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Mathematical model for Prey Predator system with Immigrant Prey

Goteti V. R. L. Sarma and Alfred Hugo
Department of Mathematics,
University of Dodoma, Dodoma, Tanzania.
E-mail: gvrlsarma@rediffmail.com, alfredhugo@ymail.com

Nanduri Lakshmi
Department of Basic Sciences and Humanities,
Pragati Engineering College, Surampalem,
E.G.Dt., A.P. India.
E-mail: nlakshmi2@rediffmail.com

Abstract

In this article a mathematical model for predator prey system with immigrant prey is proposed.In this study we found the equilibrium points and analysed their local and global stabilities. The results have been illustrated using numerical simulations along with their graphs for certain parametric values. These results can also be interpreted in terms of import of commodities and their impact on consumers.

AMS subject classification:
Keywords: Predator, prey, immigrant prey, stability.

1. Introduction

Ever since Lokta Volterra designed the system of differential equation to understand the interaction between predator and prey in any environment, researchers made many alterations by introducing different facets to Predator prey equations like harvesting prey and predators, delay in predator growth, providing additional food to predator to sustain prey population, diseases prey etc see [3–6]. In 1997 S.V. Krishna et al. [8] studied the optimal tax policy in predator prey system with harvest. Adaptive control with known and unknown parameters is studied my El-Gohary, A., and Al-Ruzaiza, A. S. [1] and [2] by introducing the control parmeters in two prey one predator system. Many authors studied the existence of Hopf bifurcation and periodic solution related to predator prey systems.

In this article we want to introduce the novice concept of immigrant prey and explore the consequence of immigrant prey when enter in prey predator system. We believe this is entirely new concept and adds a new dimension to the existing predator prey systems hence offers possibilities for further indepth studies. Our model can also viewed as Predator and two interactive prey model. We believe that this study also influence economic strategies regarding the immigration policy of nations in not only protecting the local commodities from over exploitation and also strengthening the local commodities qualitatively and quantitatively through cautiously allowing imports to country.

This research article is organized as follows: In section 2 we present our predator prey model with immigrant prey along with the assumptions made to derive the model. In section 3 we discuss the possible equilibrium points in this model along with their stability. In section 4 we discuss the global stability of the model with the construction of a suitable Lyapunov's function. In section 5 we illustrate numerical simulations of the model along with conclusions.

2. Model Formulation

We propose, formulate and analyze the prey predator system with immigrant prey to understand the interaction between local and immigrant prey and also their influence on predator. This dynamics is assumed to follow mass action theory. The model consists of prey population density denoted by $N(t) = S(t) + X(t)$ where $S(t)$ is local prey population density, $X(t)$ is immigrant prey population density and the predator population density is denoted by $Y(t)$.

We impose the following assumptions in formulating mathematical model:

(i) In the absence of predator, the local prey population grows logistically with intrinsic growth rate α_1 and having environmental carrying capacity k_1.

(ii) With availability of immigrant prey and local prey populations, the predator population growth logistically with intrinsic growth rate c_1 and c_2 while suffering loss at the rates μ_1 and μ_2

(iii) In the presence of predator, the prey population can be classified as two sub-classes, namely, the local prey $S(t)$ and the immigrant prey $X(t)$.

(iv) The local prey and immigrant prey can reproduce. Logistic law is used to model the birth process with the assumption that births should always be positive. The immigrant prey is increased at the rate α_2 and environmental carrying capacity k_2.

(v) We assume immigranr prey as the natural choice of predator. Hence the interaction between immigrant and local prey has positive effect on local prey with the force of interaction $\beta 1$while negative impact on immigrant prey with the force of interaction $\beta 2$.

(vi) It is assumed that the local prey is catched by predator the at the rate γ_1 and immigrant prey is catched by the predator at the rate ofγ_2 and it is assumed to follow the simple mass action in these interaction process.

Based on above assumptions, we propose a mathematical model that governed by the following system of the differential equations:

$$\begin{aligned}\frac{dS}{dt} &= S\left(\alpha_1 - \frac{\alpha_1 S}{k_1}\right) + \beta_1 SX - \gamma_1 SY,\\ \frac{dX}{dt} &= X\left(\alpha_2 - \frac{\alpha_2 X}{k_2}\right) - \beta_2 SX - \gamma_2 XY,\\ \frac{dY}{dt} &= c_1 SY + c_2 XY - \mu_1 Y - \mu_2 Y^2,\end{aligned} \tag{2.1}$$

with the initial populations:

$$S(0) = S_0,\, X(0) = X_0,\, Y(0) = Y_0. \tag{2.2}$$

3. Model analysis

We qualitatively analyze the model equations (2.1) and (2.2) to see the effect of immigrant prey toward the local prey and the predator.

3.1. Boundedness of the model

The boundedness of the system equations (2.1) implies that the system is well behaved.

Theorem 3.1. All solutions of the system (2.1) are uniformly bounded.

Proof. Assume W denote the total populations in the specific model, that is

$$W = S + X + Y \tag{3.1}$$

this gives

$$\frac{dW}{dt} = \frac{dS}{dt} + \frac{dX}{dt} + \frac{dY}{dt} \tag{3.2}$$

Now, substituting the model equations (2.1) into (3.2)and simplify we get

$$\frac{dW}{dt} \leq \alpha_1 S + \alpha_2 X - \mu_1 Y$$

on simplifying we get

$$\frac{dW}{dt} \leq \hat{p}_1(\alpha_1 + 1) + \hat{p}_2(\alpha_2 + 1) - hW \tag{3.3}$$

where $\hat{p}_1 = \max\{S(0), k_1\}$, $\hat{p}_2 = \max\{X(0), k_2\}$ and $h = \min\left\{1, 1, \mu_1\right\}$.
The equation (3.3) can be written as

$$\frac{dW}{dt} + hW \leq \left\{\hat{p}_1(\alpha_1 + 1) + \hat{p}_2(\alpha_2 + 1)\right\}. \tag{3.4}$$

Solving (3.4) and substituting the initial conditions we get

$$W \leq \frac{\left\{\hat{p}_1(\alpha_1 + 1) + \hat{p}_2(\alpha_2 + 1)\right\}}{h}(1 - e^{-ht}) \tag{3.5}$$

as $t \to \infty$ we have

$$W \leq \frac{\left\{\hat{p}_1(\alpha_1 + 1) + \hat{p}_2(\alpha_2 + 1)\right\}}{h}$$

which is implies that the solution is bounded for

$$0 \leq W \leq \frac{\left\{\hat{p}_1(\alpha_1 + 1) + \hat{p}_2(\alpha_2 + 1)\right\}}{h}.$$

Therefore, all solutions of the model (2.1) in $\Re^3_+$ are confined in the region

$$\Gamma = \left\{(S, X, Y) \in \Re^3_+ W \leq \frac{\left\{\hat{p}_1(\alpha_1 + 1) + \hat{p}_2(\alpha_2 + 1)\right\}}{h} + \varepsilon\right\}.$$

■

3.2. Positivity of solutions

For model (2.1) to be logically meaningful and well posed, we need to prove that all solutions of system with positive initial data will remain positive for all the times. This will be established by the following theorem.

Theorem 3.2. Let $S(0) > 0$, $X(0) > 0$, $Y(0) > 0$ this implies that $S(t)$, $X(t)$ and $Y(t)$ of system (2.1) are all positive for $\forall t \geq 0$.

Proof. To prove theorem (3.2), we use all equations of the model (2.1). From the 1st equation, we obtain the inequality expression as follows

$$\frac{dS}{dt} \leq \alpha_1 S\left(1 - \frac{S}{k_1}\right) \tag{3.6}$$

which gives

$$S \leq \frac{k_1 S(0)}{e^{-\alpha_1 t}\left\{k_1 - S(0)\right\} + S(0)} \tag{3.7}$$

As $t \to \infty$ we obtain $0 \leq S \leq k_1$. Hence all feasible solution of system (2.1) is feasible in region $\Gamma = \{S, X, Y\}$. Similar proofs can be established for the positivity of the other solutions. ■

4. Equilibria and stability analysis

4.1. Equilibrium points

The system of differential equation (2.1)has the following equilibrium points by setting

$$\frac{dS}{dt} = \frac{dX}{dt} = \frac{dY}{dt} = 0$$

The model equations (2.1) has the following equilibrium points

(i) A trial equilibrium point

$$E_T(S^*, X^*, Y^*) = (0, 0, 0),$$

(ii) The axial equilibrium point for local prey

$$E_L(S^*, X^*, Y^*) = (k_1, 0, 0),$$

(iii) The axial equilibrium point for immigrant prey

$$E_I(S^*, X^*, Y^*) = (0, k_2, 0),$$

(iv) The equilibrium point where no local prey involved in the system, that is $S^* = 0$ is

$$E(S^*, X^*, Y^*) = \left(0, \frac{k_2\left(\mu_2\alpha_2 + \gamma_2\mu_1\right)}{\gamma_2 k_2 c_2 + \mu_2\alpha_2}, \frac{\alpha_2\left(c_2k_2 - \mu_1\right)}{\gamma_2 k_2 c_2 + \mu_2\alpha_2}\right)$$

(v) The equilibrium point where no immigrant prey involved in the system, that is $X^* = 0$ is

$$E(S^*, X^*, Y^*) = \left(\frac{k_1\left(\mu_2\alpha_1 + \gamma_1\mu_1\right)}{c_1k_1\gamma_1 + \mu_2\alpha_1}, 0, \frac{\alpha_1\left(c_1k_1 - \mu_1\right)}{c_1k_1\gamma_1 + \mu_2\alpha_1}\right)$$

(vi) Endemic equilibrium point of the model equation (2.1) are

$$S^* = \frac{k_1\left(\mu_2\alpha_1\alpha_2 + c_2\alpha_1k_2\gamma_2 + \beta_1k_2\mu_2\alpha_2 + \beta_1k_2\mu_1\gamma_2 + \gamma_1\mu_1\alpha_2 - \gamma_1c_2k_2\alpha_2\right)}{c_1k_1\gamma_1\alpha_2 + \mu_2\alpha_1\alpha_2 + \mu_2\beta_2k_1k_2\beta_1 + c_1k_1\beta_1k_2\gamma_2 - c_2k_2\beta_2k_1\gamma_1 + c_2\alpha_1k_2\gamma_2}$$

$$X^* = \frac{k_2\left(c_1k_1\gamma_1\alpha_2 + \mu_2\alpha_1\alpha_2 - \mu_2\alpha_1\beta_2k_1 - c_1k_1\alpha_1\gamma_2 - \mu_1\beta_2k_1\gamma_1 + \mu_1\alpha_1\gamma_2\right)}{c_1k_1\gamma_1\alpha_2 + \mu_2\alpha_1\alpha_2 + \mu_2\beta_2k_1k_2\beta_1 + c_1k_1\beta_1k_2\gamma_2 - c_2k_2\beta_2k_1\gamma_1 + c_2\alpha_1k_2\gamma_2}$$

$$Y^* = \frac{\alpha_1c_1k_1\alpha_2 - \alpha_1\mu_1\alpha_2 - \alpha_1c_2k_2\beta_2k_1 + \alpha_1c_2k_2\alpha_2 + \alpha_2c_1k_1\beta_1k_2 - \beta_2k_1k_2\beta_1\mu_1}{c_1k_1\gamma_1\alpha_2 + \mu_2\alpha_1\alpha_2 + \mu_2\beta_2k_1k_2\beta_1 + c_1k_1\beta_1k_2\gamma_2 - c_2k_2\beta_2k_1\gamma_1 + c_2\alpha_1k_2\gamma_2}$$

Based on our assumptions and the nature of the model we can justify that all the equilibrium points are in the positive octant of the spatial region.

4.2. Local stability analysis

In this section, we study the local stability and existence criteria of the different equilibrium points by using the Jacobian matrix of the system of equations (2.1). From the model equation (2.1) we formulate the Jacobian matrix as follows:

$$J = \begin{bmatrix} \alpha_1 - 2\,\frac{\alpha_1 S}{k_1} + \beta_1 X - \gamma_1 Y & \beta_1 S & -\gamma_1 S \\ -\beta_2 X & \alpha_2 - 2\,\frac{\alpha_2 X}{k_2} - \beta_2 S - \gamma_2 Y & -\gamma_2 X \\ c_1 Y & c_2 Y & c_1 S + c_2 X - \mu_1 - 2\,\mu_2 Y \end{bmatrix} \tag{4.1}$$

We now check the stability around the different equilibrium points as follows: The trivial equilibrium point $E_T(S^*, X^*, Y^*) = (0, 0, 0)$ always exist but unstable. The axial equilibrium point for local prey $E_L(S^*, X^*, Y^*) = (k_1, 0, 0)$ will be stable if

$$c_1 k_1 < \mu_1 \quad and \quad \alpha_2 < \beta_2 k_1 \tag{4.2}$$

The axial equilibrium point for immigrant prey $E_L(S^*, X^*, Y^*) = (0, k_2, 0)$ will be stable if

$$c_2 k_2 < \mu_1 \quad and \quad \alpha_1 < -\beta_1 k_2 \tag{4.3}$$

With the absence of local prey the model will be stable if

$$\begin{bmatrix} -1/2\,\frac{\mu_1\mu_2\alpha_2 + \alpha_2\gamma_2\mu_1 + \mu_2{\alpha_2}^2 + c_2k_2\mu_2\alpha_2 - \sqrt{A_1}}{c_2k_2\gamma_2 + \mu_2\alpha_2} \\ -1/2\,\frac{-\mu_1\mu_2\alpha_2 + \alpha_2\gamma_2\mu_1 + \mu_2{\alpha_2}^2 + c_2k_2\mu_2\alpha_2 + \sqrt{A_1}}{c_2k_2\gamma_2 + \mu_2\alpha_2} \\ -\frac{\alpha_1c_2k_2\gamma_2 - \alpha_1\mu_2\alpha_2 - \beta_1k_2\mu_2\alpha_2 - \beta_1k_2\gamma_2\mu_1 + \gamma_1\alpha_2c_2k_2 - \gamma_1\alpha_2\mu_1}{c_2k_2\gamma_2 - \mu_2\alpha_2} \end{bmatrix} \tag{4.4}$$

where

$$\begin{aligned} A_1 = {}& {\mu_1}^2{\mu_2}^2{\alpha_2}^2 + 2\,\gamma_2{\mu_1}^2\mu_2{\alpha_2}^2 + 2\,\mu_1{\mu_2}^2{\alpha_2}^3 - 2\,\mu_1{\mu_2}^2{\alpha_2}^2c_2k_2 \\ & + {\alpha_2}^2{\gamma_2}^2{\mu_1}^2 + 2\,{\alpha_2}^3\gamma_2\mu_1\mu_2 + 2\,{\alpha_2}^2\gamma_2\mu_1c_2k_2\mu_2 + {\mu_2}^2{\alpha_2}^4 \\ & - 2\,c_2k_2{\mu_2}^2{\alpha_2}^3 + {c_2}^2{k_2}^2{\mu_2}^2{\alpha_2}^2 - 4\,{c_2}^2{k_2}^2\mu_2{\alpha_2}^2\gamma_2 \\ & + 4\,\alpha_2{\gamma_2}^2{\mu_1}^2c_2k_2 - 4\,\alpha_2{c_2}^2{k_2}^2{\gamma_2}^2\mu_1 \end{aligned}$$

with the absence of immigrant prey the model shows stability behaviour with the following eigenvalues

$$\left[\begin{array}{c} -1/2\,\dfrac{\mu_2{\alpha_1}^2+c_1k_1\mu_2\alpha_1-\mu_1\mu_2\alpha_1+\alpha_1\gamma_1\mu_1-\sqrt{A_2}}{c_1k_1\gamma_1+\mu_2\alpha_1} \\ -1/2\,\dfrac{\mu_2{\alpha_1}^2+c_1k_1\mu_2\alpha_1-\mu_1\mu_2\alpha_1+\alpha_1\gamma_1\mu_1+\sqrt{A_2}}{c_1k_1\gamma_1+\mu_2\alpha_1} \\ -\dfrac{\alpha_2c_1k_1\gamma_1+\beta_2k_1\gamma_1\mu_1+\gamma_2\alpha_1c_1k_1+\beta_2k_1\mu_2\alpha_1-\alpha_1\mu_2\alpha_2-\gamma_2\alpha_1\mu_1}{c_1k_1\gamma_1-\mu_2\alpha_1} \end{array}\right] \tag{4.5}$$

where

$$\begin{aligned} A_2 = {}& {\mu_2}^2{\alpha_1}^4-2\,{\mu_2}^2{\alpha_1}^3c_1k_1+2\,{\mu_2}^2{\alpha_1}^3\mu_1+2\,\mu_2{\alpha_1}^3\gamma_1\mu_1+{c_1}^2{k_1}^2{\mu_2}^2{\alpha_1}^2 \\ & -2\,c_1k_1{\mu_2}^2{\alpha_1}^2\mu_1+2\,c_1k_1\mu_2{\alpha_1}^2\gamma_1\mu_1+{\mu_1}^2{\mu_2}^2{\alpha_1}^2+2\,{\mu_1}^2\mu_2{\alpha_1}^2\gamma_1 \\ & +{\alpha_1}^2{\gamma_1}^2{\mu_1}^2+4\,c_1k_1{\gamma_1}^2\alpha_1{\mu_1}^2-4\,{c_1}^2{k_1}^2{\gamma_1}^2\alpha_1\mu_1-4\,{c_1}^2{k_1}^2\gamma_1{\alpha_1}^2\mu_2 \end{aligned}$$

The local stability of coexistence is given by the polynomial equation

$$\lambda^3+B_1\lambda^2+B_2\lambda+B_3=0 \tag{4.6}$$

where

$$B_1=-\frac{c_1S\,k_2k_1+B-k_1\gamma_2Y\,k_2+k_2\alpha_1k_1-2\,k_2\alpha_1S+k_2\beta_1\,k_1-k_2\gamma_1Y\,k_1}{k_2k_1}$$

where

$$B=c_2\,k_2k_1-\mu_1k_2k_1-2\,\mu_2Y\,k_2k_1+k_1\alpha_2k_2-2\,k_1\alpha_2-k_1\beta_2S\,k_2$$

$$B_2=-\frac{\begin{array}{l} D+c_1S^2k_1\beta_2k_2-4\,\mu_2Y\,k_1\alpha_2-2\,\mu_2Y\,k_1\beta_2S\,k_2-2\,\gamma_2Y\,k_2\alpha_1S \\ +2\,c_2\,k_2\alpha_1S+2\,\mu_2Y\,k_1\alpha_2k_2 \end{array}}{k_2k_1}$$

$$B_3=-\frac{\begin{array}{l} \mu_1\gamma_2Y\,k_2\beta_1\,k_1-\mu_1\beta_2S\,k_2\gamma_1Y\,k_1-c_2\,\beta_2S\,k_2\alpha_1k_1-c_2\,\alpha_2k_2\gamma_1Y\,k_1 \\ -2\,\mu_2Y^2\beta_2S\,k_2\gamma_1k_1+C-2\,c_2\,\alpha_2k_2\alpha_1S \end{array}}{k_2k_1}$$

where

$$\begin{aligned} D = {}& c_1S\,k_1\gamma_2Y\,k_2-c_1S\,k_2\alpha_1k_1-c_1S\,k_2\beta_1\,k_1-c_2\,k_1\alpha_2k_2+\alpha_2k_2\gamma_1Y\,k_1 \\ & +\beta_2S\,k_2\alpha_1k_1-\alpha_2k_2\beta_1\,k_1+\gamma_2Y\,k_2\alpha_1k_1-\gamma_2Y^2k_2\gamma_1k_1-\beta_2S\,k_2\gamma_1Y\,k_1 \\ & +2\,\alpha_2\,\alpha_1k_1+\gamma_2Y\,k_2\beta_1\,k_1-c_2\,k_2\alpha_1k_1+c_2\,k_1\beta_2S\,k_2-{c_2}^2k_2\beta_1k_1 \\ & +c_2\,k_2\gamma_1Y\,k_1-\mu_1k_1\beta_2S\,k_2-\mu_1k_1\gamma_2Y\,k_2+2\,{c_2}^2k_1\alpha_2 \\ & -2\,\mu_1k_1\alpha_2-c_1S\,k_1\alpha_2k_2 \end{aligned}$$

where

$$\begin{aligned} C = {} & 2\,\mu_2 Y\,\beta_2 S\,k_2\alpha_1 k_1 - 2\,\mu_2 Y\,\alpha_2 k_2 \beta_1\,k_1 + 2\,\mu_2 Y^2 \gamma_2 k_2 \beta_1\,k_1 \\ & - 2\,c_1 S\,\alpha_2\,\alpha_1 k_1 - 2\,\mu_1 \alpha_2\,\gamma_1 Y\,k_1 + 2\,c_2\,\beta_2 S^2 k_2 \alpha_1 + 2\,Y\,c_2\,k_1 \beta_2 \gamma_1 S\,k_2 \\ & - 2\,\mu_1 \gamma_2 Y\,k_2 \alpha_1 S + c_2\,\alpha_2 k_2 \alpha_1 k_1 - 8\,\mu_2 Y\,\alpha_2\,\alpha_1 S + 4\,\mu_2 Y\,\alpha_2\,\alpha_1 k_1 \\ & - 2\,\mu_2 Y\,\alpha_2 k_2 \alpha_1 k_1 + 2\,\mu_2 Y^2 \alpha_2 k_2 \gamma_1 k_1 - 4\,\mu_2 Y\,\beta_2 S^2 k_2 \alpha_1 \\ & + 4\,\mu_2 Y\,\alpha_2 k_2 \alpha_1 S + 4\,\mu_2 Y\,{\alpha_2}^2 \beta_1 k_1 \end{aligned}$$

Using the Routh-Hurwitz criteria, the coexistence equilibrium point will be stable if the equation (4.6) will obey

$$B_1 > 0,\, B_2 > 0,\, B_3 > 0;\, B_2 B_1 > B_3. \tag{4.7}$$

Otherwise the coexistence equilibrium point is unstable.

4.3. Global stability Analysis

We perform global stability analysis of the system (2.1) around the positive equilibrium point of the coexistence $E(S^*, X^*, Y^*)$. We consider the following theorem on the Lyapunov function U.

Theorem 4.1. Let

$$U = \frac{1}{2}(S - S^*)^2 + \frac{1}{2}\xi_1 (X - X^*)^2 + \frac{1}{2}\xi_2 (Y - Y^*)^2 \tag{4.8}$$

where $\xi_1, \xi_2 > 0$ to be chosen carefully such that $U^{'}(E) = 0$ then $E(S^*, X^*, Y^*)$ and

$$U = (S, X, Y) > 0, \forall S, X, Y | \{E\} \tag{4.9}$$

The time derivative of U is $\dfrac{dU}{dt} \leq 0, \forall S, X, Y \in \Omega^+$ then

$$\frac{dU}{dt} = 0, \forall S, X, Y \in \Omega^+ \tag{4.10}$$

implies that E^* of the system is Lyapunov stable and

$$\frac{dU}{dt} < 0, \forall S, X, Y \in \Omega^+ \tag{4.11}$$

near E^* is global stable.

Proof. Let,

$$\frac{dU}{dt} = \left(S - S^*\right) \frac{dS}{dt} + \xi_1 \left(X - X^*\right) \frac{dX}{dt} + \xi_2 \left(Y - Y^*\right) \frac{dY}{dt} \tag{4.12}$$

substituting the model equations (2.1) and simplifying we get

$$\frac{dU}{dt} = -\left(S - S^*\right)^2 \left\{\frac{\alpha_1 S}{k_1} - \alpha_1 - \beta_1 X + \gamma_1 Y\right\}$$
$$-\xi_1 \left(X - X^*\right)^2 \left\{\frac{\alpha_2 X}{k_2} - \alpha_2 + \beta_2 S + \gamma_2 Y\right\}$$
$$-\xi_2 \left(Y - Y^*\right)^2 \left\{\mu_1 + \mu_2 Y - c_2 X - c_1 S\right\}$$

Thus it is possible to set $\xi_1, \xi_2 > 0$ such that $U^{'} \leq 0$ and endemic positive equilibrium point is globally stable. Therefore, the parameters k_1 and k_2 play important roles in controlling the stability aspects of the system. ■

5. Numerical simulation

In this section we present a numerical simulations of the model (2.1) using Rung-Kutta iteration scheme with a set of reasonable parameter values given in *Table 1*. These parameter values are mainly hypothetical but they are chosen following realistic and ecological observations.

Table 1: Table for parameter values for the model.

Parameter symbol	Parameter value
α_1	0.12
k_1	50
β_1	0.2
β_2	0.1
γ_1	0.01
γ_2	0.9
k_2	30
c_1	0.9
c_2	0.8
μ_1	0.01
μ_2	0.01

In Figure 1, shows the distribution of population with time in all classes, it is observed that immigrant populations decreases with time due to the strengthening of the local prey masses and its availability to the predator than the immigrant prey.It also shows the variation of the immigrant prey population with high amplitude to low as time increases. It is also observed that the variation of the local prey with high oscillation amplitude at the beginning and slowly decreases with time and eventually reached the steady

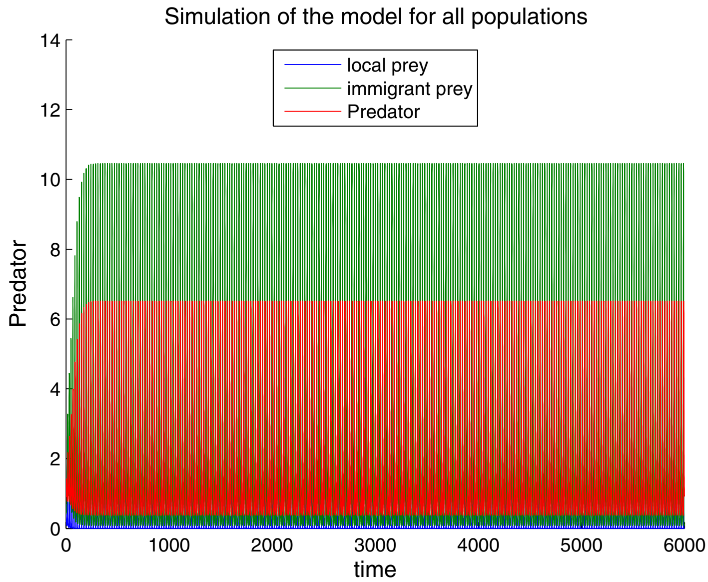

Figure 1: Total variation of population around the parameter values in table 1.

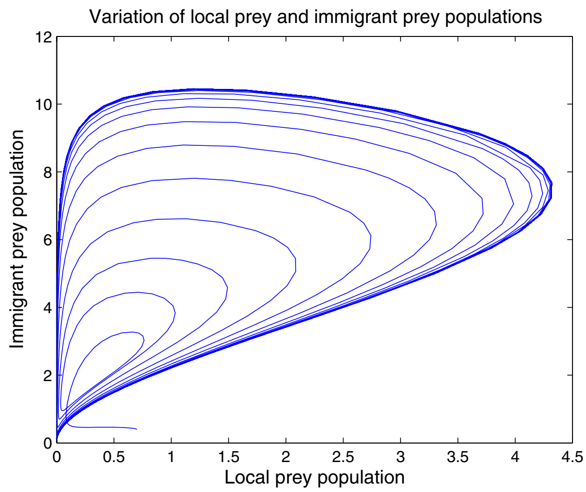

Figure 2: variation of immigrant and local prey populations with parameter values in table 1.

state. We can also interpret how the predator population vary with time depending on the interaction with local and immigrant prey populations. The sharply increase and decrease of the population occurs as the result of high or low availability of local prey. Figure 2 shows the interactions between local and immigrant prey, at the $t = 0$ predator choice is based on immigrant prey and as time increases the predator could choose the local prey. Figure 3 shows how the predator population interact with immigrant prey and show spiral in indicating that the model is locally asymptotic stable. Figure 4 shows

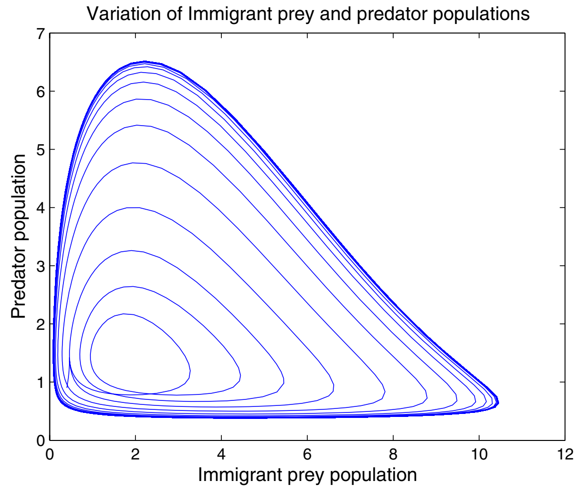

Figure 3: Variation of immigrant prey population against predator around parameter values in table 1.

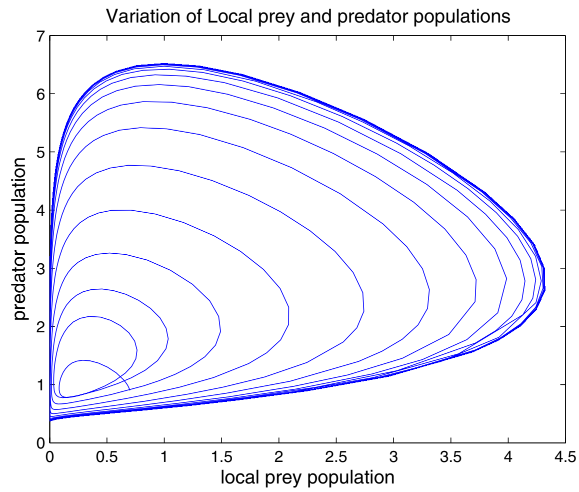

Figure 4: Variation of local prey population against predator around parameter values in table 1.

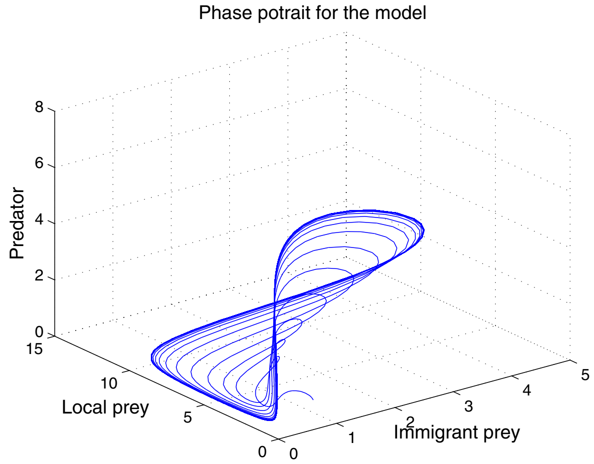

Figure 5: Variation of total population around parameter values in table 1.

how the predator population interact with local prey and show spiral in indicating that the model is locally asymptotic stable compared with Figure 5.

6. Summary and Conclusions

In this article we introduced a mathematical model for predator prey system with immigrant prey. The study indicates that immigrant prey helps for the survival and self sustainability of local prey with out causing much damage to the existing predator in the system. We also observe that predator population grows steadily without much fluctuations. From this study we recommend that the system should facilitate some feasible conditions for the immigrant prey to enter into the system which ensures the long time survival and coexistence of prey and predator thus helps the peaceful coexistence of the natural habitat.

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