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Root Square Mean Labeling of Subdivision of Some More Graphs

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Abstract

A graph $G = (V, E)$ with p vertices and q edges is called a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, \ldots, q+1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case f is called Root Square Mean Labeling of G. In this paper we prove that subdivision of some graphs are Root Square Mean graphs.

Key Words: Graph, Root Square Mean graph, Triangular Snake, Quadrilateral Snake, Alternate Triangular Snake, Alternate Quadrilateral Snake.

1. Introduction

All graphs in this paper are finite, simple, and undirected graph $G = (V, E)$ with p vertices and q edges. For all detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. Root Square Mean labeling was introduced by S.S.Sandhya, S.Somasundaram, and S.Anusa in [3] and studied their behavior in[4], [5], [6], [7], [8], [9], [10].In this paper we prove that subdivision of some graphs are Root Square Mean graphs. The following definitions and theorems are necessary for our present study.

Definition1.1: A graph $G = (V, E)$ with p vertices and q edges is called a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1,2, \dots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the edge labels are distinct. In this case f is called Root Square Mean Labeling of G.

Definition1.2: A Triangular Snake T_n is obtained from a path $u_1 u_2 \cdots u_n$ by joining u_i and u_{i+1} to a new vertex $v_i, 1 \le i \le n - 1$.

Definition1.3: An Alternate Triangular Snake $A(T_n)$ is obtained from a path $u_1 u_2 \cdots u_n$ by joining u_i and u_{i+1} (Alternatively) to a new vertex v_i.

Definition1.4: A Quadrilateral Snake Q_n is obtained from a path $u_1 u_2 \cdots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i , $1 \le i \le n - 1$ respectively and then joining v_i and w_i .

Definition1.5: An Alternate Quadrilateral Snake $A(Q_n)$ is obtained from a path $u_1 u_2 \cdots u_n$ by joining u_i and u_{i+1} (Alternatively) to new vertices v_i and w_i respectively and then joining v_i and w_i .

Theorem1.6: A Triangular snake T_n is a Root Square Mean graph.

Theorem1.7: Alternate Triangular Snake $A(T_n)$ is a Root Square Mean graph.

Theorem1.8: A Quadrilateral Snake Q_n is a Root Square Mean graph.

Theorem1.9: Alternate Quadrilateral Snake $A(Q_n)$ is a Root Square Mean graph.

2.Main Results

Theorem2.1: $S(T_n)$ is a Root Square Mean graph.

Proof: Let $u_1 u_2 \cdots u_n$ be a path of length n. Let T_n be the triangular snake obtained by joining u_i and u_{i+1} to a new vertex v_i , $1 \le i \le n - 1$ .Let us subdivide the edges of T_n. Here we consider the following cases.

Case(1): G is obtained by subdividing each edge of the path.
Let $t_1, t_2, \cdots, t_{n-1}$ be the vertices which subdivide the edge $u_i u_{i+1}$.
Define a function $f: V(G) \to \{1,2, \dots, q + 1\}$ by
$f(u_i) = 4i - 3, 1 \le i \le n$
$f(v_i) = 4i - 2, 1 \le i \le n - 1$
$f(t_i) = 4i, 1 \le i \le n - 1$

Then the edges are labeled as
$f(u_i v_i) = 4i - 3, 1 \le i \le n - 1$
$f(u_i t_i) = 4i - 2, 1 \le i \le n - 1$
$f(t_i u_{i+1}) = 4i, 1 \le i \le n - 1$
$f(v_i u_{i+1}) = 4i - 1, 1 \le i \le n - 1$

Then we get distinct edge labels. Hence f is a Root Square Mean labeling of G.
The labeling pattern of $S(T_5)$ is shown below.

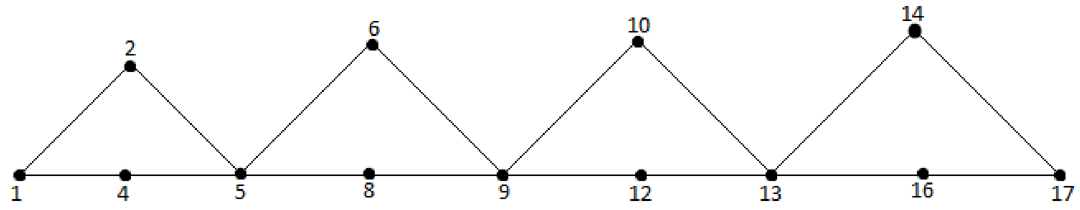

Figure1

Case(2): G is obtained by subdividing the edges $u_i v_i$ and $u_{i+1} v_i$.
Let x_i and y_i be the two vertices which subdivide the edges $u_i v_i$ and $u_{i+1} v_i, 1 \le i \le n - 1$ respectively. Define a function $f: V(G) \to \{1,2, \dots, q + 1\}$ by
$f(u_i) = 5i - 4, 1 \le i \le n$
$f(v_i) = 5i - 2, 1 \le i \le n - 1$
$f(x_i) = 5i - 3, 1 \le i \le n - 1$
$f(y_i) = 5i - 1, 1 \le i \le n - 1$

Then the edges are labeled as
$f(u_i x_i) = 5i - 4, 1 \le i \le n - 1$
$f(x_i v_i) = 5i - 3, 1 \le i \le n - 1$
$f(v_i y_i) = 5i - 2, 1 \le i \le n - 1$
$f(y_i u_{i+1}) = 5i, 1 \le i \le n - 1$
$f(u_i u_{i+1}) = 5i - 1, 1 \le i \le n - 1$

Then we get distinct edge labels. Hence f is a Root Square Mean labeling of G.
The labeling pattern of $S(T_5)$ is shown below.

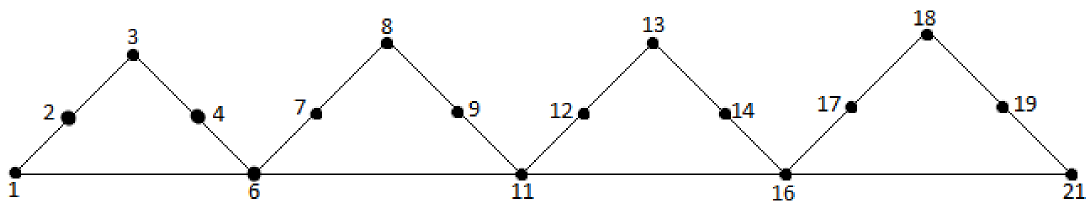

Figure2

Case(3): G is obtained by subdividing all the edges of T_n.
Let x_i, y_i and t_i be the vertices which subdivide the edges u_iv_i, v_iu_{i+1} and u_iu_{i+1} respectively. Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by
$f(u_i) = 6i - 5, 1 \le i \le n$
$f(v_i) = 6i - 3, 1 \le i \le n - 1$
$f(x_i) = 6i - 4, 1 \le i \le n - 1$
$f(y_i) = 6i - 1, 1 \le i \le n - 1$
$f(t_i) = 6i - 2, 1 \le i \le n - 1$

Then the edges are labeled as
$f(u_it_i) = 6i - 3, 1 \le i \le n - 1$
$f(u_ix_i) = 6i - 5, 1 \le i \le n - 1$
$f(x_iv_i) = 6i - 4, 1 \le i \le n - 1$
$f(t_iu_{i+1}) = 6i - 1, 1 \le i \le n - 1$
$f(v_iy_i) = 6i - 2, 1 \le i \le n - 1$
$f(y_iu_{i+1}) = 6i, 1 \le i \le n - 1$

Then we get distinct edge labels. Hence f is a Root Square Mean labeling of G.
The labeling pattern of $S(T_5)$ is shown below.

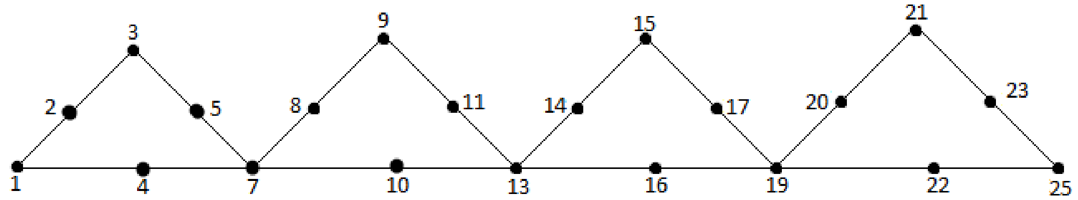

Figure3

From case(1) case(2), case(3), it can be seen that $S(T_n)$ is a Root Square Mean graph.

Theorem2.2: $S(Q_n)$ is a Root Square Mean graph.

Proof: Let $u_1u_2 \cdots u_n$ be a path P_n .Join u_i and u_{i+1} to new vertices v_i and w_i, $1 \le i \le n - 1$ respectively and then joining v_i and w_i . The resulting graph is a Quadrilateral snake Q_n . Let G be the graph obtained by subdividing the edges of Q_n . Here we consider the following cases.

Case(1): G is obtained by subdividing the edges of the path.
Let $t_1, t_2, \cdots, t_{n-1}$ be the vertices which subdivide the edge $u_iu_{i+1}, 1 \le i \le n - 1$.
Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by
$f(u_i) = 5i - 4, 1 \le i \le n$
$f(v_i) = 5i - 3, 1 \le i \le n - 1$
$f(w_i) = 5i - 2, 1 \le i \le n - 1$
$f(t_i) = 5i, 1 \le i \le n - 1$
Then the edges are labeled as

$f(u_iv_i) = 5i - 4, 1 \leq i \leq n - 1$
$f(v_iw_i) = 5i - 3, 1 \leq i \leq n - 1$
$f(w_iu_{i+1}) = 5i - 1, 1 \leq i \leq n - 1$
$f(u_it_i) = 5i - 2, 1 \leq i \leq n - 1$
$f(t_iu_{i+1}) = 5i, 1 \leq i \leq n - 1$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(Q_5)$ is shown below.

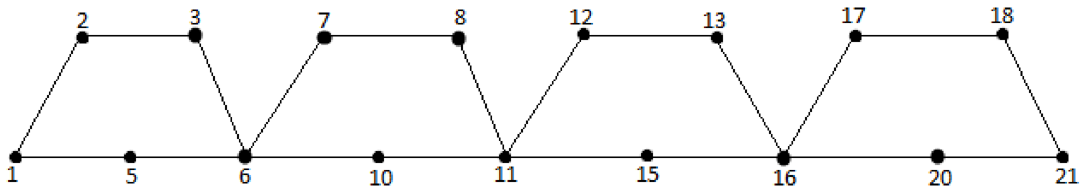

Figure4

Case(2): G is obtained by subdividing all the edges of Q_n.
Let t_i, x_i, s_i, y_i be the vertices which subdivide the edges $u_iu_{i+1}, u_iv_i, v_iw_i$ and w_iu_{i+1} respectively. Define a function $f: V(G) \rightarrow \{1,2, \dots, q + 1\}$ by
$f(u_i) = 8i - 7, 1 \leq i \leq n$
$f(x_i) = 8i - 6, 1 \leq i \leq n - 1$
$f(v_i) = 8i - 5, 1 \leq i \leq n - 1$
$f(s_i) = 8i - 4, 1 \leq i \leq n - 1$
$f(w_i) = 8i - 3, 1 \leq i \leq n - 1$
$f(y_i) = 8i - 2, 1 \leq i \leq n - 1$
$f(t_i) = 8i, 1 \leq i \leq n - 1$

Then the edges are labeled as
$f(u_ix_i) = 8i - 7, 1 \leq i \leq n - 1$
$f(x_iv_i) = 8i - 6, 1 \leq i \leq n - 1$
$f(v_is_i) = 8i - 5, 1 \leq i \leq n - 1$
$f(s_iw_i) = 8i - 4, 1 \leq i \leq n - 1$
$f(w_iy_i) = 8i - 2, 1 \leq i \leq n - 1$
$f(y_iu_{i+1}) = 8i - 1, 1 \leq i \leq n - 1$
$f(u_{i+1}t_i) = 8i, 1 \leq i \leq n - 1$
$f(t_iu_i) = 8i - 3, 1 \leq i \leq n - 1$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(Q_4)$ is shown below.

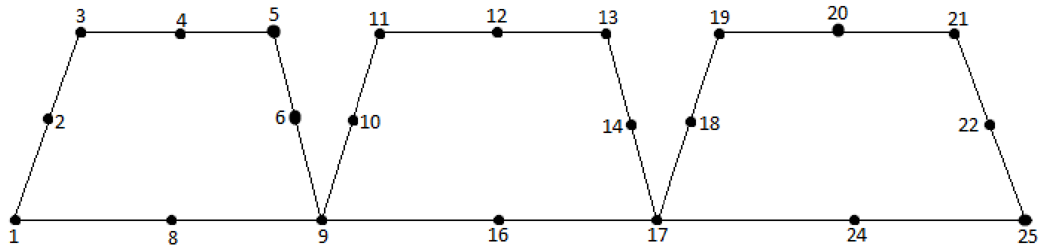

Figure5

From case(1), case(2), it is clear that, $S(Q_n)$ is a Root Square Mean graph.

Theorem2.3: $S(A(T_n))$ is a Root Square Mean graph.

Proof: Let $u_1 u_2 \cdots u_n$ be the path. Let $A(T_n)$ be the alternate triangular snake obtained by joining u_i and u_{i+1} (Alternatively) to a new vertex $v_i, 1 \leq i \leq n-1$. Let G be the graph obtained by subdividing the edges of $A(T_n)$. Here we consider two cases

Case(1): If the triangle starts from u_1 .
Let t_i, x_i, y_i be the vertices which subdivide the edges $u_i u_{i+1}, u_i v_i, v_i u_{i+1}$ respectively.
Here we have to consider two sub cases

Sub case(1.a): If n is odd
Define a function $f: V(G) \rightarrow \{1,2, \dots, q+1\}$ by

$$f(u_i) = \begin{cases} 4i-3, & i = 1,3,5, \dots, n \\ 4i-1, & i = 2,4,6, \dots, n-1 \end{cases}$$

$$f(v_i) = 8i-5, 1 \leq i \leq \frac{n-1}{2}$$

$$f(x_i) = 8i-6, 1 \leq i \leq \frac{n-1}{2}$$

$$f(y_i) = 8i-4, 1 \leq i \leq \frac{n-1}{2}$$

$$f(t_i) = \begin{cases} 4i+2, & i = 1,3,5, \dots, n-2 \\ 4i, & i = 2,4,6, \dots, n-1 \end{cases}$$

Then the edges are labeled as

$$f(u_i t_i) = \begin{cases} 4i, & i = 1,3,5, \dots, n-2 \\ 4i-1, & i = 2,4,6, \dots, n-1 \end{cases}$$

$$f(t_i u_{i+1}) = \begin{cases} 4i+2, & i = 1,3,5, \dots, n-2 \\ 4i, & i = 2,4,6, \dots, n-1 \end{cases}$$

$$f(u_{2i-1} x_i) = 8i-7, 1 \leq i \leq \frac{n-1}{2}$$

$$f(x_i v_i) = 8i-6, 1 \leq i \leq \frac{n-1}{2}$$

$f(v_iy_i) = 8i - 5, 1 \le i \le \frac{n-1}{2}$

$f(y_iu_{2i}) = 8i - 3, 1 \le i \le \frac{n-1}{2}$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(A(T_5))$ is shown below.

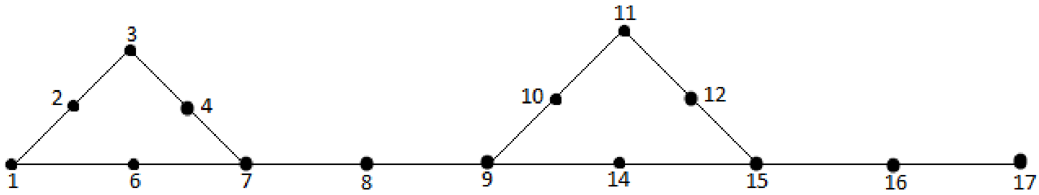

Figure6

Sub Case(1.b): If n is even
Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$$f(u_i) = \begin{cases} 4i-3, & i = 1,3,5,\dots,n-1 \\ 4i-1, & i = 2,4,6,\dots,n \end{cases}$$

$f(v_i) = 8i - 5, 1 \le i \le \frac{n}{2}$

$f(x_i) = 8i - 6, 1 \le i \le \frac{n}{2}$

$f(y_i) = 8i - 4, 1 \le i \le \frac{n}{2}$

$$f(t_i) = \begin{cases} 4i+2, & i = 1,3,5,\dots,n-1 \\ 4i, & i = 2,4,6,\dots,n-2 \end{cases}$$

Then the edges are labeled as

$$f(u_it_i) = \begin{cases} 4i, & i = 1,3,5,\dots,n-1 \\ 4i-1, & i = 2,4,6,\dots,n-2 \end{cases}$$

$$f(t_iu_{i+1}) = \begin{cases} 4i+2, & i = 1,3,5,\dots,n-1 \\ 4i, & i = 2,4,6,\dots,n-2 \end{cases}$$

$f(u_{2i-1}x_i) = 8i - 7, 1 \le i \le \frac{n}{2}$

$f(x_iv_i) = 8i - 6, 1 \le i \le \frac{n}{2}$

$f(v_iy_i) = 8i - 5, 1 \le i \le \frac{n}{2}$

$f(y_iu_{2i}) = 8i - 3, 1 \le i \le \frac{n}{2}$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(A(T_6))$ is shown below.

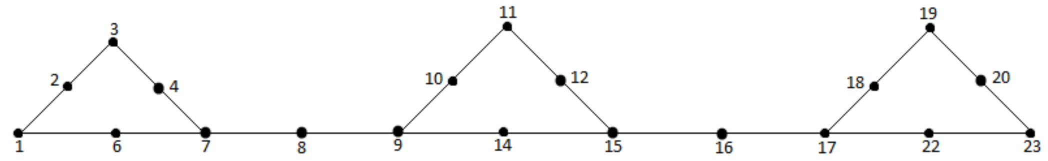

Figure7

Case(2) If the triangle starts from u_2.
Let t_i, x_i, y_i be the vertices which subdivide the edges $u_iu_{i+1}, u_iv_i, v_iu_{i+1}$ respectively.
Here we have to consider two sub cases

Sub case(2.a): If n is odd
Define a function $f: V(G) \rightarrow \{1,2,\ldots,q+1\}$ by

$$f(u_i) = \begin{cases} 4i-3, & i = 1,3,5,\ldots,n \\ 4i-5, & i = 2,4,6,\ldots,n-1 \end{cases}$$

$$f(v_i) = 8i-3, 1 \le i \le \frac{n-1}{2}$$

$$f(x_i) = 8i-4, 1 \le i \le \frac{n-1}{2}$$

$$f(y_i) = 8i-2, 1 \le i \le \frac{n-1}{2}$$

$$f(t_i) = \begin{cases} 4i-2, & i = 1,3,5,\ldots,n-2 \\ 4i, & i = 2,4,6,\ldots,n-1 \end{cases}$$

Then the edges are labeled as

$$f(u_it_i) = \begin{cases} 4i-3, & i = 1,3,5,\ldots,n-2 \\ 4i-2, & i = 2,4,6,\ldots,n-1 \end{cases}$$

$$f(t_iu_{i+1}) = \begin{cases} 4i-2, & i = 1,3,5,\ldots,n-2 \\ 4i, & i = 2,4,6,\ldots,n-1 \end{cases}$$

$$f(u_{2i}x_i) = 8i-5, 1 \le i \le \frac{n-1}{2}$$

$$f(x_iv_i) = 8i-4, 1 \le i \le \frac{n-1}{2}$$

$$f(v_iy_i) = 8i-3, 1 \le i \le \frac{n-1}{2}$$

$$f(y_iu_{2i+1}) = 8i-1, 1 \le i \le \frac{n-1}{2}$$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(A(T_5))$ is shown below.

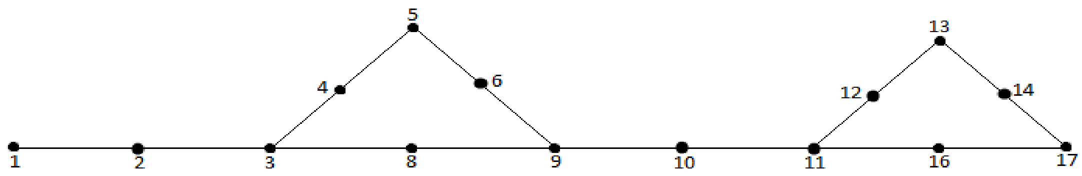

Figure8

Sub case(2.b): If n is even

Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$$f(u_i) = \begin{cases} 4i-3, & i = 1,3,5,\dots,n-1 \\ 4i-5, & i = 2,4,6,\dots,n \end{cases}$$

$$f(v_i) = 8i-3, 1 \le i \le \frac{n-2}{2}$$

$$f(x_i) = 8i-4, 1 \le i \le \frac{n-2}{2}$$

$$f(y_i) = 8i-2, 1 \le i \le \frac{n-2}{2}$$

$$f(t_i) = \begin{cases} 4i-2, & i = 1,3,5,\dots,n-1 \\ 4i, & i = 2,4,6,\dots,n-2 \end{cases}$$

Then the edges are labeled as

$$f(u_i t_i) = \begin{cases} 4i-3, & i = 1,3,5,\dots,n-1 \\ 4i-2, & i = 2,4,6,\dots,n-2 \end{cases}$$

$$f(t_i u_{i+1}) = \begin{cases} 4i-2, & i = 1,3,5,\dots,n-1 \\ 4i, & i = 2,4,6,\dots,n-2 \end{cases}$$

$$f(u_{2i} x_i) = 8i-5, 1 \le i \le \frac{n-2}{2}$$

$$f(x_i v_i) = 8i-4, 1 \le i \le \frac{n-2}{2}$$

$$f(v_i y_i) = 8i-3, 1 \le i \le \frac{n-2}{2}$$

$$f(y_i u_{2i+1}) = 8i-1, 1 \le i \le \frac{n-2}{2}$$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.

The labeling pattern of $S(A(T_6))$ is shown below.

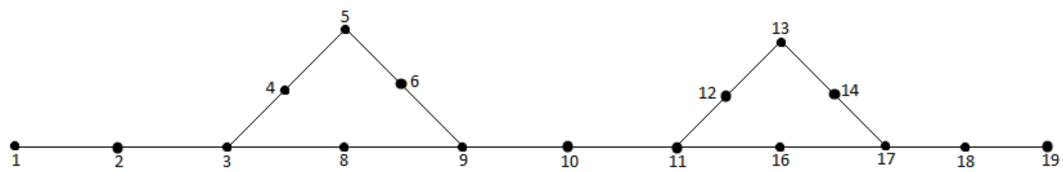

Figure9

From case(1) and case(2), $S(A(T_n))$ is a Root Square Mean graph.

Theorem2.4: $S(A(Q_n))$ is a Root Square Mean graph.

Proof: Let $u_1u_2\cdots u_n$ be the path . $A(Q_n)$ is obtained by joining u_i and u_{i+1} (Alternatively) to two new vertices $v_i$and w_i respectively and then joining $v_i$and w_i . Let G be the graph obtained by subdividing the edges of $A(Q_n)$. Here we consider two cases.

Case(1): If the Quadrilateral snake starts from u_1 .
Let t_i, x_i, y_i, s_i be the vertices which subdivide the edges $u_iu_{i+1}, u_iv_i, w_iu_{i+1}, v_iw_i$ respectively.
Here we have to consider two sub cases.

Sub case(1.a): If n is odd.
Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$$f(u_i) = \begin{cases} 5i-4, & i = 1,3,5,\dots,n \\ 5i-1, & i = 2,4,6,\dots,n-1 \end{cases}$$

$$f(v_i) = 10i-7, 1 \le i \le \frac{n-1}{2}$$

$$f(w_i) = 10i-5, 1 \le i \le \frac{n-1}{2}$$

$$f(x_i) = 10i-8, 1 \le i \le \frac{n-1}{2}$$

$$f(y_i) = 10i-4, 1 \le i \le \frac{n-1}{2}$$

$$f(s_i) = 10i-6, 1 \le i \le \frac{n-1}{2}$$

$$f(t_i) = \begin{cases} 5i+2, & i = 1,3,5,\dots,n-2 \\ 5i, & i = 2,4,6,\dots,n-1 \end{cases}$$

Then the edges are labeled as

$$f(u_it_i) = \begin{cases} 5i, & i = 1,3,5,\dots,n-2 \\ 5i-1, & i = 2,4,6,\dots,n-1 \end{cases}$$

$$f(t_iu_{i+1}) = \begin{cases} 5i+3, & i = 1,3,5,\dots,n-2 \\ 5i, & i = 2,4,6,\dots,n-1 \end{cases}$$

$$f(u_{2i-1}x_i) = 10i-9, 1 \le i \le \frac{n-1}{2}$$

$$f(x_iv_i) = 10i-8, 1 \le i \le \frac{n-1}{2}$$

$$f(v_is_i) = 10i-7, 1 \le i \le \frac{n-1}{2}$$

$$f(s_iw_i) = 10i-6, 1 \le i \le \frac{n-1}{2}$$

$$f(w_iy_i) = 10i-4, 1 \le i \le \frac{n-1}{2}$$

$f(y_i u_{2i}) = 10i - 3, 1 \leq i \leq \frac{n-1}{2}$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(A(Q_5))$ is shown below.

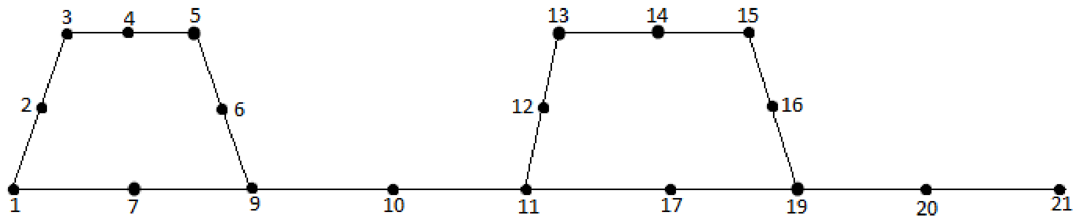

Figure10

Sub case(1.b) If n is even
Define a function $f: V(G) \rightarrow \{1,2, \dots, q+1\}$ by

$f(u_i) = \begin{cases} 5i-4, & i = 1,3,5, \dots, n-1 \\ 5i-1, & i = 2,4,6, \dots, n \end{cases}$

$f(v_i) = 10i - 7, 1 \leq i \leq \frac{n}{2}$

$f(w_i) = 10i - 5, 1 \leq i \leq \frac{n}{2}$

$f(x_i) = 10i - 8, 1 \leq i \leq \frac{n}{2}$

$f(y_i) = 10i - 4, 1 \leq i \leq \frac{n}{2}$

$f(s_i) = 10i - 6, 1 \leq i \leq \frac{n}{2}$

$f(t_i) = \begin{cases} 5i+2, & i = 1,3,5, \dots, n-1 \\ 5i, & i = 2,4,6, \dots, n-2 \end{cases}$

Then the edges are labeled as

$f(u_i t_i) = \begin{cases} 5i, & i = 1,3,5, \dots, n-1 \\ 5i-1, & i = 2,4,6, \dots, n-2 \end{cases}$

$f(t_i u_{i+1}) = \begin{cases} 5i+3, & i = 1,3,5, \dots, n-1 \\ 5i, & i = 2,4,6, \dots, n-2 \end{cases}$

$f(u_{2i-1} x_i) = 10i - 9, 1 \leq i \leq \frac{n}{2}$

$f(x_i v_i) = 10i - 8, 1 \leq i \leq \frac{n}{2}$

$f(v_i s_i) = 10i - 7, 1 \leq i \leq \frac{n}{2}$

$f(s_i w_i) = 10i - 6, 1 \leq i \leq \frac{n}{2}$

$f(w_i y_i) = 10i - 4, 1 \leq i \leq \frac{n}{2}$

$f(y_i u_{2i}) = 10i - 3, 1 \le i \le \frac{n}{2}$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(A(Q_6))$ is shown below.

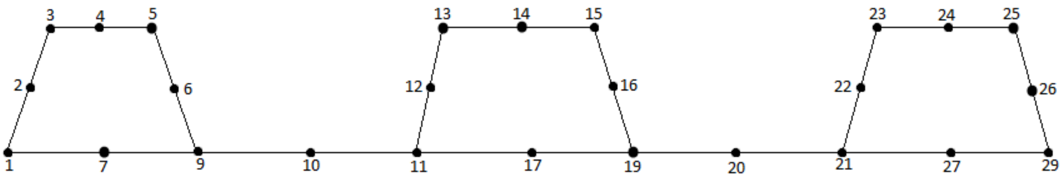

Figure11

Case(2): If the triangle starts from u_2.
Let t_i, x_i, y_i, s_i be the vertices which subdivide the edges $u_i u_{i+1}, u_i v_i, w_i u_{i+1}, v_i w_i$ respectively.
Here we consider two sub cases

Sub case(1.a): If n is odd.
Define a function $f: V(G) \to \{1,2, \ldots, q + 1\}$ by

$f(u_i) = \begin{cases} 5i - 4, & i = 1,3,5, \ldots, n \\ 5i - 7, & i = 2,4,6, \ldots, n - 1 \end{cases}$

$f(v_i) = 10i - 5, 1 \le i \le \frac{n-1}{2}$

$f(w_i) = 10i - 3, 1 \le i \le \frac{n-1}{2}$

$f(x_i) = 10i - 6, 1 \le i \le \frac{n-1}{2}$

$f(y_i) = 10i - 2, 1 \le i \le \frac{n-1}{2}$

$f(s_i) = 10i - 4, 1 \le i \le \frac{n-1}{2}$

$f(t_i) = \begin{cases} 5i - 3, & i = 1,3,5, \ldots, n - 2 \\ 5i - 1, & i = 2,4,6, \ldots, n - 1 \end{cases}$

Then the edges are labeled as

$f(t_i u_{i+1}) = \begin{cases} 5i - 3, & i = 1,3,5, \ldots, n - 2 \\ 5i, & i = 2,4,6, \ldots, n - 1 \end{cases}$

$f(u_i t_i) = 5i - 4, 1 \le i \le n - 1$

$f(u_{2i} x_i) = 10i - 7, 1 \le i \le \frac{n-1}{2}$

$f(x_i v_i) = 10i - 6, 1 \le i \le \frac{n-1}{2}$

$f(v_i s_i) = 10i - 5, 1 \le i \le \frac{n-1}{2}$

$f(s_iw_i) = 10i - 3, 1 \le i \le \frac{n-1}{2}$

$f(w_iy_i) = 10i - 2, 1 \le i \le \frac{n-1}{2}$

$f(y_iu_{2i+1}) = 10i - 1, 1 \le i \le \frac{n-1}{2}$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(A(Q_5))$ is shown below.

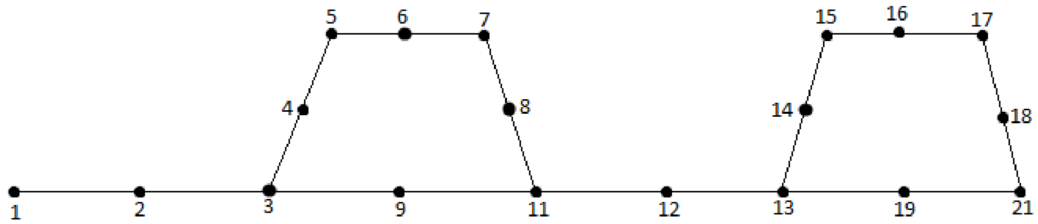

Figure12

Sub case(1.a): If n is even.
Define a function $f: V(G) \to \{1,2,\dots,q+1\}$ by

$f(u_i) = \begin{cases} 5i - 4, & i = 1,3,5,\dots,n-1 \\ 5i - 7, & i = 2,4,6,\dots,n \end{cases}$

$f(v_i) = 10i - 5, 1 \le i \le \frac{n-2}{2}$

$f(w_i) = 10i - 3, 1 \le i \le \frac{n-2}{2}$

$f(x_i) = 10i - 6, 1 \le i \le \frac{n-2}{2}$

$f(y_i) = 10i - 2, 1 \le i \le \frac{n-2}{2}$

$f(s_i) = 10i - 4, 1 \le i \le \frac{n-2}{2}$

$f(t_i) = \begin{cases} 5i - 3, & i = 1,3,5,\dots,n-1 \\ 5i - 1, & i = 2,4,6,\dots,n-2 \end{cases}$

Then the edges are labeled as

$f(t_iu_{i+1}) = \begin{cases} 5i - 3, & i = 1,3,5,\dots,n-1 \\ 5i, & i = 2,4,6,\dots,n-2 \end{cases}$

$f(u_it_i) = 5i - 4, 1 \le i \le n - 1$

$f(u_{2i}x_i) = 10i - 7, 1 \le i \le \frac{n-2}{2}$

$f(x_iv_i) = 10i - 6, 1 \le i \le \frac{n-2}{2}$

$f(v_is_i) = 10i - 5, 1 \le i \le \frac{n-2}{2}$

$f(s_i w_i) = 10i - 3, 1 \leq i \leq \frac{n-2}{2}$
$f(w_i y_i) = 10i - 2, 1 \leq i \leq \frac{n-2}{2}$
$f(y_i u_{2i+1}) = 10i - 1, 1 \leq i \leq \frac{n-2}{2}$

Then the edge labels are distinct. Hence G is a Root Square Mean graph.
The labeling pattern of $S(A(Q_6))$ is shown below.

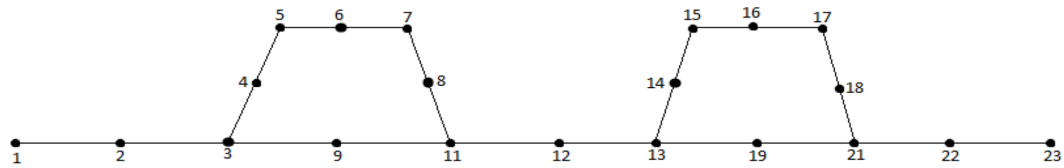

Figure13

From case(1) and case(2), $S(A(Q_n))$ is a Root Square Mean graph.

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