randers change above metric becomes

(4.17) $L^* = (\alpha + \beta + \frac{\alpha^2}{\beta})$.

The conformal change of metric (4.17) is metric of same type. Take

(4.18) $A_{ij} = S_{ij} - \frac{1}{b^2}(b_i S_j - b_j S_i) + \frac{4}{(n-1)} V_{ij} = 0$ and

(4.19) $W_{ij} = r_{ij} - \frac{1}{2}(b_i S_j + b_j S_i) + 4a_{ij} = 0$.

Assume $(M^n, \bar{L} = e^{\sigma}(\alpha + \beta + \frac{\alpha^2}{\beta}))$ is Douglas type. Then $\bar{A}_{ij} = 0$ and $\bar{W}_{ij} = 0$.But $\bar{A}_{ij} = e^{\sigma} A_{ij}$, $\bar{V}_{ij} = e^{\sigma} V_{ij}$ so we get $\bar{A}_{ij} = 0$ if $A_{ij} = 0$. Also

$$\bar{W}_{ij} = W_{ij} + e^{\sigma}(\rho\, a_{ij} + \frac{1}{2} b_i b_j - \frac{2+b^2}{4} (b_i \sigma_j + b_j \sigma_i) \tag{4.19}$$

In view of (4.19), $\bar{W}_{ij} = 0$ implies

$$\rho\, a_{ij} + \frac{1}{2} b_i b_j = \frac{2+b^2}{4} (b_i \sigma_j + b_j \sigma_i) \tag{4.20}$$

Contracting by b^j we get $\rho\, b_i = \sigma_i b^2$. Thus we have

Theorem (4.4) A Finsler space $\bar{F}^n$ (n > 2) which is obtained by conformal Randers change of a $(M^n, L = \alpha + \frac{\alpha^2}{\beta})$ of Douglas type remains to be Douglas type if and only if $\rho\, b_i = \sigma_i b^2$ where $\rho = b_r \sigma^r$.

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