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Graceful Labeling for Cycle of Graphs

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Abstract

We investigate a new graph which is called cycle of graphs. We prove that cycle of cycles $C_t(C_n)$, $t \equiv 0 \pmod 2$, $n \equiv 0 \pmod 4$ is graceful graph. We also prove that cycle of complete bipartite graphs $C_t(K_{n,n})$, $t \equiv 0 \pmod 2$, $n \in N$ is graceful graph.

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Keywords: Labeling, Graceful labeling, Cycle, Complete bipartite graph, Cycle of graphs.

1. Introduction

Let $G = (V, E)$ be a simple, undirected, finite graph with p vertices and q edges. In this work we introduce a new graph which is called cycle of graphs and it is denoted by $C(G_1, G_2, \ldots, G_n)$. C_n denotes a cycle on n vertices and $K_{m,n}$ denotes a complete bipartite graph on $m + n$ vertices. For all terminology and notations we follows Harary (Harary 1972). We will give brief summary of definitions which are useful for this paper.

Definition 1.1. If the vertices of the graph are assigned values subject to certain conditions then it is known as *graph labeling*.

Definition 1.2. Given a graph $G = (V, E)$, the set N of non$-$negative integers and a commutative binary operation $* : N \times N \longrightarrow N$, every vertex $f : V \longrightarrow N$ induces an edge function $f^* : E \longrightarrow N$ such that $f^*((u, v)) = |f(u) - f(v)|$, for all $(u, v) \in E$. A function f is called *graceful labeling* of a graph G if $f : V \longrightarrow \{0, 1, 2, \dots, q\}$ is injective and the induced function $f^* : E \longrightarrow \{1, 2, \dots, q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective, where $e = (u, v)$. A graph which admits graceful labeling is called *graceful graph*.

Definition 1.3. A graph $G = (V, E)$ is said to be *bipartite* if the vertex set can be partitioned into two disjoint subsets V_1 and V_2 such that for every edge $e = (v_i, v_j) \in E$, $v_i \in V_1$ and $v_j \in V_2$.

A *complete bipartite graph* is a bipartite graph such that two vertices are adjacent if and only if they are in different partite sets. If partite sets are having m and n vertices then the related complete bipartite graph is denoted by $K_{m,n}$.

Definition 1.4. For a cycle C_n, each vertices of C_n is replace by connected graphs $G_1, G_2, \dots, G_n$ is known as *cycle of graphs* and it is denoted by $C(G_1, G_2, \dots, G_n)$. If we replace each vertices by a graph G i.e. $G_1 = G, G_2 = G, \dots G_n = G$, such cycle of graphs is denoted by $C_n(G)$.

For the detail survey of graph labeling one can refer Gallian (Gallian 2013). Labeled graphs have many diversified applications. The graceful labeling was introduced by Rosa (Rosa 1967, p. 349–355).

In present paper we have proved that cycle of cycles $C_t(C_n)$ and cycle of complete bipartite graphs $C_t(K_{n,n})$ are graceful graphs.

2. Main Results

Theorem 2.1. Cycle of cycles $C_t(C_n)$, $t \equiv 0 \pmod 2$, $n \equiv 0 \pmod 4$ is graceful graph.

Proof. Let $u_{i,j}$; $j = 1, 2, \dots, n$ be vertices of i^{th} copy of cycle C_n, $\forall\, i = 1, 2, \dots, t$. We join $u_{i,n}$ vertex of i^{th} copy of C_n with $u_{i+1,1}$ vertex of $(i + 1)^{th}$ copy of C_n by an edge, $\forall\, i = 1, 2, \dots, t - 1$ and join $u_{t,n}$ with $u_{1,1}$ to form cycle of graphs $C_t(C_n)$. ■

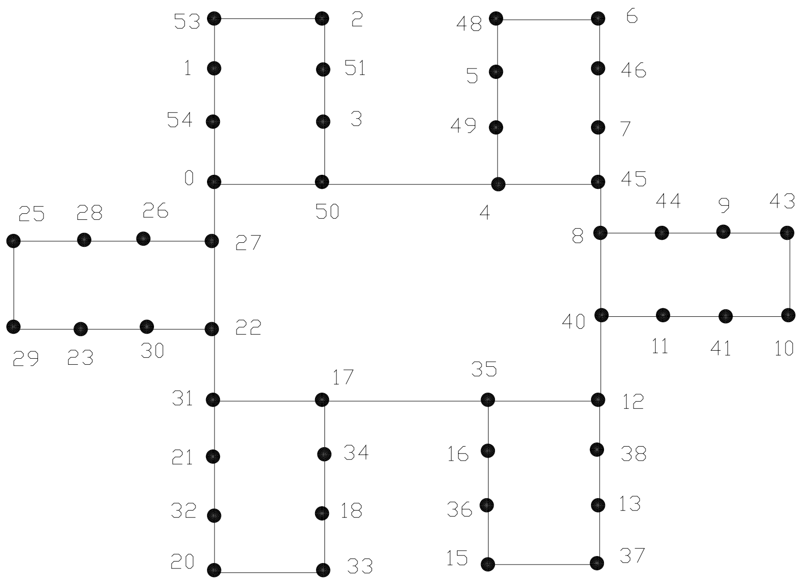
We define the labeling function $f : V \longrightarrow \{0, 1, \dots, q\}$, where $q = (n + 1)t$ as follows:

$$\begin{aligned} f(u_{1,j}) &= \frac{j-1}{2}, && \forall j = 1, 3, \dots, n-1 \\ &= q - \frac{j-2}{2}, && \forall j = 2, 4, \dots, \frac{n}{2} \\ &= q - \frac{j}{2}, && \forall j = \frac{n}{2} + 2, \frac{n}{2} + 4, \dots, n \end{aligned}$$

$$
\begin{aligned}
f(u_{l,j}) &= f(u_{l-1,j}) + \frac{n}{2}, && \forall j = 1, 3, \dots, n-1\\
&= f(u_{l-1,j}) - \left(\frac{n}{2}+1\right), && \forall j = 2, 4, \dots, n;\\
& && \forall l = 2, 3, \dots, \frac{t}{2},\\
f\left(u_{\frac{t}{2}+1,j}\right) &= f\left(u_{\frac{t}{2},n-1}\right) + \frac{j+1}{2}, && \forall j = 1, 3, \dots, \frac{n}{2}-1\\
&= f(u_{\frac{t}{2},n-1}) + \frac{j+3}{2}, && \forall j = \frac{n}{2}+1, \frac{n}{2}+3, \dots, n-1\\
&= f\left(u_{\frac{t}{2},n}\right) - \frac{j+2}{2}, && \forall j = 2, 4, \dots, n\\
f(u_{l,j}) &= f(u_{l-1,j}) + \left(\frac{n}{2}+1\right), && \forall j = 1, 3, \dots, n-1\\
&= f(u_{l-1,j}) - \frac{n}{2}, && \forall j = 2, 4, \dots, n;\\
& && \forall l = \frac{t}{2}+2, \frac{t}{2}+3, \dots, t.
\end{aligned}
$$

Above labeling pattern give rise graceful labeling to $C_t(C_n)$.

Illustration - 2.2: $C_6(C_8)$ and its graceful labeling shown in following figure.



Theorem 2.2. Cycle of complete bipartite graphs $C_t(K_{n,n})$, $t \equiv 0 \pmod 2$, $n \in N$ is graceful graph.

Proof. Let $u_{i,j}$, $w_{i,j}$; $j = 1, 2, \dots, n$ be vertices of i^{th} copy of $K_{n,n}$, $\forall\, i = 1, 2, \dots, t$.

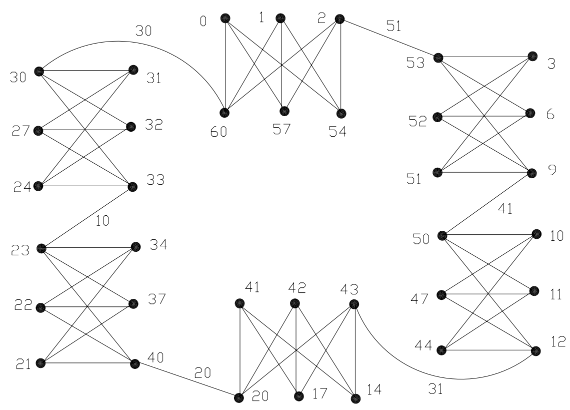
We join $u_{i,n}$ vertex of i^{th} copy of $K_{n,n}$ with $w_{i+1,1}$ vertex of $(i+1)^{th}$ copy of $K_{n,n}$ by an edge, $\forall\, i = 1, 2, \ldots, t-1$ and join $u_{t,n}$ with $w_{1,1}$ to form cycle of graphs $C_t(K_{n.n})$. ■

We define the labeling function $f : V \longrightarrow \{0, 1, \ldots, q\}$, where $q = (n^2+1)t$ as follows:

$$
\begin{aligned}
f(u_{1,j}) &= j-1, && \forall j = 1, 2, \ldots, n\\
f(w_{1,j}) &= q-(j-1)n, && \forall j = 1, 2, \ldots, n\\
f(u_{2,j}) &= f(u_{1,n}) + 1 + (j-1)n, && \forall j = 1, 2, \ldots, n\\
f(w_{2,j}) &= f(w_{1,n}) - j, && \forall j = 1, 2, \ldots, n\\
f(u_{l,j}) &= f(u_{l-2,j}) + (n^2+1), && \forall j = 1, 2, \ldots, n\\
f(w_{l,j}) &= f(w_{l-2,j}) - (n^2+1), && \forall j = 1, 2, \ldots, n;\\
& && \forall l = 3, 4, \ldots, \frac{t}{2},\\
f(u_{\frac{t}{2}+1,j}) &= f(u_{\frac{t}{2}-1,j}) + (n^2+2), && \forall j = 1, 2, \ldots, n\\
f(w_{\frac{t}{2}+1,j}) &= f(w_{\frac{t}{2}-1,j}) - (n^2+1), && \forall j = 1, 2, \ldots, n\\
f(u_{\frac{t}{2}+2,j}) &= f(u_{\frac{t}{2},j}) + (n^2+2), && \forall j = 1, 2, \ldots, n\\
f(w_{\frac{t}{2}+2,j}) &= f(w_{\frac{t}{2},j}) - (n^2+1), && \forall j = 1, 2, \ldots, n\\
f(u_{l,j}) &= f(u_{l-2,j}) + (n^2+1), && \forall j = 1, 2, \ldots, n\\
f(w_{l,j}) &= f(w_{l-2,j}) - (n^2+1), && \forall j = 1, 2, \ldots, n;\\
& && \forall l = \frac{t}{2}+3, \frac{t}{2}+4, \ldots, t.
\end{aligned}
$$

Above labeling pattern give rise graceful labeling to $C_t(K_{n,n})$.

Illustration - 2.4: $C_6(K_{3,3})$ and its graceful labeling shown in following figure.



3. Concluding Remarks

We introduced here a new graph is called cycle of graphs. We discussed here graceful labeling for cycle of graphs. Present work contributes two new results. The labeling pattern is demonstrated by means of illustrations, which provide better understanding of derived results.

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