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Coefficient Inequalities For Certain Classes Of Generalized Sakaguchi Type Functions With Respect To Symmetric Points

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Abstract

In the present investigation, we introduce a new class $K^{\lambda}(A,B,s,t)$ of certain coefficient inequalities and subordination properties for the class of generalized Sakaguchi type functions. Further we have obtained various inequalities for this class.

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1. INTRODUCTION

Let A be the class of analytic functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (z \in \Delta = \{z \in \mathbb{C} : |z| < 1\}). \tag{1.1}$$

and S be the subclass of A consisting of univalent functions. For two functions $f, g \in A,$ we say that the function $f(z)$ is subordinate to $g(z)$ in Δ and write $f \prec g, or\ f(z) \prec g(z);\ z \in \Delta$ if there exists an analytic functions $w(z)$ with $w(0) = 0$ and $|w(z)| < 1\ (z \in \Delta)$, such that $f(z) = g(wz)$, $z \in \Delta$. In particular if the function g is univalent in $\Delta,$ the above subordination is equivalent to $f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$.

Recently B. A. Frasin [4] introduced and studied a generalized Sakaguchi type class $S(\alpha,s,t)$ *and* $T(\alpha,s,t)$. A function $f(z)\in A$ is said to be in the class $S(\alpha,s,t)$ if it satisfies

$$\Re\left\{\frac{(s-t)zf^{\prime}(z)}{f(sz)-f^{\prime}(tz)}\right\}<\alpha \tag{1.2}$$

for some $0\le\alpha<1, s,t\in C$ with $s\ne t$ and for all $z\in\Delta.$ We also denote by the subclass $T(\alpha,s,t)$ the subclass of A consisting of all functions $f(z)$ such that $zf^{\prime}(z)\in S(\alpha,s,t)$. The class $S(\alpha,s,t)$ that was introduced and studied by Owael. Al. [7] and by taking $t=-1,$ the class $S(\alpha,s,t)\in S_s(\alpha)$ was introduced by Sakaguchi [4] and is called Sakaguchi function of order $\alpha,$ where as $S_s(0)=S_s$ is the class of starlike functions with respect to symmetrical points in $\Delta.$

Also, we note that $S(\alpha,1,0)=S^*(\alpha)$ *and* $T(\alpha,1,0)\equiv C(\alpha)$ which are respectively, the similar classes of starlike function of order $\alpha\ (0\le\alpha\le1)$ and convex functions of order $\alpha\ (0\le\alpha\le1)$.

Let $S_s^{\lambda}(A,B,s,t)$ denote the class of functions of the form (1.1) and satisfying the condition

$$e^{i\lambda}\frac{(s-t)zf^{\prime}(z)}{f(sz)-f(tz)}\prec\frac{1+Az}{1+Bz}; \tag{1.3}$$

for $s,t\in C,$ with $t\ne s$ where $(-1\le N<A\le1)$.

In this paper, we consider the class $K_s^{\lambda}(A,B,\mathrm{s},\mathrm{t})$ of funciotns of the form (1.1) and satisfying the condition

$$e^{i\lambda}\frac{(s-t)zf^{\prime}(z)}{g(sz)-g(tz)}\prec\frac{1+Az}{1+Bz}; \tag{1.4}$$

for $s,t\in C,$ with $t\ne s, \mathrm{g}\in S_s^{\lambda}(A,s,t)$, $(-1\le B<A\le1)$, $z\in\Delta.$

By the definition of subordination it follows that if $f\in K_s^{\lambda}(A,B,s,t)$ if and only if

$$e^{i\lambda}\frac{(s-t)zf^{\prime}(z)}{g(sz)-g(tz)}=\frac{1+Az}{1+Bz}=\wp(z); \tag{1.5}$$

for $s,t\in C,$ with $t\ne s, |w(z)|<1,\ \mathrm{w}\in\mathrm{D},\ \mathrm{g}\in S_s^{\lambda}(A,s,t)$, $(-1\le B<A\le1)$, $z\in\Delta.$ where

$$\wp(z)=1+\sum_{n=1}^{\infty}\wp_n z^n. \tag{1.6}$$

In this paper, we obtain the coefficient estimates of the class $K_s^{\lambda}(A,B,s,t)$.

2. PRELIMINARY RESULT

In the present paper, we obtain the coefficient inequality for the functions in the class $K_s^{\lambda}\ \ A,B,s,t$. To prove our main results, need the following Lemma:

Lemma 2.1. If $\wp\ \ z$ is given by (1.6) then

$$|\wp| \le A-B, \qquad n=1,2,3,4,.... \tag{2.1}$$

Lemma 2.2. Let $f \in S_s^{\lambda}\ \ A,s,t$ then $n \ge 1$

$$|a_n| \le \frac{A-B}{|n-u_n|}\left\{\begin{array}{l} 1+\ A-B\ \displaystyle\sum_{i=2}^{n-1}\frac{|u_i|}{|\ i-u_i\ |}+\ A-B\ ^2\sum_{i_2>i_1}^{n-1}\sum_{i_1=2}^{n-2}\frac{|u_{i_1}u_{i_2}|}{|\ i_1-u_{i_1}\ |\,|\ i_2-u_{i_2}\ |}+....+ \\ A-B\ ^{n-2}\displaystyle\prod_{i=2}^{n-1}\frac{|u_i|}{|\ i-u_i\ |} \end{array}\right\} \tag{2.2}$$

where $u_i = \dfrac{s^i-t^i}{s-t}$.

3. MAIN RESULT

The coefficient inequalities for the class $K_s^{\lambda}\ \ A,B,s,t$.

Theorem 3.1. Let $f \in K_s^{\lambda}\ \ A,B,s,t$ then for $n \ge 1$,

Proof. Since $g \in K_s^{\lambda}\ \ A,B,s,t$, then

$$e^{i\lambda}\ \ s-t\ \ zg^{\prime}\ \ z\ \ = \left[g\ \ sz\ \ -g\ \ tz\ \ \right]\alpha\cos\lambda K\ \ s\ \ , \qquad z\in\Delta$$

with $\Re\ \ K\ \ z\ \ > 0$, where $K\ \ z\ \ = 1+c_1z+c_2z+c_3z+....$

Now equating the coefficients of above equation, we get

$$b_2\ \ 2-u_2\alpha\ \ = c_1\alpha \tag{3.1}$$

$$b_3\ \ 3-u_3\alpha\ \ = c_2\alpha+\frac{c_1^2u_2\alpha^2}{2-u_2\alpha} \tag{3.2}$$

$$b_4\ \ 4-u_4\alpha\ \ = c_3\alpha+\frac{c_1c_2u_2\alpha^2}{2-u_2\alpha}+\frac{c_1c_2u_3\alpha^2}{3-u_3\alpha}+\frac{c_1^2u_2u_3\alpha^3}{2-u_2\alpha\ \ 3-u_3\alpha} \tag{3.3}$$

$$\begin{aligned} b_5\ \ 5-u_5\alpha\ \ &= c_4\alpha+\frac{c_1c_3u_2\alpha^2}{2-u_2\alpha}+\frac{c_2^2u_3\alpha^2}{3-u_3\alpha}+\frac{c_1c_3u_4\alpha^2}{4-u_4\alpha}+\frac{c_1^2c_2u_2u_3\alpha^3}{2-u_2\alpha\ \ 3-u_3\alpha}+ \\ &\frac{c_1^2c_2u_2u_4\alpha^3}{4-u_4\alpha\ \ 2-u_2\alpha}+\frac{c_1^2u_3u_4\alpha^3}{4-u_3\alpha\ \ 4-u_4\alpha}+\frac{c_1^4u_3u_4\alpha^3}{2-u_2\alpha\ \ 3-u_3\alpha\ \ 4-u_4\alpha} \end{aligned} \tag{3.4}$$

Now from (1.4), we get

$$e^{i\lambda}\ s-t\ zf^{|}\ z\ =\left[g\ \ sz\ -g\ \ tz\ \right]P\ \ z\ \ \alpha\cos\lambda \tag{3.5}$$

Using the value from (1.5) and (1.6), we get

$$z+2a_2z^2+3a_3z^3+...+2na_{2n}z^{2n}+\ \ 2n+1\ \ a_{2n+1}z^{2n+1}+...\ e^{i\lambda}$$

$$=\left[z+b_2z^2u_2+b_3u_3z^3+...+b_{2n}u_{2n}z^{2n}+b_{2n+1}u_{2n+1}z^{2n+1}+...\right].$$

$$\left[1+p_1z+p_2z^2+p_3z^3+...+p_{2n}z^{2n}+p_{2+1}z^{2+1}+...\right].$$

Equating the coefficients of various powers of z, we have,

$$2a_2=\ \ p_1+u_2b_2\ \ \alpha, \tag{3.6}$$

$$3a_3=\ \ p_2+p_1u_2b_2+u_3b_3\ \ \alpha, \tag{3.7}$$

$$4a_4=\ \ p_3+p_1u_3b_3+p_2u_2b_2+u_4b_4\ \ \alpha, \tag{3.8}$$

$$5a_5=\ \ p_4+p_1u_4b_4+p_2u_3b_3+p_3u_2b_2+u_5b_5\ \ \alpha, \tag{3.9}$$

Similarly we can get,

$$na_n=p_{n-1}+p_1u_{n-1}b_{n-1}+...+\mathrm{b}_2\,u_2\,p_{n-2}b_nu_n \tag{3.10}$$

Where $u_i=\dfrac{s^i-t^i}{s-t}.$

From Lemma 2.1 and Lemma 2.2 in (3.7) and (3.8) respectively, we get

$$|2a_2|\le a^2\left[1+\frac{|u_2\alpha|}{|\ 2-u_2\alpha\ |}\right], \tag{3.11}$$

$$|3a_3|\le\alpha^2\left[1+\frac{u_3\alpha}{|\ 3-u_3\alpha\ |}\right]\left\{1+\frac{|u_2|\alpha}{|\ 2-u_2\alpha\ |}\right\}, \tag{3.12}$$

Similarly from Lemma (2.1) and Lemma (2.2) in (3.9) and (3.10) respectively, we get

$$|4a_4|\le\alpha^2\left[1+\frac{u_4\alpha}{|\ 4-u_4\alpha\ |}\right]1+\frac{|u_2|}{|\ 2-u_2\alpha\ |}+\frac{|u_3|}{|\ 3-u_3\alpha\ |}+\frac{|u_2u_3|}{|\ 2-u_2\alpha\ ||\ 3-u_3\ |}, \tag{3.13}$$

$$|5a_5|\le\alpha^2\left[1+\frac{u_5\alpha}{|\ 5-u_5\alpha\ |}\right]\begin{cases}1+\alpha\left(\dfrac{|u_2|}{|\ 2-u_2\alpha\ |}+\dfrac{|u_3|}{|\ 3-u_3\alpha\ |}+\dfrac{|u_4|}{|\ 4-u_4\alpha\ |}\right)+\\ \alpha^2\left(\dfrac{|u_2u_3|}{|\ 2-u_2\alpha\ ||\ 3-u_3\alpha\ |}+\dfrac{|u_2u_4|}{|\ 2-u_2\alpha\ ||\ 4-u_4\alpha\ |}+\dfrac{|u_3u_4|}{|\ 3-u_3\alpha\ ||\ 3-u_4\alpha\ |}\right)+\\ \alpha^3\dfrac{|u_2u_3u_4|}{|\ 2-u_2\alpha\ ||\ 3-u_3\alpha\ ||\ 3-u_4\alpha\ |}.\end{cases} \tag{3.14}$$

It follows that from above equations Theorem 3.1 holds for $n=2,3,4\,and\,5.$ Now by mathematical induction, we can easily prove Theorem 3.1.

From 3.11 and Lemma 2.1 and Lemma 2.2, we get,

$$|a_n| \le \frac{\alpha^2}{n}\left(1+\frac{|u_n|}{|n-u_n\alpha|}\right)\left[\begin{array}{l} 1+\alpha\sum_{i=2}^{n-1}\frac{|u_i|}{|\ i-\alpha u_i\ |}+\alpha^2\sum_{i_2>i_1}^{n-1}\sum_{i_1=2}^{n-2}\frac{|u_{i_1}||u_{i_2}|}{|\ i_1-\alpha u_{i_1}\ ||\ i_2-\alpha u_{i_2}\ |} \\ +\ldots+\alpha^{n-2}\prod_{i=2}^{n-1}\frac{|u_i|}{|\ i-\alpha u_i\ |}, \end{array}\right. \tag{3.15}$$

Where $\alpha = A-B$ and $u_i = \dfrac{s^i - t^i}{s-t}$

Corollary 3.2. Let $f \in K_s \;\; A,B,t \;\;$, then for $n \ge 1$,

$$|a_n| \le \frac{\alpha^2}{n}\left(1+\frac{|u_n|}{|n-u_n\alpha|}\right)\left[\begin{array}{l} 1+\alpha\sum_{i=2}^{n-1}\frac{|u_i|}{|\ i-\alpha u_i\ |}+\alpha^2\sum_{i_2>i_1}^{n-1}\sum_{i_1=2}^{n-2}\frac{|u_{i_1}||u_{i_2}|}{|\ i_1-\alpha u_{i_1}\ ||\ i_2-\alpha u_{i_2}\ |} \\ +\ldots+\alpha^{n-2}\prod_{i=2}^{n-1}\frac{|u_i|}{|\ i-\alpha u_i\ |}, \end{array}\right. \tag{3.16}$$

Where $\alpha = A-B$ and $u_i = \dfrac{1-t^i}{1-t}$.

Proof. Let $S_s^{\lambda} \;\; A,B,t \;\;$, the class of functions of the form (1.1) and satisfying the condition

$$\frac{1-t \;\; zf^{\prime} \;\; z}{f \;\; z \;\; - f \;\; tz} \prec \phi \;\; z \;\; ; |t| \le 1, \mathrm{t} \ne 1, \text{where} \, \phi \;\; z \;\; = \frac{1+Az}{1+Bz}, -1 \le B < A \le 1$$

then

$$|a_n| \le \frac{\alpha^2}{n}\left(1+\frac{|u_n|}{|n-u_n\alpha|}\right)\left[\begin{array}{l} 1+\alpha\sum_{i=2}^{n-1}\frac{|u_i|}{|\ i-\alpha u_i\ |}+\alpha^2\sum_{i_2>i_1}^{n-1}\sum_{i_1=2}^{n-2}\frac{|u_{i_1}||u_{i_2}|}{|\ i_1-\alpha u_{i_1}\ ||\ i_2-\alpha u_{i_2}\ |} \\ +\ldots+\alpha^{n-2}\prod_{i=2}^{n-1}\frac{|u_i|}{|\ i-\alpha u_i\ |}. \end{array}\right. \tag{3.18}$$

Remark 3.3. Put $s=1$ and $t=-1$ in Theorem 3.1 we get [6].

Remark 3.4. Put $t=-1$ in Corollary 3.2 we get [6].

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