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Bianchi Type II Stiff Fluid Tilted Cosmological Model in General Relativity

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Abstract

In this paper, we have investigated tilted LRS Bianchi Type II cosmological model for stiff fluid distribution with heat conduction in General Relativity. To get the deterministic solution in terms of cosmic time t, we have also assume a supplementary conditions $R = S^n$ metric potential R and S, n being a constant. The physical and geometrical aspects of the model are also discussed.

Key Words: Tilted, Bianchi II, Heat Conduction, Stiff Fluid.

Introduction

Spatially homogeneous and anisotropic cosmological models in which the fluid flow is not normal to the hyper-space of homogeneity create a more interest in the study. These models are called Tilted models. King and Ellis [1], Ellis and King [2] have studied the general dynamics of tilted universe. Bradley and Sviestine [3] have shown that heat flow is expected for tilted cosmological models with heat flux have been investigated by number of authors like Roy and Banerjee [4], Banerjee and Santos [5], Coley andTupper [6], Bali and Sharma [7], Bali and Meena [8]. Recently Bali and Banerjee [9] have investigated spatially homogeneous and LRS (Locally Rotationally Symmetric) Bianchi Type II space-time for perfect fluid distribution in General Relativity.

To get the deterministic solution in terms of cosmic time t, we have also assumed a condition $\sigma \alpha \theta$ where σ is the shear and θ the expansion in the model.

We consider the Bianchi Type II metric in the form

$$ds^2 = -dt^2 + R^2 dx^2 + S^2 (dy - x\, dz)^2 + R^2 dz^2 \qquad \ldots(1)$$

where R and S are functions of t alone.

The energy momentum tensor for perfect fluid distribution with heat conduction is taken into the form given by Ellis [10] as

$$T_i^j = (\in + p)v_i v^j + p g_i^j + q_i v^j + v_i q^j \qquad \ldots(2)$$

together with

$$g_{ij} v^i v^j = -1 \qquad \ldots(3)$$

$$q_i v^i = 0 \qquad \ldots(4)$$

$$q_i q^j > 0 \qquad \ldots(5)$$

where p is the isotropic pressure, $\in$ density and q_i the heat conduction vector orthogonal to $v^i\left(\frac{\sinh\lambda}{R}, 0, 0, \cosh\lambda\right)$.

Einstein's field equations

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi T_i^j$$

For the metric (1) leads to

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} + \frac{S^2}{4R^4} = -8\pi[(\in + p)\sinh^2\lambda + p + 2Rq^1 \sinh\lambda] \qquad \ldots(6)$$

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} - \frac{3S^2}{4R^4} = -8\pi p \qquad \ldots(7)$$

$$\frac{2R_4 S_4}{RS} + \frac{R_4^2}{R^2} - \frac{S^2}{4R^4} = -8\pi[-(\in + p)\cosh^2\lambda + p - 2Rq^1 \sinh\lambda] \qquad \ldots(8)$$

$$(\in + p)R \sinh\lambda \cosh\lambda + R^2 q^1 \frac{\cosh 2\lambda}{\cosh\lambda} = 0 \qquad \ldots(9)$$

where the suffix '4' after R and S denotes differentiation with respect to 't'.

Equation (6) – (9) are four equations in six unknowns R, S, $\in$, p, q and λ.

For the complete determination of these quantities, we assume two extra conditions. We assume the model is filled with stiff fluid which leads to

$$\in = p \qquad \ldots(10)$$

and

$$R = S^n \qquad \ldots(11)$$

Equations (6) and (8) lead to

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{3R_4 S_4}{RS} + \frac{R_4^2}{R^2} = -8\pi(p - \in) \qquad \ldots(12)$$

for stiff fluid $\in = p$

$$\frac{R_{44}}{R}+\frac{S_{44}}{S}+\frac{3R_4S_4}{RS}+\frac{R_4^2}{R^2}=0 \qquad \ldots(13)$$

(11) and (13) lead to

$$\frac{S_{44}}{S}+2n\frac{S_4^2}{S^2}=0 \qquad \ldots(14)$$

which leads to

$$f\,f^1+\frac{2n}{S}f^2=0 \qquad \ldots(15)$$

where

$$S_4=f, \qquad f^1=df/ds$$

(15) leads to

$$S=(at+b)^{\frac{1}{2n+1}} \qquad \ldots(16)$$

From (11) and (16)

$$R=(at+b)^{\frac{n}{2n+1}} \qquad \ldots(17)$$

where

$$a=\frac{\ell}{2n+1}$$

Hence metric (1) reduces to the form

$$ds^2=-\ell(2n+1)dT+T^{\frac{2n}{2n+1}}dX^2+T^{\frac{2}{2n+1}}(dY^2-XdZ)^2+T^{\frac{2n}{2n+1}}dZ^2 \qquad \ldots(18)$$

where ℓ is the constant of integration.

Some Physical and Geometrical Features

From equations (7), (16), (17), we have

$$16\pi p=\frac{4n\ell^2+3T^{\frac{4}{2n+1}}}{2T^2} \qquad \ldots(19)$$

$$\in=\frac{4n\ell^2+3T^{\frac{4}{2n+1}}}{32\pi T^2} \qquad \ldots(20)$$

$$q^1=-\frac{2p\sinh\lambda\cosh^2\lambda}{R\cosh^2\lambda}$$

and

$$\cosh 2\lambda=\frac{16\pi p}{\frac{R_4S_4}{RS}+\frac{R_4^2}{R^2}-\frac{R_{44}}{R}-\frac{S_{44}}{S}-\frac{S^2}{2R^4}} \qquad \ldots(21)$$

where

$$\frac{R_4 S_4}{RS} + \frac{R_4^2}{R^2} - \frac{R_{44}}{R} - \frac{S_{44}}{S} - \frac{S^2}{2R^4} = \frac{4n\ell^2(n+2) - T^{\frac{4}{2n+1}}}{2T^2} \quad \ldots(22)$$

Equations (20) and (22) lead to

$$\cosh 2\lambda = \frac{4n\ell^2 + 3T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}} \quad \ldots(23)$$

$$\cosh^2\lambda = \frac{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}} \quad \ldots(24)$$

$$\sinh^2\lambda = \frac{-2n(n+1)\ell^2 + 2T^{\frac{4}{2n+1}}}{4n\ell^2(n+2) - T^{\frac{4}{2n+1}}} \quad \ldots(25)$$

$$q_1 = -\frac{\{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}{16\pi T^{\frac{3n+2}{2n+1}}} \sqrt{\frac{-2n(n+1)\ell^2 + 2T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}} \quad \ldots(26)$$

$$v^1 = \frac{1}{T^{\frac{n}{2n+1}}} \sqrt{\frac{-2n(n+1)\ell^2 + 2T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}} \quad \ldots(27)$$

$$v^4 = \sqrt{\frac{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}} \quad \ldots(28)$$

$$q_4 = \frac{-2n(n+1)\ell^2 + 2T^{\frac{4}{2n+1}}}{16\pi T^2} \sqrt{\frac{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}} \quad \ldots(29)$$

We know

$$q_1 v^1 + q_4 v^4 = 0$$

Using (26), (27), (28) and (29), we have

$$-\left[\left\{\frac{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}{16\pi T^{\frac{3n+2}{2n+1}}}\right\}\sqrt{\frac{-2n(n+1)\ell^2+2T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}}}\right]$$

$$\times\left\{\frac{1}{T^{\frac{n}{2n+1}}}\sqrt{\frac{-2n(n+1)\ell^2+2T^{\frac{4}{2n}}}{4n(n+2)\ell^2-T^{\frac{4}{2n}}}}\right\}+\left[\left\{\frac{-2n(n+1)\ell^2+2T^{\frac{4}{2n+1}}}{16\pi T^2}\right\}\right.$$

$$\left.\times\sqrt{\frac{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}}}\right]\left\{\sqrt{\frac{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-T^{\frac{4}{2n}}}}\right\}$$

$$= 0 \qquad \ldots(30)$$

$$\sigma_{11}=\frac{R^2}{3}\cosh\lambda\left[\left(\frac{R_4}{R}-\frac{S_4}{S}\right)\cosh^2\lambda+2\cosh\lambda\frac{\partial}{\partial t}(\cosh\lambda)\right]$$

$$\frac{\partial}{\partial t}\{\cosh\lambda\}=\frac{\partial}{\partial t}\left\{\frac{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}}\right\}^{1/2}$$

$$=\frac{\partial}{\partial t}\left\{\frac{2n(n+3)\ell^2+(at+b)^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-(at+b)^{\frac{4}{2n+1}}}\right\}^{1/2} \quad |\text{since } T=at+b$$

$$=\left[\frac{\left\{4n(n+2)\ell^2-(at+b)^{\frac{4}{2n+1}}\right\}\dfrac{2a(at+b)^{\frac{3-2n}{2n+1}}}{2n+1}}{\{4n(n+2)\ell^2-}\right.$$

$$\left.\frac{\left\{2n(n+3)\ell^2+(at+b)^{\frac{2}{2n+1}}\right\}\left\{\dfrac{-4}{2n+1}a(at+b)^{\frac{3-2n}{2n+1}}\right\}}{(at+b)^{\frac{4}{2n+1}}\}^2}\right]$$

$$\times\frac{1}{2}\left\{\frac{2n(n+3)\ell^2+(at+b)^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-(at+b)^{\frac{4}{2n+1}}}\right\}^{-\frac{1}{2}}$$

$$\frac{\partial}{\partial t}(\cosh\lambda)=\frac{2a(at+b)^{\frac{3-2n}{2n+1}}\{2n(3n+1)\ell^2}{\left\{4n(n+2)\ell^2-(at+b)^{\frac{4}{2n+1}}\right\}^{\frac{2}{n}}}\sqrt{\frac{4n(n+2)\ell^2-(at+b)^{\frac{4}{2n+1}}}{2n(n+3)\ell^2+(at+b)^{\frac{4}{2n+1}}}}$$

$$\frac{\partial}{\partial t}\cosh\lambda=\frac{2\ell T^{\frac{3-2n}{2n+1}}2n(3n+7)\ell^2}{\{4n(n+2)\ell^2T^{\frac{4}{2n+1}}\}^2}\sqrt{\frac{4n(n+2)\ell^2\ T^{\frac{4}{2n+1}}}{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}} \quad \ldots(31)$$

Since

$$\sigma_{11}=\frac{R^2}{3}\cosh\lambda\left[\left(\frac{R_4}{R}-\frac{S_4}{S}\right)\cosh^2\lambda+2\cosh\lambda\frac{\partial(s)}{\partial t}\right]$$

Using equations (16), (17), (24) and (31), we have

$$\sigma_{11}=\frac{T^{\frac{2n}{2n+1}}}{3}\left\{\frac{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}}\right\}$$

$$\times\left[\frac{(n-1)\ell}{T}\sqrt{\frac{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}}}+\frac{8\ell^3(3n+7)T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}\right\}}\right.$$

$$\left.\times\sqrt{\frac{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}}{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}}\right]$$

$$\sigma_{11}=\frac{T^{\frac{2n}{2n+1}}}{3}\left\{\frac{2n(n+3)\ell^2+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2-T^{\frac{4}{2n+1}}}\right\}^{3/2}$$

$$\times\left[\frac{(n-1)\ell}{T}+\frac{8\ell^{3}n(3n+7)\,T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}\right\}\left\{2n(n+3)\ell^{2}+T^{\frac{4}{2n+1}}\right\}}\right] \qquad \ldots(32)$$

$$v^{1}=\frac{1}{T^{\frac{n}{2n+1}}}\sqrt{\frac{-2n(n+1)\ell^{2}+2\,T^{\frac{4}{2n+1}}}{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}}}$$

$$\sigma_{11}v^{1}=\frac{T^{\frac{n}{2n+1}}}{3}\,\frac{\sqrt{\left\{-2n(n+1)\ell^{2}+2T^{\frac{4}{2n+1}}\right\}\left\{2n(n+3)\ell^{2}+T^{\frac{4}{2n+1}}\right\}}}{\left\{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}\right\}}$$

$$\times\left[\frac{(n-1)\ell}{T}\left\{\frac{2n(n+3)\ell^{2}+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}}+\frac{8\ell^{3}n(3n+7)T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}\right\}^{2}}\right\}\right] \qquad \ldots(33)$$

$$\sigma_{14}=-\frac{R}{3}\sinh\lambda\cosh\lambda\left[\left(\frac{R_{4}}{R}-\frac{S_{4}}{S}\right)\cosh\lambda+2\frac{\partial}{\partial t}(\cosh\lambda\right]$$

Putting values of (16), (17), (24), (25) and (31), we have

$$\sigma_{14}=-T^{\frac{n}{2n+1}}\sqrt{\frac{\left\{-2n(n+1)\ell^{2}+3T^{\frac{4}{2n+1}}\right\}}{\left\{4n\ell^{2}(n+2)-T^{\frac{4}{2n+1}}\right\}}}\sqrt{\frac{2n(n+3)\ell^{2}+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}}}$$

$$\left[\frac{(n-1)\ell}{T}\sqrt{\frac{2n(n+3)\ell^{2}+T^{\frac{4}{2n+1}}}{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}}}+\frac{8\ell^{3}n(3n+7)T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^{2}-T^{\frac{4}{2n+1}}\right\}^{2}}\right.$$

$$\left.\sqrt{\frac{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}}\right]$$

$$\sigma_{14} = -\frac{T^{\frac{n}{2n+1}}}{3}\frac{\left\{-2n(n+1)\ell^2 + 2T^{\frac{4}{2n+1}}\right\}}{\left\{4n\,\ell^2(n+2) - T^{\frac{4}{2n+1}}\right\}}$$

$$\times\left[\frac{(n-1)\ell}{T}\left\{\frac{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}\right\} + \frac{8\ell^3 n(3n+7)\,T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}\right\}}\right] \qquad \ldots(34)$$

$$v^4 = \sqrt{\frac{2n(n+3) + T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}}$$

$$\sigma_{14}\,v^4 = -\frac{T^{\frac{n}{2n+1}}}{3}\frac{\sqrt{\left\{-2n(n+1)\ell^2 + 2T^{\frac{4}{2n+1}}\right\}}\left\{2n(n+3) + T^{\frac{4}{2n+1}}\right\}}{\left\{4n\,\ell^2(n+2) - T^{\frac{4}{2n+1}}\right\}}$$

$$\times\left[\frac{(n-1)\ell}{T}\left\{\frac{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}\right\} + \frac{8\ell^3 n(3n+7)\,T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}\right\}}\right] \qquad \ldots(35)$$

When adding (33) and (35), we have

$$\sigma_{11}v^1 + \sigma_{14}v^4 = 0$$

we know

$$\theta = \frac{\partial}{\partial t}(\cosh\lambda) + \cosh\lambda\left(\frac{2R_4}{R} + \frac{S_4}{S}\right)$$

From (24), (31) and (16), (17), we have

$$\theta = \frac{4\ell^3 n(3n+7)\, T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}\right\}^2} \sqrt{\frac{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}}$$

$$+ \frac{\ell(2n+1)}{T} \sqrt{\frac{2n(n+3)\ell^2 - T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}}$$

$$\theta = \sqrt{\frac{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}}{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}}} \times \left[\frac{4\ell^3 n(3n+7)\, T^{\frac{3-2n}{2n+1}}}{\left\{4n(n+2)\ell^2 - T^{\frac{4}{2n+1}}\right\}\left\{2n(n+3)\ell^2 + T^{\frac{4}{2n+1}}\right\}} + \frac{(2n+1)\ell}{T} \right] \qquad \ldots(36)$$

Discussion

The model (18) starts with a big bang at T = 0 and expansion in the model decreases as time increases where 2n+1 > 0.

The matter density $\in \to 0$ when $T \to \infty$ and 2n+1 > 0. The fluid velocity v^i satisfies trace free condition $\sigma_{ij}\, v^j = 0$ and $w_{ij}\, v^j = 0$. The model in general represents tilted, shearing and non-rotating universe. The model (18) has Point Type singularity at T = 0 (MacCallum [1971][11].

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