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Higher Dimensional Spherical Symmetric Metric Models In A New Scalar Tensor Theory Of Gravitation

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Abstract

The field equations for spherical symmetric metric with mass less scalar field are solved with conditions $\rho = \lambda_s$ and $\rho = -\lambda_s$ for five dimensional spaces time in General Theory of Relativity. Also various physical and geometrical properties of the model have been discussed.

Keywords: Five dimensions, mass less scalar field, energy momentum tensor for a cloud of massive string.

1. Introduction

Any physical theory can be studied easily through the exact solutions of its mathematical structure. Therefore, the exact solutions of relativistic model play an important role than those obtained through approximations scheme and numerical computations. Moreover, the use of various symmetries leads to physical viable information from the complicated structure of the field equations in Einstein's theory. The gravitational effect of cylindrically symmetric interacting mass less scalar field is a subject of current interest because of it's possible applications to nuclear physics. The causes of coupled sources free electromagnetic fields and stiff fluid distributions are equivalent to mass less scalar fields was investigated by Mohanty, Tiwari and Rao[8]. The origin of structure in the universe is one of the greatest cosmological mysteries even today. The existence of a large scale network of strings in the early universe is not a contradiction with the present day observations of the universe. The present day observations indicate that the universe at large scale is homogeneous and isotropic and it is accelerating phase of the universe (recently detected experimentally by Gasperni [1]. It is well known that exact solutions of general theory of relativity for homogeneous space-time belong to either Bianchi type or Kantowski-Sachs by Roy and Choudhary[17]. Weber [20,21] had done a qualitative study of Kantowski-Sachs. Lorenz [5, 6] has obtained exact Kantowski-Sachs vacuum models in Brans-Dicke theory, while Sing and Agrawal discussed Kantowski-Sachs type models in Seaz and Ballester. In particular Reddy discussed some string models in Seaz and Ballester scalar- tensor theory of gravitation in four dimensions.

Recently, Reddy [16, 17] presented a string cosmological model in Brans-Dicke and Seaz-Ballester Scalar-tensor theories of gravitation. Also the mass less scalar field in relativistic mechanics yields some significant results regarding both the singularities involved and Mach's principle. Panigrahi and Sahu [11] studied a micro and macro cosmological model in the presence of mass less scalar field interacted with the perfect fluid.

In this paper, we have studied cosmological model generated by energy momentum tensor for a cloud of massive string with mass less scalar field for five dimensional space-time in general theory of relativity. For solving field equations, we use the condition of cloud and geometric string. Some physical and geometrical properties of the determinate model are also discussed.

2. Metric and Field Equations

We consider the five dimensional spherical symmetric metric in the form

$$ds^2 = dt^2 - e^{\lambda}(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2) + e^{\mu} dy^2 \tag{1}$$

Where λ and μ are function of t only. The field equation given by Sen and Dunn [22] for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2} g_{ij} R = \phi^{-2}\left(\omega U_{ij} - T_{ij}\right) \tag{2}$$

Where ω is a dimensionless constant , R_{ij} Ricci tensor and R is the Riemann-curvature of scalar, T_{ij} is the energy momentum tensor for a cloud of massive string, U_{ij} is the stress energy tensor corresponding to mass less scalar field, $\phi = \phi(t)$ and g_{ij} is the metric tensor.

$$T_{ij} = \rho u_i u_j - \lambda_s x_i x_j \tag{3}$$

$$U_{ij} = \phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij}\phi_{,k}\phi^{,k} \tag{4}$$

ρ is the rest energy density of the cloud strings with particles attached to them. λ_s is the tension density of the strings and $\rho = \rho_p + \lambda_s$, ρ_p being the energy density of the particles. The velocity u^i describes the five velocity which has components (1,0,0,0,0) for a cloud particles and x^i represents the direction of string which will satisfy

$$u^i u_i = -x^i x_i = 1 \quad \text{and} \quad u^i x_i = 0 \tag{5}$$

The direction of the strings is taken to be along x_4 axis.
Field equation for the metric can be written as

$$-\frac{3}{4}\left(\dot{\lambda}^2 + \dot{\lambda}\dot{\mu}\right) = \phi^{-2}\left(\omega\frac{\dot{\phi}^2}{2} - \rho\right) \tag{6}$$

$$\ddot{\lambda} + \frac{3}{4}\dot{\lambda}^2 + \frac{\ddot{\mu}}{2} + \frac{\dot{\mu}^2}{4} + \frac{\dot{\lambda}\dot{\mu}}{2} = \phi^{-2}\omega\frac{\dot{\phi}^2}{2} \tag{7}$$

$$\frac{3}{2}\left(\ddot{\lambda} + \dot{\lambda}^2\right) = -\phi^{-2}\left(\frac{\omega\dot{\phi}^2}{2} + \lambda_s\right) \tag{8}$$

Where the overhead dot denotes ordinary differentiation with respect to 't' .

3. Solutions Of The Field Equations

Here there are five unknowns $\lambda, \mu, \phi, \rho, \lambda_s$ involved in three equations (6), (7) and (8). In order to avoid the insuffi1ciency of field equation for solving five unknowns through three field equations we consider $\mu = a\lambda$ where $a(a \neq 0)$ is a parameter.

3.1 Geometric strings or Nambu strings

Here $\rho = \lambda_s$

Now from eqn(8)-eqn(6)+2eqn(7)

$$2(7+2a)\ddot{\lambda} + (2a^2 + 7a + 15)\dot{\lambda}^2 = 0 \tag{9}$$

$$\lambda(t) = \frac{2(7+2a)}{2a^2+7a+15}\ln\left[\left(\frac{2(7+2a)}{2a^2+7a+15}\right)t + k_1\right] + k_2$$

$$\mu(t) = a\lambda(t) = a\left\{\frac{2(7+2a)}{2a^2+7a+15}\ln\left[\left(\frac{2(7+2a)}{2a^2+7a+15}\right)t + k_1\right] + k_2\right\}$$

$$\phi(t) = k_3 e^{\sqrt{\frac{2a^3+12a^2+21a+33}{2\omega}}\lambda(t)}$$

$$\phi(t) = k_3 e^{\sqrt{\frac{2a^3+12a^2+21a+33}{2\omega}}\left\{\frac{2(7+2a)}{2a^2+7a+15}\ln\left[\left(\frac{2(7+2a)}{2a^2+7a+15}\right)t+k_1\right]+k_2\right\}}$$

$$\rho = (a^3 + 3a^2 + 6a + 9)\dot{\lambda}^2\phi^2$$

$$\rho = (a^3 + 3a^2 + 6a + 9)\left\{\frac{1}{\left(\frac{(2a^2+7a+15)}{2(7+2a)}\right)t+k_1}\right\}^2\left[k_3 e^{\sqrt{\frac{2a^3+12a^2+21a+33}{2\omega}}\left\{\frac{2(7+2a)}{2a^2+7a+15}\ln\left[\left(\frac{2(7+2a)}{2a^2+7a+15}\right)t+k_1\right]+k_2\right\}}\right]^2$$

$$\lambda_s = \rho$$

$$\lambda_s = (a^3 + 3a^2 + 6a + 9)\left\{\frac{1}{\left(\frac{(2a^2+7a+15)}{2(7+2a)}\right)t+k_1}\right\}^2 \left[k_3 e^{\sqrt{\frac{2a^3+12a^2+21a+33}{2\omega}}\left\{\frac{2(7+2a)}{2a^2+7a+15}\ln\left[\left(\frac{2(7+2a)}{2a^2+7a+15}\right)t+k_1\right]+k_2\right\}}\right]^2$$

Where k_1, k_2, k_3 are integrating constants.

3.2 Massive strings or Cloud strings

Here $\rho + \lambda_s = 0$

Now from eqn(6)+eqn(8)

$$\lambda(t) = \frac{2}{1-a}\ln\left\{\frac{1}{2}(1-a)t + k_4\right\} + k_5$$

$$\mu(t) = a\lambda(t) = a\left[\frac{2}{1-a}\ln\left\{\frac{1}{2}(1-a)t + k_4\right\} + k_5\right]$$

$$\phi(t) = k_6 e^{\sqrt{\frac{(2a^2+1)(a+1)}{2\omega}}\lambda(t)}$$

$$\phi(t) = k_6 e^{\sqrt{\frac{(2a^2+1)(a+1)}{2\omega}}\left\{\frac{2}{1-a}\ln\left\{\frac{1}{2}(1-a)t+k_4\right\}+k_5\right\}}$$

$$\rho = \frac{(a+1)(a+2)}{4}\dot{\lambda}^2\phi^2$$

$$\rho = \frac{(a+1)(a+2)}{4}\left[\frac{1}{\left(\frac{1-a}{2}\right)t+k_4}\right]^2\left[k_6 e^{\sqrt{\frac{(2a^2+1)(a+1)}{2\omega}}\left\{\frac{2}{1-a}\ln\left\{\frac{1}{2}(1-a)t+k_4\right\}+k_5\right\}}\right]^2$$

$$\lambda_s = -\rho = -\left\{\frac{(a+1)(a+2)}{4}\left[\frac{1}{\left(\frac{1-a}{2}\right)t+k_4}\right]^2\left[k_6 e^{\sqrt{\frac{(2a^2+1)(a+1)}{2\omega}}\left\{\frac{2}{1-a}\ln\left\{\frac{1}{2}(1-a)t+k_4\right\}+k_5\right\}}\right]^2\right\}$$

Where k_4, k_5, k_6 are integrating constants.

4. Discussion

Some Physical and geometrical properties of the models:

For Geometric String

(a) The anisotropy σ is defined as (Raychaudhuri, 1955)

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{g_{00,0}}{g_{00}} - \frac{g_{11,0}}{g_{11}}\right)^2 + \left(\frac{g_{11,0}}{g_{11}} - \frac{g_{22,0}}{g_{22}}\right)^2 + \left(\frac{g_{22,0}}{g_{22}} - \frac{g_{33,0}}{g_{33}}\right)^2 + \left(\frac{g_{33,0}}{g_{33}} - \frac{g_{44,0}}{g_{44}}\right)^2 + \left(\frac{g_{44,0}}{g_{44}} - \frac{g_{00,0}}{g_{00}}\right)^2\right]$$

i.e. $$\sigma^2 = 2(a^2 + 1 - a)\dot{\lambda}^2 = 2(a^2 + 1 - a)\left\{\frac{1}{\left(\frac{(2a^2+7a+15)}{2(7+2a)}\right)t+k_1}\right\}^2$$

$$\sigma = \sqrt{2(a^2 + 1 - a)}\dot{\lambda} = \sqrt{2(a^2 + 1 - a)}\,\frac{1}{\left(\frac{(2a^2+7a+15)}{2(7+2a)}\right)t+k_1}$$

When $t \to \infty, \sigma = 0$ the model is anisotropy and $t \to 0, \sigma = \infty$ the model is isotropy.

(b) Spatial Volume

$$V = (-g)^{\frac{1}{2}} = \left(e^{3\lambda}r^4 sin^2\theta\right)^{\frac{1}{2}}$$
$$= e^{\frac{3}{2}\lambda}r^2\sin\theta\, e^{\frac{\mu}{2}} = e^{\left(\frac{3+a}{2}\right)\lambda}r^2\sin\theta$$

When $t \to \infty, V = \infty$ and $t \to 0, V = constant$ so the universe is constant volume at the initial stage but when the time increases the volume also increases and at $t = \infty$ the volume of the universe expands as large as possible.

(c) Expansion Scalar

$$\theta = u^l{}_{;l} = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\frac{1}{\left(\frac{(2a^2+7a+15)}{2(7+2a)}\right)t+k_1}$$

When $t \to \infty, \theta =$ has no expansion the universe and $t \to 0, \theta = \infty$.

(d) $$\frac{\sigma}{\theta} = \frac{\sqrt{2(a^2+1-a)}\dot{\lambda}}{\left(\frac{3+a}{2}\right)\dot{\lambda}} = \frac{2\sqrt{2(a^2+1-a)}}{3+a}$$

The universe remains anisotropic throughout the evolution.

For Cloud String

(a) The anisotropy σ is defined as (Raychaudhuri, 1955)

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{g_{00,0}}{g_{00}} - \frac{g_{11,0}}{g_{11}}\right)^2 + \left(\frac{g_{11,0}}{g_{11}} - \frac{g_{22,0}}{g_{22}}\right)^2 + \left(\frac{g_{22,0}}{g_{22}} - \frac{g_{33,0}}{g_{33}}\right)^2 + \left(\frac{g_{33,0}}{g_{33}} - \frac{g_{44,0}}{g_{44}}\right)^2 + \left(\frac{g_{44,0}}{g_{44}} - \frac{g_{00,0}}{g_{00}}\right)^2\right]$$

i.e. $$\sigma^2 = 2(a^2+1-a)\dot{\lambda}^2 = 2(a^2+1-a)\left\{\frac{1}{\left(\frac{1-a}{2}\right)t+k_4}\right\}^2$$

$$\sigma = \sqrt{2(a^2+1-a)}\dot{\lambda} = \sqrt{2(a^2+1-a)}\ \frac{1}{\left(\frac{1-a}{2}\right)t+k_4}$$

When $t \to \infty, \sigma = 0$ the model is anisotropy and $t \to 0, \sigma = \infty$ the model is isotropy.

(b) Spatial Volume

$$V = (-g)^{\frac{1}{2}} = \left(e^{3\lambda}r^4 sin^2\theta\right)^{\frac{1}{2}}$$
$$= e^{\frac{3}{2}\lambda}r^2 \sin\theta\, e^{\frac{\mu}{2}} = e^{\left(\frac{3+a}{2}\right)\lambda}r^2 \sin\theta$$

When $t \to \infty, V = \infty$ and $t \to 0, V = constant$ so the universe is constant volume at the initial stage but when the time increases the volume also increases and at $t = \infty$ the volume of the universe as large as possible.

(c) Expansion Scalar

$$\theta = u^l{}_{;l} = \left(\frac{3+a}{2}\right)\dot{\lambda} = \left(\frac{3+a}{2}\right)\frac{1}{\left(\frac{1-a}{2}\right)t+k_4}$$

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The universe remains anisotropic throughout the evolution.

Conclusions

In this paper, we have considered higher dimensional spherical symmetric metric in Sen Dunn theory of gravitation in the context of cosmic strings.The solutions of the field equations are obtained and discussed in two different.

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