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The Wiener Related Indices of Some Graph Operations

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Abstract

The Wiener index of a connected graph G, denoted by $W(G)$, is defined as $\frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$. Similarly, hyper-Wiener index of a connected graph G, denoted by $WW(G)$, is defined as $\frac{1}{2}W(G)+\frac{1}{4}\sum_{u,v\in V(G)} d_G^2(u,v)$. In this paper, we present the explicit formulae for the Wiener, hyper-Wiener and reverse Wiener indices of some graph operations. Using the results obtained here, the exact formulae for Wiener, hyper-Wiener and reverse Wiener indices of some important classes of graphs are obtained.

Keywords: Wiener index, hyper-Wiener index, reverse Wiener index
MSC: 05C12, 05C76.

1. Introduction

For a vertex v of G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The maximum eccentricity is its diameter, denoted by $D(G)$. A topological index is a numerical quantity related to a graph that is invariant under graph automorphisms. The distance between the vertices u and v of G is denoted by $d_G(u,v)$ and it is defined as the number of edges in a minimal path connecting them. A topological index related to distance function $d(-,-)$ is called a"distance based topological index." In theoretical chemistry, molecular structure descriptors (also called topological indices) are used for modeling physicochemical, pharmacologic, toxicologic, biological and other properties of chemical compounds [2]. There exist several types of such indices, especially those based on vertex and edge distances. One of the most intensively studied topological indices is the Wiener index.

The Wiener index [14] is one of the oldest molecular-graph-based structure-descriptors [13]. Its chemical applications [11] and mathematical properties are well studied [1]. Let *G* be a connected graph. Then Wiener index of *G* is defined as

$$W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$$

with the summation going over all pairs of vertices of *G*. The hyper-Wiener index of acyclic graph was first introduced by Randic [12]. Then, as a generalization of the Wiener index, Klein *et al.* [7] generalized Randic's definition for all connected graphs. The hyper - Wiener index of a connected graph *G*, denoted by *WW*(*G*), is defined as

$$WW(G) = \frac{1}{2}W(G) + \frac{1}{4}\sum_{u,v\in V(G)} d_G^2(u,v),$$

where $d_G^2(u,v) = (d_G(u,v))^2$. Applications of the hyper-Wiener index as well as its calculation are well explained in [6, 8, 9].

The reverse Wiener index was proposed by Balaban *et al.* in 2000[15], it turns out that this index is important for a reverse problem and also found applications in modeling of structure-property relations [15, 16]. The reverse-Wiener index is defined as follows

$$\Lambda(G) = \frac{1}{2}n(n-1)D(G) - W(G),$$

where *n* is the number of vertices and *D*(*G*) is the diameter of *G*. Some mathematical properties of the reverse Wiener index may be found in [18, 19].

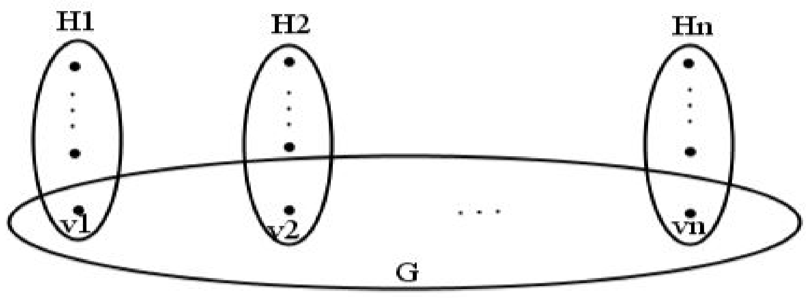


Fig. 1: Cluster of two graphs G and H with root vertex vi=h, where Hi is the copy of H, for i in {1,2,..,N}

The cluster *G*{*H*} of two graphs *G* and *H* is defined as the graph obtained by taking one copy of *G* and | *V*(*G*) | copies of a rooted graph *H* and by identifying the root of the *i*- th copy of *H* with *i* - th vertex of *G*, for *i* = 1. In this paper, we present the explicit formulae for the Wiener, hyper-Wiener and reverse Wiener indices of some graph operations. Using the results obtained here, the exact formulae for Wiener, hyper-Wiener and reverse Wiener indices of some important classes of graphs are obtained.

2. Wiener Index

In this section we compute the Wiener index of cluster of two graphs.

We denote the root vertex of *H* by h and the copy of H whose root is identified with the vertex $x \in V(G)\ by H^x$. The distance between *u,v* of $G\{H\}$is given by

$$d_{G\{H\}}(u,v) = \begin{cases} d_H(u,h) + d_G(x,y) + d_H(h,v), if\ u \in V(H^x), v\epsilon V(H^y)\ x \neq y \\ d_H(u,v), if\ u, v\epsilon V(H^x). \end{cases}$$

Theorem 2.1. Let *G* and *H* be graphs with n_1 and n_2 vertices and let the copies of *H* used in the construction of $G\{H\}$ be rooted in vertex h. Then $W(G\{H\})$=$n_2^2 W(G) + n_1 W(H) + n_1(n_1-1)n_2 D_H(h), where\ D_H(h) = \sum_{u\epsilon V(H)} d_H(u,h)$.

Proof.

$$W(G\{H\}) = \frac{1}{2}\sum_{u,v\epsilon V(G\{H\})} d_{G\{H\}}(u,v)$$

$$= \frac{1}{2}\Big(\sum_{x\epsilon V(G)}\sum_{u\epsilon v(H^x)}\sum_{v\epsilon V(H^x)} d_H(u,v)$$

$$+ \sum_{x\epsilon V(G)}\sum_{y\epsilon V(G)-x}\sum_{u\epsilon v(H^x)}\sum_{v\epsilon V(H^y)} (d_H(u,h) + d_G(x,y) + d_H(h,v))\Big)$$

$$= n_1 W(H) + \frac{n_1(n_1-1)}{2} n_2 D_H(h) + n_2^2 W(G) + \frac{n_1(n_1-1)}{2} D_H(h)$$

$$= n_2^2 W(G) + n_1 W(H) + n_1(n_1-1)n_2 D_H(h).$$

Using above theorem we have following corollary.

Corollary 2.2 Let *G* be graph of *n* vertex and let the copies of *G* used in the construction of $G\{G\}$ be rooted in vertex *h*. Then *W*(*G*{*G*})=*n*(*n*+1) *W*(*G*) +n^2(*n*-1) $D_G(h)$, where $D_G(h)$, = $\sum_{u\epsilon V(G)} d_G(u,h)$.

We quote the following lemma for our future reference.

Lemma 2.3. Let$P_n, C_n, S_n, and\ W_n$denote the path, cycle, star and wheel on n vertices respectively.

(1) For $n \geq 3, W(C_n) = \begin{cases} \frac{n^3}{8}, n\ is\ even \\ \frac{n(n^2-1)}{8}, n\ is\ odd. \end{cases}$

(2)For $n \geq 2, W(P_n) = \frac{n(n^2-1)}{6}$.

(3) For $n \geq 1, W(S_n) = (n-1)^2$
(4) For $n \geq 4, W(W_n) = (n-1)(n-2)$.

Consider the square comb lattice Cq(N) with open ends where N=n^2 is the number of vertices of this graph, see Fig. 2.This graph can be represented as the cluster graph $P_n\{P_n\}$, where the root of P_n is on its vertex of degree 1.

Now using Corollary 2.2 and Lemma 2.3, we obtain the exact Wiener index of the square comb lattice Cq(N).

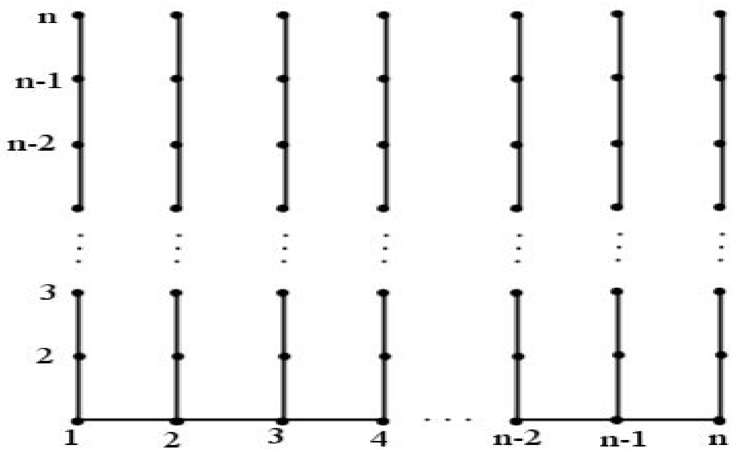
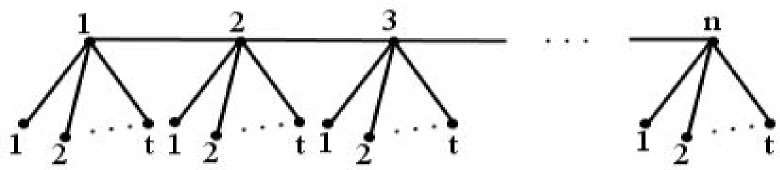


Fig. 2: The square comb lattice Cq (N)

Example 2.4. W(Cq(N)= $\frac{n^2(n-1)}{6}(4n^2 - n + 1)$.

For a given graph G, its t-fold bristled graph $Brs_t(G)$ is obtained by attaching t vertices of degree 1 to each vertex of G. This graph can be represented as the cluster of G and the star on t+1 vertices S_{t+1}, where the root of $S_{t+1}$is on its vertex of degree t. the t–fold bristled graph of a given graph is also known as its t- thorny graph. By using Theorem 3.1 and Lemma 3.3, we obtain the Wiener index of the t-fold bristled graph $Brs_t(G)$.

Example 2.5. $W(Brs_t(G)) = (1+t)^2 W(G) + nt\,(nt+n-1)$.



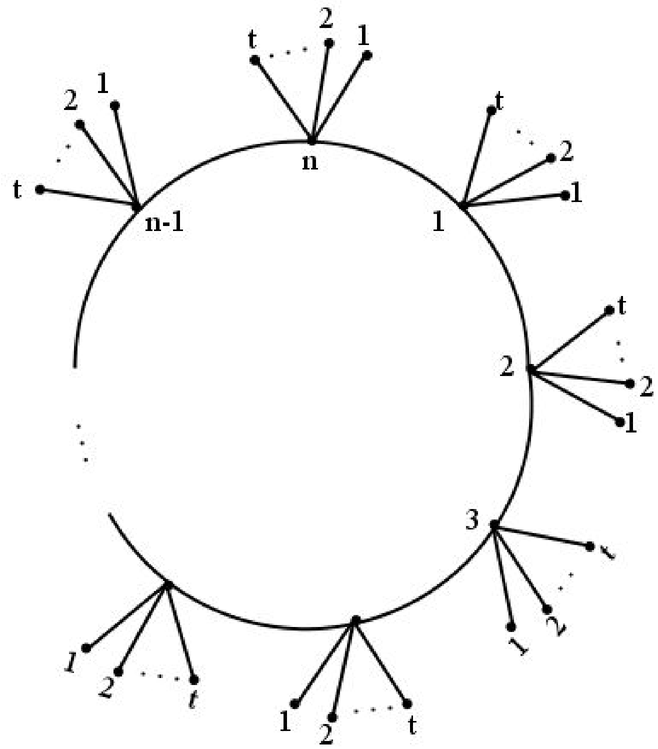


Fig. 3: The t-fold bristled graph of Path and Cycle on n vertices.

From the above formula and Lemma 2.3, the exact Wiener indices of the *t*-fold bristled graph of P_n, $and\ C_n$, (see Fig. 3) are computed.

Example 2.6.1. W(Brs$_t$(P_n)) = $\frac{n}{6}(1+t)^2(n^2-1)$+6t (nt+n-1).

2. W (Brs$_t$(C_n)) = $\begin{cases} \frac{n}{8}n^2(1+t)^2 + 8t(nt+n-1), if\ n\ is\ even \\ \frac{n}{8}(n^2-1)(1+t)^2 + 8t(nt+n-1), if\ n\ is\ odd. \end{cases}$

Let $\{G_i\}_{i=1}^n$ be a set of finite pairwise disjoint graphs with $v_i \in V(G_i)$. The bridge graph $B\{G_1, G_2, \ldots, G_n; v_1, v_2, \ldots, v_n\}\ of\ \{G_i\}_{i=1}^n$ with respect to the vertices $\{v_i\}_{i=1}^n$ is the graph obtained from the graphs $G_1, G_2, \ldots, G_n$ by connecting the vertices v_i and v_{i+1} by an edge for all i= 1, 2,…, n–1, see Fig. 4.

We define $G_n(H, v) = B\{\underbrace{H, H, \ldots H;}_{n\ times} \underbrace{v, v, \ldots, v}_{n\ times}\}$, (n times) which is the special case of bridge graph. For example, let $p_n$be the path on n vertices $v_1, v_2, \ldots, v_n$, define $B_n = G_n(P_3, v_2)$, see Fig.5 (Polyethene when n = 4). As another example, let C_k be the cycle with k vertices and define $T_n = G_n(C_k, v_1)$, see Fig. 6 (when k = 3 and n = 5). As a final example, define the bridge graph $J_{n,r} = G_n(W_r, v_1)$, where W_r is the

Wheel graph on r vertices $v_1, v_2, \ldots, v_r$, such that $\deg(v_1) = r - 1$ and $\deg(v_i) = 3, i = 1,2, \ldots, r$. By the definition of cluster, $B_n = P_n\{S_3\}, T_{n,3} = P_n\{K_3\}$and $J_{n,r} = P_n\{W_r\}$.

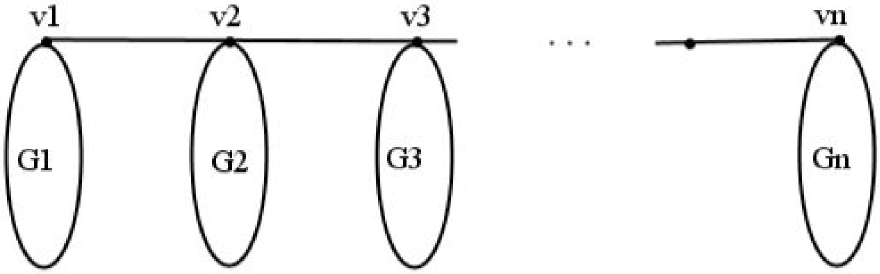


Fig.4 The bridge graph

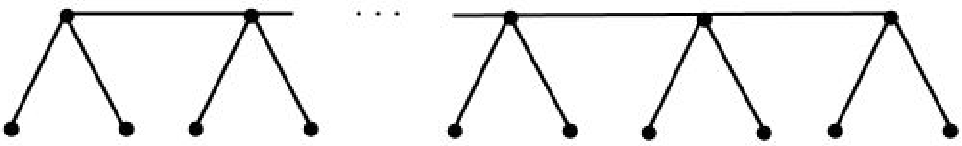


Fig.5 The graph Bn

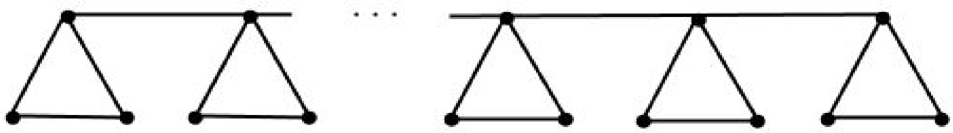


Fig.6 The graph T(n,3)

Example 2.7. Using Theorem 2.1, we obtain the Wiener indices of the following graphs.

1. $W(B_n) = \frac{n}{2}(3n^2 + 12n - 7)$.
2. $W(P_n\{K_r\}) = \frac{nr}{2}(rn^2 + 6nr - 6n - 4r + 3)$.
3. $W(T_{n,3}) = \frac{3n}{2}(n^2 + 4n - 3)$.
4. $W(J_{n,r}) = \frac{n}{6}(r^2n^2 + 4r^2 + nr^2 - nr - 17r - 12), r \geq 4$.

Let *G* be a connected graph. If we put two similar graphs *G* side by side, and any vertex of the first graph *G* is connected by edges with those vertices which are adjacent to the corresponding vertex of the second graph *G* and the resultant graph is denoted by $K_2 \bullet G$, then we have $V(K_2 \bullet G) = 2V(G)$ and $E(K_2 \bullet G) = 4E(G)$. Moreover, $K_2 \bullet G$ is the graph of K_2 and G with the vertex set $V(K_2 \bullet G) = V(K_2) \times V(G)$ and $(u_i, v_j)(u_k, v_r)$ is an edge of $K_2 \bullet G$ whenever $u_i = u_k$and $v_j v_k \in E(G) or\, u_i \neq u_k$ and $v_j v_k \in E(G)$.

Theorem 2.8. Let *G* be a connected graph with *n* vertices. Then $W(K_2 \bullet G) = 4W(G) + 2n$.

Proof. By the definition of the Wiener index, we have

$$W(K_2 \bullet G) =$$

$$\sum_{(u_i,v_j),(u_k,v_r)\in V(K_2\bullet G),(u_i,v_j)\neq(u_k,v_r)} d_{K_2\bullet G}\big((u_i, v_j), (u_k, v_r)\big)$$

$$= \sum_{(u_i,v_j),(u_k,v_r)\in V(K_2\bullet G),j\neq r} d_{K_2\bullet G}\big((u_i, v_j), (u_i, v_r)\big)$$

$$+ \sum_{(u_i,v_j),(u_k,v_r)\in V(K_2\bullet G),i\neq k} d_{K_2\bullet G}\big((u_i, v_j), (u_k, v_r)\big)$$

$$= \sum_{u_i\in V(K_2),v_j} \sum_{v_k\in V(G),j\neq r} d_G(v_j, v_r) + \sum_{v_j\in V(G)}\Big(2 + \sum_{v_r\in V(G),j\neq r} d_G(v_j, v_r)\Big)$$

$$= 2 \sum_{v_j v_k\in V(G),j\neq r} d_G(v_j, v_r) + \sum_{v_j\in V(G),} \sum_{v_r\in V(G),j\neq r} d_G(v_j, v_r) + 2n$$

$$= 4W(G) + 2n. \blacksquare$$

Let *G* be a connected graph. If we put two similar graphs *G* side by side, and any vertex of the first graph *G* is connected by edges with those vertices which are nonadjacent to the corresponding vertex (including the corresponding vertex itself) of the second graph *G* and the resultant graph is denoted by $K_2 \star G$, then we have $V(K_2 \star G) = 2V(G)$ and $E(K_2 \star G) = |V(G)|^2$. Moreover, $K_2 \star G$ is the graph of K_2 and *G* with the vertex set $V(K_2 \star G) = V(K_2) \times V(G)$ and $(u_i, v_j)(u_k, v_r)$ is an edge of $K_2 \star G$ $u_i$whenever $= u_k$and $v_j v_k \in E(G) or\, u_i \neq u_k$ and $v_j v_k \in E(G)$.

Theorem 2.9. Let *G* be a connected graph with *n* vertices. Then $W(K_2 \star G) = n(3n - 2)$.

Proof. By the definition of the Wiener index, we have

$$W(K_2 \star G) =$$

$$\sum_{(u_i,v_j),(u_k,v_r)\in V(K_2\star G),(u_i,v_j)\neq(u_k,v_r)} d_{K_2\star G}\left(\left(u_i,v_j\right),(u_k,v_r)\right)$$

$$= \frac{1}{2}\sum_{u_i\in V(K_2)}\sum_{v_j\in V(G)}\left(d_{K_2\star G}\left(u_i,v_j\right) + 2(2n - d_{K_2\star G}\left(u_i,v_j\right) - 1\right)$$

since for very vertex $\left(u_i,v_j\right) \in V(K_2 \star G)$, there are$d_G\left(v_j\right) + n - 1 - d_G\left(v_j\right) + 1 = n$ neighbors, and $d_G\left(v_j\right) + n - 1 - d_G\left(v_j\right) = n - 1$ vertices with the distance 2 from itself

$$= \frac{1}{2}\sum_{u_i\in V(K_2)}\sum_{v_j\in V(G)}(n + 2(n-1)) = n(3n-2).$$

3. Hyper–Wiener index

In this section, first we compute the hyper Wiener index of $G\{H\}$.

Theorem 3.1. Let G and H be graphs with n_1 and n_2 vertices and let the copies of H used in the construction of $G\{H\}$ be rooted in vertex h. Then $WW(G\{H\}) = n_2^2 WW(G) + n_1 WW(H) + 2n_2 W(G) D_H(h) + \frac{n_1(n_1-1)}{2}(n_2 D_H^2(h) + (D_H(h))^2)$,

Where

$$D_H(h) = \sum_{u\in V(H)} d_H(u,h) \ and\ D_H^2(h) = \sum_{u\in V(H)} (d_H(u,h) + d_H^2(u,h)).$$

Proof. $WW(G\{H\}) = \frac{1}{2}W(G) + \frac{1}{4}\sum_{u,v\epsilon V(G\{H\})} d_{G\{H\}}^2(u,v)$

$$= \frac{1}{2}\left(\sum_{x\epsilon V(G)}\sum_{u\epsilon v(H^x)}\sum_{v\epsilon V(H^x)}(d_H(u,v) + d_H^2(u,v))\right.$$

$$+ \sum_{x\epsilon V(G)}\sum_{y\epsilon V(G)-x}\sum_{u\epsilon v(H^x)}\sum_{v\epsilon V(H^y)}(d_H(u,h) + d_G(x,y) + d_H(h,v)$$

$$+ (d_H(u,h) + d_G(x,y) + d_H(h,v))^2$$

$$= \frac{n_1}{2}\sum_{u\in V(H^x)}\sum_{v\in V(H^x)}(d_H(u,v) + d_H^2(u,v))$$

$$+ \frac{1}{2}\sum_{x\epsilon V(G)}\sum_{y\epsilon V(G)-x}\sum_{u\epsilon v(H^x)}\sum_{v\epsilon V(H^y)}(d_H(u,h) + d_H^2(u,h) + d_G(x,y)$$

$$+ d_G^2(x,y) + d_H(h,v) + d_H^2(h,v)2d_H(u,h)d_G(x,y) +$$
$$2d_H(u,h)d_H(h,v)+2d_G(x,y)d_H(h,v))$$
$$n_2^2 ww(G) + n_1 WW(H) + 2n_2 W(G)D_H(h) +$$
$$\frac{n_1(n_1-1)}{2}(n_2 D_H^2(h) + (D_H(h))^2).$$

Using above theorem we have the following corollary.

Corollary 3.2. Let G be a graph of n vertex and let the copies of G used in the construction of G{G} be rooted in vertex h. Then

WW(G{G})=n(n+1)WW(G)+2n$D_G(h)W(G) + \frac{n(n-1)}{2}(nD_G^2(h) + \left(D_G(h)\right)^2$, $where\ D_G(h) = \sum_{u\epsilon V(G)} d_G(u,h)\ and\ D_G^2(h) = \sum_{u\epsilon V(G)}(d_G(u,h) + d_G^2(u,h))$.

For our reference we quote the following lemma from[5].

Lemma 3.3. (1) For $t \geq 1$, WW $(S_{t+1}) = \frac{t(3t-1)}{2}$.

(2) For $n \geq 2$, WW$(P_n) = \frac{1}{24}(n^4 + 2n^3 - n^2 - 2n)$.

(3) For $n \geq 3$, WW $(C_n) = \begin{cases} \frac{n^2(n+1)(n+2)}{48}, if\ n\ is\ even \\ \frac{n(n^2-1)(n+3)}{48}, if\ n\ is\ odd. \end{cases}$

Now using corollary 3.2 and Lemma 3.3, we obtain the hyper Wiener index of the square comb lattice $Cq(N)$.

Example 3.4. WW $(Cq(N))= \frac{n^2(n-1)}{24}(12n^3 - 2n^2 + 2)$.

By using Theorem 3.1 and Lemma 3.3, we obtain the hyper Wiener index of the *t*-fold bristled graph $Brs_t(G)$.

Example3.5. WW(Brs_t(G))=$(1+t)^2 WW(G) + 2t(t+1)W(G) + \frac{nt}{2}(3nt + 2n - 3)$.

From the above formula and Lemma 3.3, the exact hyper–Wiener indices of the t-fold bristled graph of P_n and C_n are computed.

Example 3.6 1. WW(Brs$_t$(P_n)) = $\frac{n(n^2-1)(t+1)}{24}(nt+n+10t+2)+\frac{nt}{2}(3nt+2n-3)$.

2. WW(Brs$_t$(C_n))=

$$\begin{cases}\frac{n^2(t+1)}{48}(n^2t+15nt+2t+n^2+3n+2)+\frac{nt}{2}(3nt+2n-3), if\ n\ is\ even\\ \frac{n(n^2-1)(t+1)}{48}(nt+15t+n+3)+\frac{nt}{2}(3nt+2n-3), if\ n\ is\ odd.\end{cases}$$

3. WW$(\boldsymbol{P_n}\{\boldsymbol{S_3}\}) = \frac{\boldsymbol{n}}{\boldsymbol{8}}(\boldsymbol{3n^3}+\boldsymbol{22n^2}+\boldsymbol{61n}-\boldsymbol{46})$.
4. WW$(\boldsymbol{P_n}\{\boldsymbol{K_r}\}) = \frac{\boldsymbol{n}}{\boldsymbol{24}}(\boldsymbol{r}(\boldsymbol{n^2}-\boldsymbol{1})(\boldsymbol{nr}+\boldsymbol{10r}-\boldsymbol{8})+\boldsymbol{12}(\boldsymbol{r}-\boldsymbol{1})(\boldsymbol{3nr}-\boldsymbol{n}-\boldsymbol{2r}+\boldsymbol{1}))$.
5. WW$(\boldsymbol{P_n}\{\boldsymbol{K_3}\}) = \frac{\boldsymbol{n}}{\boldsymbol{8}}(\boldsymbol{3n^3}+\boldsymbol{22n^2}+\boldsymbol{61n}-\boldsymbol{62})$.

Theorem 3.7. Let G be a connected graph with n vertices. Then $WW(K_2 \bullet G) = 4WW(G) + 3n$.

Proof. By the definition of the hyper–Wiener index, we have $WW(K_2 \bullet G) = \frac{W(K_2\bullet G)}{2} + \frac{1}{2}\sum_{(u_i,v_j),(u_k,v_r)\epsilon V(K_2\bullet G),(u_i,v_j)\neq(u_k,v_r)} d^2_{(K_2\bullet G)}((u_i,v_j),(u_k,v_r))$

$$= \frac{W(K_2\bullet G)}{2} + \frac{1}{2}\Big(\sum_{(u_i,v_j),(u_i,v_r)\epsilon V(K_2\bullet G),j\neq r} d^2_{(K_2\bullet G)}((u_i,v_j),(u_i,v_r))$$

$$+ \sum_{(u_i,v_j)(u_k,v_r)\epsilon V(K_2\bullet G),i\neq k} d^2_{(K_2\bullet G)}((u_i,v_j),(u_k,v_r))\Big)$$

$$= \frac{W(K_2\bullet G)}{2} + \frac{1}{2}\left(\sum_{u_i\epsilon V(K_2)}\sum_{v_jv_k\epsilon V(G),j\neq r} d^2_G(v_j,v_r) + \sum_{v_j\epsilon V(G)}\Big(4 + \sum_{v_r\in V(G),j\neq r} d^2_G(v_j,v_r)\Big)\right)$$

$$= 2W(G) + n + \frac{1}{2}\left(2\sum_{v_jv_k\in V(G),j\neq r} d^2_G(v_j,v_r) + \sum_{v_j\epsilon V(G)}\sum_{v_r\in V(G),j\neq r} d^2_G(v_j,v_r) + 4n\right)$$

= 4*WW*(G) +3*n*.

Theorem 3.8. Let *G* be a connected graph with *n* vertices. Then

$$\mathrm{WW}\,(K_2 * G) = \frac{n(8n-5)}{2}.$$

4. Reverse–Wiener index

By definition it is easy to see that $D(G\{H\}) = D(G) + 2e(h)$, then by Theorem 2.1 and by a simple calculation, we have the following theorem on the reverse wiener index of the cluster of two graphs.

Theorem 4.1. Let *G* and *H* be graphs with n_1 and n_2 vertices and let the copies of *H* used in the construction of $G\{H\}$ be rooted in vertex *h*. Then $\Lambda(G\{H\}) = \frac{n_1n_2}{2}\Big((n_1n_2-1)\big(2e(h)+D(G)\big) - 2(n_1-1)D_H(h)\Big) - n_2^2W(G) - n_1W(H)$, where $D_H(h) = \sum_{u\epsilon V(H)} d_H(u,h)$.

Corollary 4.2. Let *G* be graph of *n* vertex and let the copies of *G* used in the construction of $G\{G\}$ be rooted in vertex *h*.Then $\Lambda(G\{G\}) = \frac{n^2(n-1)}{2}\Big((n+1)\big(2e(h)+D(G)\big) - 2D_G(h)\Big) - n(n+1)W(G)$, where $D_G(h) = \sum_{u\epsilon V(G)} d_H(u,h)$.

By using Theorem 4.1, we obtain the reverse Wiener index of the *t*-fold bristled graph $Brs_t(G)$.

Example 4.3. $\Lambda(\mathrm{Brs}_t(G)) = \frac{n}{2}\big((t+1)(nt+3n+2t-3) - 2t^2\big) - (t+1)^2\,W(G)$.

Example 4.4. 1. $\Lambda(Cq(N)) = \frac{n^2(n-1)}{6}(5n^2+n-10)$.

2. $\Lambda(\mathrm{Brs}_t(P_n)) = \frac{n}{6}\big((t+1)(3nt+9n+7t-n^2-tn^2-8) - 6t^2\big)$.

3. $\Lambda(\mathrm{Brs}_t(C_n)) = \begin{cases} \frac{n}{8}\big((t+1)(4nt+12n+8t-n^2-tn^2-12) - 8t^2\big), & if\ n\ is\ even \\ \frac{n}{8}\big((t+1)(4nt+12n+9t-n^2-tn^2-11) - 8t^2\big), & if\ n\ is\ odd. \end{cases}$

4. $\Lambda(P_n\{S_3\}) = \frac{n}{2}(15n-3n^2-2)$.

5. $\Lambda(P_n\{K_r\}) = \frac{nr}{6}(2rn^2-3nr+3n+4r-6)$.

6. $\Lambda(P_n\{K_3\}) = 3n(n^2-n+1)$.

7. $\wedge(P_n\{W_r\}) = \frac{n}{6}(2r^2n^2 - 3nr^2 + 3nr + 13r^2 - 27r + 12), r \geq 4.$

Using theorem 2.1, $D(K_2 \bullet G) = D(G)$ and $D(K_2 * G)=2$, we have the following theorems.

Theorem 4.5. Let G be a connected graph with n vertices. Then $\wedge(K_2 \bullet G)$=n(2n-1)D(G)-4W(G)-2n.

Theorem 4.6. Let G be a connected graph with n vertices. Then $\wedge(K_2 * G) = n^2$.

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