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Proving Random Fixed Point Theorem for Two Random Operators Using Random Mann Iteration Scheme

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ABSTRACT

In present paper, it is proved that if a random Mann iteration scheme defined by two operators is convergent under a rational inequality the limit point is a common fixed point of each of two random operators in a Banach space.

KEY WORDS: Mann iteration, fixed point, measurable mappings, Banach space.

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1. Introduction and preliminaries

In paper [8], Kasahara had shown that if an iterated sequence defined by using a continuous linear mapping is convergent under certain assumptions, then the limit point is a common fixed point of each of two non-linear mappings. Ganguly [6] arrived at the same conclusion under the same contractive definition by taking the sequence of Mann iterates [10]. In paper [9] R. Parsai, M.S. Rathore and R. S. Chandel have proved the random version of Ganguly's result. In present paper, we extend the result of R. Parsai, M.S. Rathore and R. S. Chandel for rational expression. The study of random fixed points has been an active area of contemporary research in Mathematics. Random iteration scheme has been elaborately discussed by Choudhury ([1], [2], [3], [4]). Looking to the immense applications of iterative algorithms in signal processing and image reconstruction, it is essential to venture upon random iteration.

We first review the following concepts, which are essential for our study in this paper.

Throughout this paper, $(\Omega, \sum)$ denotes a measurable space consisting of a set Ω and sigma algebra $\sum$ of subsets of Ω, X stands for a separable Banach space and C is a nonempty subset of X.

A mapping $f: \Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \sum$ for every Boral subset B of X.

A mapping $F: \Omega \times C \to C$ is said to be a random operator, if $F(.,x): \Omega \to C$ is measurable for every $x \in C$.

A measurable mapping $g : \Omega \to C$ is said to be a random fixed point of random operator F: $\Omega\times C \to C$, if $F\big(t, g(t)\big) = g(t)$ for all $\in \Omega$.

A random operator $F : \Omega \times C \to C$ is said to be continuous if, for fixed $t \in \Omega, F(t,.): C \to C$ is continuous.

Definition 1 (Random Mann iteration Scheme). Let $S, T: \Omega \times C \to C$ be two operators on a non-empty convex subset C of a separable Banach Space X. Then the sequence $\{x_n\}$ of random Mann iterates associated with S or T is defined as follows:

(1) Let $x_0 : \Omega \to C$ be any given measurable mapping.

(2) $x_{n+1}(t) = (1 - c_n)\, x_n(t) + c_n\, S(t, x_n(t))$ for $n > 0, t \in \Omega$

or

(3) $x_{n+1}(t) = (1 - c_n)\, x_n(t) + c_n\, T(t, x_n(t))$ for $n > 0, t \in \Omega$

where c_n satisfies:

(4) $c_0 = 1$ for $n = 0$,

(5) $0 < c_n \le 1$ for $n > 0$,

(6) $\lim_n c_n = h > 0$.

Since C is convex it follows from the above construction that x_n is a mapping from Ω to C for all n=0, 1, 2, ….

2. Main Result

Theorem 2.1. Let $S, T : \Omega \times C \to C$, where C is a nonempty closed convex subset of a separable Banach Space X, be two continuous random operators which satisfy the following inequality : for all $x, y \in C$ and $t \in \Omega$

(7) $$\|S(t,x) - T(t,y)\| \le \frac{a\,\|x-T(t,y)\|^2 + b\,\|x-S(t,x)\|^2}{\|x-T(t,y)\| + \|x-S(t,x)\|} + \frac{c\,\|y-T(t,y)\|^2 + d\,\|y-S(t,x)\|^2}{\|y-T(t,y)\| + \|y-S(t,x)\|}$$

where $a, b, c, d > 0, a + c < 1, b + d < 1$.

If the sequence $\{x_n\}$ of random Mann iterates associated with S or T satisfying (1)-(6) converges, then it converges to a common random fixed point of both S and T.

Proof. We may assume that the sequence $\{x_n\}$ defined by (2) is pointwise convergent, that is,

for all $t \in \Omega$,

(8) $x(t) = \lim_{n\to\infty} x_n(t)$

Since X is a separable Banach Space, for any continuous random operator $A : \Omega \times C \to C$ and any measurable mapping $f : \Omega \to C$, the mapping $x(t) = A\big(t, f(t)\big)$ is a measurable mapping [7].

Since $x(t)$ is measurable and C is convex, it follows that $\{x_n\}$ constructed in the random iteration from (2)-(6) is a sequence of measurable mappings. Hence, $x : \Omega \to C$ being limit of measurable mapping sequence is also measurable. For fixed $t \in \Omega$, from (2) it follows that

(9) $\| x(t) - T(t, x(t))\| \leq \| x(t) - x_{n+1}(t)\| + \| x_{n+1}(t) - T(t, x(t))\|$

$\leq \| x(t) - x_{n+1}(t)\| + (1-c_n) \| x_n(t) - T(t, x(t))\| + c_n \| S(t, x_n(t)) - T(t, x(t))\|$

$\leq \| x(t) - x_{n+1}(t)\| + (1-c_n) \| x_n(t) - T(t, x(t))\|$

$$+c_n \left\{ \frac{a\|x_n(t)-T(t,x(t))\|^2 + b\,\|x_n(t)-S(t,x_n(t))\|^2}{\|x_n(t)-T(t,x(t))\|+\|x_n(t)-S(t,x_n(t))\|} + \frac{c\,\|x(t)-T(t,x(t))\|^2 + d\,\|x(t)-S(t,x_n(t))\|^2}{\|x(t)-T(t,X(t))\|+\|x(t)-S(t,x_n(t))\|} \right\}$$

by equations (6) and (7).

Now,

$$c_n\left(S\big(t, x_n(t)\big) - x_n(t)\right) = c_n S\big(t, x_n(t)\big) - c_n x_n(t) = x_{n+1}(t) - x_n(t)$$

by (2), so that

$$\|S\big(t, x_n(t)\big) - x_n(t)\| \leq \frac{1}{c_n} \|x_{n+1}(t) - x_n(t)\|.$$

This shows that for $t \in \Omega, S\big(t, x_n(t)\big) - x_n(t) \to 0$ and so $S\big(t, x_n(t)\big) \to x(t)$ as $n \to \infty$, as S is a continuous random operator and x is a measurable mapping. Consequently from (9) on taking the limit as $n \to \infty$, we obtain

$\| x(t) - T(t, x(t))\| \leq (1-h)\| x(t) - T(t, x(t))\| + h(a+c)\|x(t) - T(t, x(t))\|$

$\Rightarrow h\,[1-(a+c)]\, \| x(t) - T(t, x(t))\| \leq 0$

$\Rightarrow T(t, x(t)) = x(t)$ (as $h > 0, a + c < 1$), for all $t \in \Omega$

As T is a continuous random operator and x is measurable. Therefore,

$\| S(t, x(t)) - x(t)\| = \| S(t, x(t)) - T(t, x(t))\|$

$$\leq \frac{a\,\|x(t)-T(t,x(t))\|^2 + b\,\|x(t)-S(t,x(t))\|^2}{\|x(t)-T(t,x(t))\|+\|x(t)-S(t,x(t))\|} + \frac{c\,\|x(t)-T(t,x(t))\|^2 + d\,\|x(t)-S(t,x(t))\|^2}{\|x(t)-T(t,x(t))\|+\|x(t)-S(t,x(t))\|}$$

$\Rightarrow \| S(t, x(t)) - x(t)\| \leq (b+d)\, \| x(t) - S(t, x(t))\|$

$\Rightarrow [1-(b+d)]\, \| x(t) - S(t, x(t))\| \leq 0$

Since $(b+d) < 1$, it follows that for all $t \in \Omega$, and x is measurable, $S(t, x(t)) = x(t)$.

This completes the proof of the theorem 2.1.

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