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Some Results On Super Geometric Mean Labeling

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ABSTRACT

Let f:V(G)→{1,2,. . . ,p+q} be an injective function. For a vertex labeling “f” the induced edge labeling f*(e=uv) is defined by,

$$f^*(e) = \left\lceil \sqrt{f(u)f(v)} \right\rceil or \left\lfloor \sqrt{f(u)f(v)} \right\rfloor$$

then “f” is called a **Super Geometric Mean labeling** if {f(V(G)} U {f(e):e∈E(G)} = {1,2,. . . p+q}.A graph which admits Super Geometric Mean labeling is called **Super Geometric Mean Graph.**
In this paper we investigate Super Geometric Mean Labeling for Some Standard Graphs.

Key words: Graph, Mean graph, Geometric mean graph, Super Geometric mean graph, Ladder, Comb.

1.Introduction

We begin with simple finite connected and undirected graph G=(V,E) with p vertices and q edges. The vertex set is denoted by V(G) and the edge set is denoted by E(G). A **Path** of length ‘n’ is denoted by P_n and the Cycle of length ‘n’ is denoted by C_n. For all other standard terminology and notations we follow Harary[2] and for the detailed survey of graph labeling we follow Gallian J.A[1].
The Concept of "Geometric Mean Labeling" has introduced by S. Somasundaram, R. Ponraj, and P. Vidhyarani in [6].
Somasundaram. S and Ponraj.R introduced **“Mean Labeling”** in [4]. The concept of

“**Harmonic Mean Labeling”** has introduced by Somasundaram.S, Ponraj.R and Sandhya.S.S in[5]. Jeyasekaran.C, Sandhya.S.S. and David Raj.C has introduced “**Super Harmonic Mean Labeling”** in [3].
In this paper we discuss **“Super Geometric Mean Labeling”** behavior for some standard graphs.
The definitions which are useful for the present investigation are given below.

Definition 1.1:
A graph G=(V,E) with p vertices and q edges is called a **Geometric Mean Graph** if it is possible to label vertices x∈V with distinct label f(x) from 1,2, . . . , q+1 in such a way that when each edge e=uv is labeled with,

$$f(e=uv)=\left\lceil\sqrt{f(u)f(v)}\right\rceil \text{ or } \left\lfloor\sqrt{f(u)f(v)}\right\rfloor$$

then the edge labels are distinct. In this case, “f” is called **Geometric Mean Labeling** of G.

Definition 1.2:
Let f:V(G)→{1,2, …, p+q} be an injective function. For a vertex labeling “f”, the induced edge labeling f*(e=uv) is defined by,

$$f^*(e)=\left\lceil\sqrt{f(u)f(v)}\right\rceil \text{ or } \left\lfloor\sqrt{f(u)f(v)}\right\rfloor$$

Then “f” is called a **Super Geometric Mean labeling** if
{*f*(V(G)} U {f(e):e∈E(G)}= {1,2,…,p+q}.
A graph which admits Super Geometric Mean labeling is called **Super Geometric Mean Graph.**

Example1.3: A Super Geometric Mean labeling of a graph G is shown below.

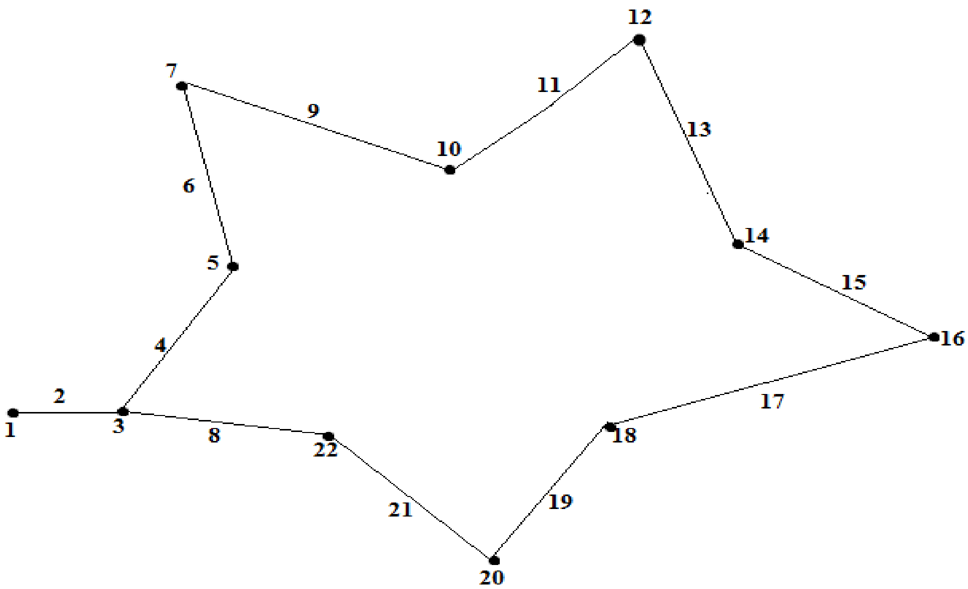

Figure: 1

Definition 1.4
The **Ladder** L_n, $n\geq 2$, is the product graph P_n x P_2 and contains 2n vertices and 3n-2 edges.

Definition 1.5
The graph obtained by joining a single pendant edge to each vertex of a Path is called a **Comb**($P_n \odot K_1$).
Now we shall use frequent reference to the following theorems.

Theorem 1.6[6]
Crowns are Geometric mean graphs.

Theorem 1.7[6]
Ladders are Geometric mean graphs.

Theorem 1.8[6]
Combs are Geometric mean graphs.

Theorem 1.9[3]
Crowns are Super Harmonic mean graphs.

Theorem 1.10[3]
Comb is a Super Harmonic mean graph.

Remarks 1.11
In a Super Geometric mean labeling, the labels of edges and vertices are together from 1,2, …,p+q.

2.Main Results

Theorem 2.1
Let G be a graph obtained from a Ladder L_n, $n\geq 2$ by joining a pendant vertex with a vertex of degree two on both sides of upper and lower Path of the Ladder. Then G is Super Geometric mean Graph.

Proof:
Let $L_n=P_n$ x P_2 be a Ladder.
Let G be a graph obtained from a Ladder by joining pendant vertices *u,w,x,z* with v_1, v_n,u_1,u_n (vertices of degree 2) respectively on both sides of upper and lower Path of the Ladder.
Define a function $f:V(G)\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(u) = 1$

$f(v_1) = 5$

$f(v_i) = 5i-1,\ 2\leq i\leq n$

$f(w) = 5n+5$

$f(x) = 3$
$f(u_i) = 5i+3,\ 1 \le i \le n$
$f(z) = 5n+6$
Edges are labeled with,
$f(v_iv_{i+1}) = 5i+2,\quad 1 \le i \le n-1$
$f(uv_1) = 2$
$f(v_nw) = 5n+2$
$f(xu_1) = 4$
$f(u_iu_{i+1}) = 5i+5,\ 1 \le i \le n-1$
$f(u_nz) = 5n+4$
$f(v_iu_i) = 5i+1,\ 1 \le i \le n$

In view of the above labeling pattern, *f* provides a Super Geometric Mean labeling of G.
Hence G is a Super Geometric mean Graph.

Example: 2.2
A Super Geometric mean labeling of G when n=5 is shown below.

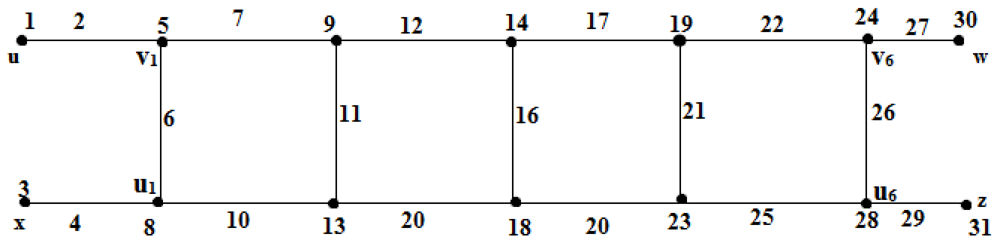

Figure : 2

Theorem 2.3:
Let G be a graph obtained by joining a pendant vertex with a vertex, of degree two of a Comb graph. Then G is a Super Geometric mean graph.

Proof:
Comb ($P_n \odot K_1$) is a graph obtained from a Path $P_n=v_1v_2v_3\ldots v_n$ by joining a vertex u_i to v_i, $1 \le i \le n$.
Let G be a graph obtained by joining a pendant vertex w to v_n(a vertex of degree 2)
Define a function $f:V(G) \rightarrow \{1,2,3,\ldots,p+q\}$ by,
$f(v_i) = 4i-1,\ 1 \le i \le n$
$f(w) = 4n+1$
$f(u_i) = 4i-3, 1 \le i \le n$
Edges are labeled with,
$f(v_iv_{i+1}) = 4i,\ 1 \le i \le n-1$
$f(v_nw) = 4n$
$f(v_iu_i) = 4i-2,\ 1 \le i \le n$

Then the edge labels are distinct.
Hence G is a Super Geometric mean Graph.

Example:2.4
A Super Geometric mean labeling of G when n=6 is given below.

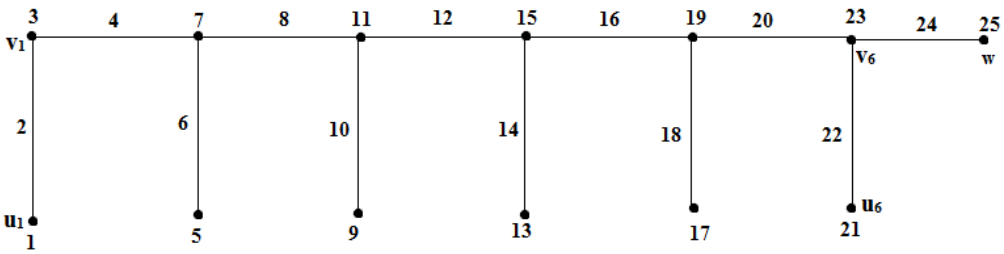

Figure: 3

Theorem 2.5
Let G be a graph obtained by joining a pendant vertex with a vertex of degree two on both sides of a Comb graph. Then G is a Super Geometric mean Graph.

Proof:
Comb ($P_n \odot K_1$) is a graph obtained from a Path $P_n=v_1v_2v_3\ldots v_n$ by joining a vertex u_i to v_i, $1\leq i\leq n$.
Let G be a graph obtained by joining pendant vertices w and z to v_1 and v_n respectively.
Define a function $f:V(G)\rightarrow\{1,2,3,\ldots,p+q\}$ by,

$f(w) = 1$
$f(v_1) = 3$
$f(v_i) = 4i+1,\ 2 \leq i \leq n$
$f(z) = 4n+3$
$f(u_1) = 5$
$f(u_i) = 4i-1,\ 2 \leq i \leq n$

Edges are labeled with,

$f(wv_1)= 2$
$f(v_iv_{i+1})= 4i+2,\ 1 \leq i \leq n-1$
$f(v_nz) = 4n+2$
$f(v_iu_i)=4i,\ 1 \leq i \leq n$

$\therefore f(V(G))U\ \{f(e):e\in E(G)\} = \{1,2,\ldots..,p+q\}$
Thus f is a Super Geometric mean Labeling of G.
Hence G is a Super Geometric mean Graph.

Example:2.6
A Super Geometric mean labeling of G when n=5 is given below.

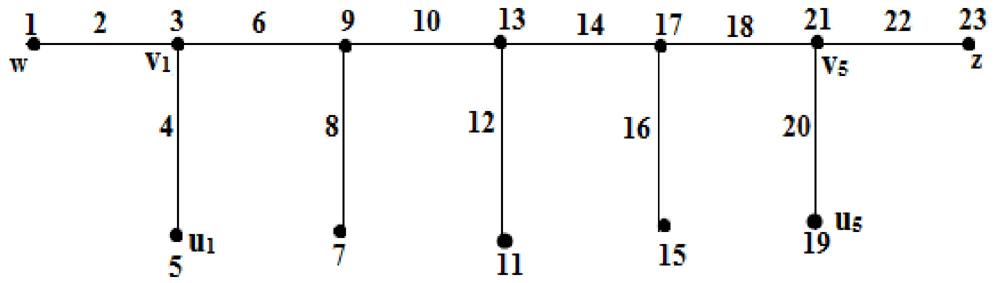

Figure: 4

Theorem 2.7

Let P_n be a Path and G be the graph obtained from P_n by attaching C_3 in both end edges of P_n. Then G is a Super Geometric mean graph.

Proof:

Let P_n be a Path $u_1u_2\ldots u_n$ and $v_1u_1u_2$, $v_2\, u_{n-1}u_n$ be the triangles at the end edges of P_n.
Define a function $f:V(G)\rightarrow\{1,2,3,\ldots p+q\}$ by,

$f(v_1) = 4$
$f(u_1) = 1$
$f(u_i) = 2i+2,\ 2 \le i \le n-1$
$f(u_n) = 2n+5$
$f(v_2) = 2n+2$

Edges are labeled with,

$f(v_1u_1) = 2$
$f(v_1u_2) = 5$
$f(u_1u_2) = 3$
$f(u_iu_{i+1}) = 2i+3,\ 2 \le i \le n-2$
$f(u_{n-1}u_n) = 2n+3$
$f(v_2u_{n-1}) = 2n+1$
$f(v_2u_n) = 2n+4$

Thus both vertices and edges together get distinct lables from $\{1,2,\ldots,p+q\}$.
Hence G is a Super Geometric mean Graph.

Example: 2.8

A Super Geometric mean labeling of G obtained from P_8 is given below.

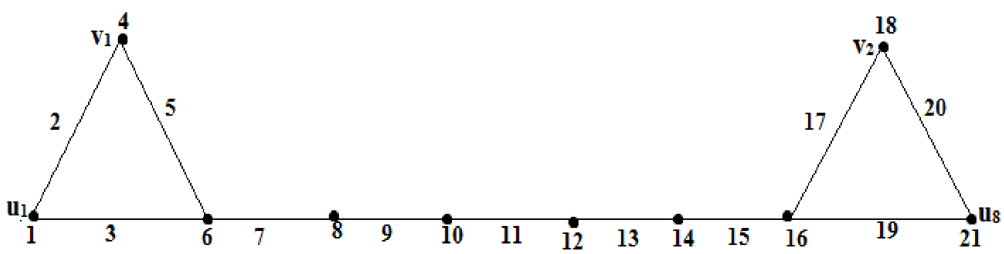

Figure: 5

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