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Geometric Mean Labeling on Double Quadrilateral Snake Graphs

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Abstract

A Graph G= (p,q) is a called a Geometric Mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2…..q+1 in such a way that when each edge e=uv is labeled with f(e=uv) = $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$, then the edge labels are distinct. In this case f is called a Geometric mean labeling of G and G is a Geometric mean graph. In this paper we prove that Double Quadrilateral snakes and Alternate Double Quadrilateral snake graphs are Geometric mean graphs.

Keywords: Geometric mean graph, Double Quadrilateral snakes, Alternate Double Quadrilateral snakes.

1. Introduction

The graphs considered here will be finite, undirected and simple. The symbols V(G) and E(G) denote the vertex set and edge set of G. A path of length n is denoted by P_n and a cycle of length n is denoted by C_n. A Triangular snake T_n is obtained from a path $v_1, v_2 \ldots\ldots\ldots v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1 \leq i \leq n-1$. The Quadrilateral snake Q_n is obtained from a path $u_1, u_2 \ldots\ldots u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and then joining v_i and w_i. An Alternate Quadrilateral snake $A(Q_n)$ is obtained from a path $u_1, u_2 \ldots u_n$ by joining u_i, u_{i+1} (Alternatively) to new vertices v_i, w_i respectively and then joining v_i and w_i. The Double Quadrilateral snake $D(Q_n)$ consists of two Quadrilateral snakes that have a common path. The Alternate Double Quadrilateral snake $A(D(Q_n))$ consists of two alternative Quadrilateral snakes that have a common path. Terms and Terminology are used here in the sense of Gallian [1] and Harary [2]. The notion of Geometric mean labeling has been introduced in [3]. In [4] we have investigated Geometric mean labeling behaviour of some standard graphs. In this paper we show that every Double Quadrilateral snakes and Alternate Double Quadrilateral snakes are Geometric mean graphs.

2. Geometric Mean Labeling

A Graph G= (V, E) with p vertices and q edges is called a Geometric mean graph if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2……q+1 in such a way that when each edge e=uv is labeled with f(e=uv) = $\lceil \sqrt{f(u)f(v)} \rceil$ (or) $\lfloor \sqrt{f(u)f(v)} \rfloor$, then the edge labels are distinct. In this case f is a Geometric mean labeling of G.

We need the following theorems for proving our results.

Theorem 2.1 [3]: Triangular snakes ad Quadrilateral snakes are Geometric mean graphs.

Theorem 2.2 [5]: Double Triangular and Alternate Double Triangular Snakes are Harmonic mean graphs.

Theorem 2.3 [5]: Double Quadrilateral, Alternate Double Quadrilateral snakes are Harmonic mean graphs.

Theorem 2.4 [5]: Double Triangular snakes and Alternate Double Triangular snakes are Geometric Mean graphs.

3. Main Results

Theorem 3.1: Double Quadrilateral snakes $D(Q_n)$ is a Geometric mean graph

Proof: Let P_n be the path u_1, u_2 ……..u_n. To construct $D(Q_n)$ from path P_n we join u_i, u_{i+1} to four new vertices v_i, w_i and x_i, y_i by edges u_iv_i, u_{i+1} w_i, u_ix_i, u_{i+1} y_i, x_iy_i and for i=1, 2…., n-1.

Define a function f: $V(D(Q_n)) \rightarrow \{1,2….q+1\}$

by $f(u_1) = 2$

$f(u_i) = 7(i-1)+1, 2 \leq i \leq n$

$f(v_1) = 1$

$f(v_i) = 7(i-1), 2 \leq i \leq n-1$

$f(w_1) = 6$

$f(w_i) = 7i-3, 1 \leq i \leq n-1$

$f(x_1) = 4$

$f(x_i) = 7i-2, 1 \leq i \leq n-1$

$f(y_1) = 5$

$f(y_i) = 7i-1, 1 \leq i \leq n-1$

Then we get distinct edge labels

Hence f provides Geometric mean labeling for the graph $D(Q_n)$

Example 3.2: The Geometric mean labeling of $D(Q_4)$ is given below

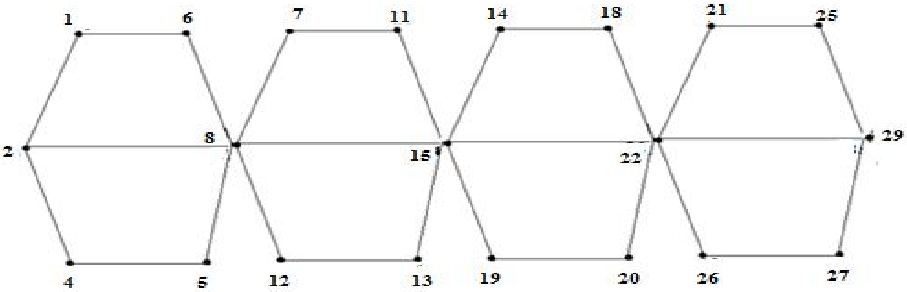

Fig. 1

Theorem 3.3: Alternate Double Quadrilateral snake $A(D(Q_n))$ is a Geometric mean graph.
Proof: Let G be the Double Quadrilateral snake A $(D(Q_n))$
Consider the path $u_1, u_2 \dots\dots\dots u_n$.
Join u_i, u_{i+1} (alternatively) with four new vertices v_i, w_i, x_i and y_i
Here we consider two different cases
Case (1): If the Quadrilateral snake starts from u_1, then
Define a function $f:V(G) \rightarrow \{1,2\dots..q+1\}$
by $f(u_i) = 4i-1, \quad 1 \le i \le n$
$f(v_i) = 8i-7, \quad 1 \le i \le n/2$
$f(w_i) = 8i-3, \quad 1 \le i \le n/2$
$f(x_i) = 8i-6, \quad 1 \le i \le n/2$
$f(y_i) = 8i-2, \quad 1 \le i \le n/2$
The labeling pattern is shown below

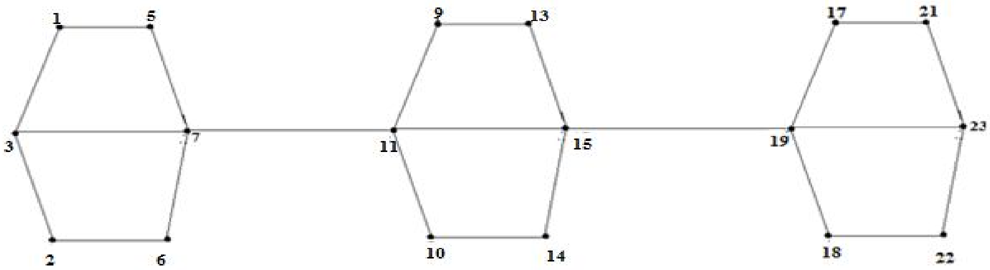

Fig. 2

Then we get distinct edge labels. In this case, f provides a Geometric mean labeling.
Case (ii): The Double Quadrilateral snakes starts from u_2, then
Define $f: V(G) \rightarrow \{1,2\dots\dots q+1\}$
by $f(u_1) = 1 \qquad f(u_i) = 4i-4, \ 2 \le i \le n$

$f(v_i) = 8i-6$, $1 \le i \le \frac{n-1}{2}$

$f(w_i) = 8i-1$, $1 \le i \le \frac{n-1}{2}$

$f(x_1) = 8i-5$, $1 \le i \le \frac{n-1}{2}$

$f(y_i) = 8i+1$, $1 \le i \le \frac{n-1}{2}$

Then we get distinct edge labels

Then labeling pattern is given in the following figure

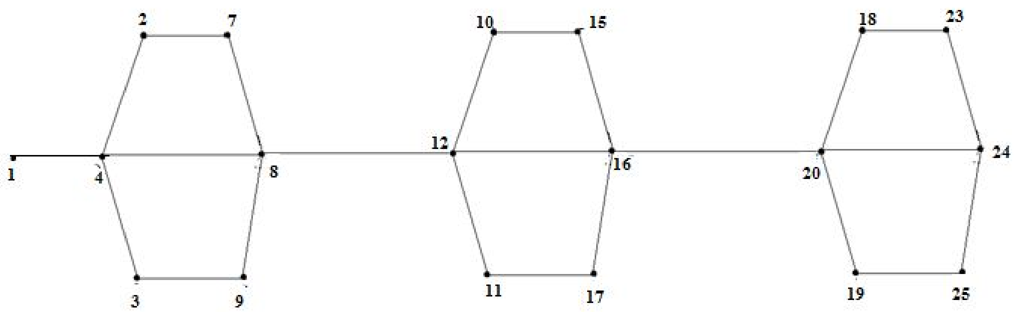

Fig. 3

This makes f a Geometric mean labeling

From case (i) and case (ii) we conclude that Alternate Double Quadrilateral snakes are Geometric mean graphs.

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