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# Anisotropic Radiating Model As A Mutual Effect Of Perfect Fluid, Cloud Massive String And Magnetic Field In Bimetric Relativity

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**Abstract**

A homogeneous spherically symmetric cosmological model has been considered in presence of prefect fluid, cloud massive string and magnetic field in Bimetric relativity. The cosmological solution is obtained for radiating model $(\rho = 3P)$. Moreover the variation of cosmic string wave length $(\lambda)$, proper pressure $(P)$and density $(\rho)$ of perfect fluid and the magnetic field$(\eta)$ decrease rapidly as the fourth power of the parameter '$r$' for existance of the model. The volume expansion is independent of time and the model shows anisotropy throughout the evolution.

**Key words:** Symmetric, perfect fluid, cloud massive string, magnetic field and radiating model.

## 1. Introduction:

The Rosen's Bimetric theory is the theory of gravitation based on two metrics $ds^2 = g_{ij}\, dx^i\, dx^j$, where $g_{ij}$ is the Riemannian metric tensor of curved space time and a back-ground metric $ds^2 = \gamma_{ij}\, dx^i\, dx^j$, where $\gamma_{ij}$ correspond to flat space time of empty universe describing inertial forces.

The Bimetric theory also satisfied covariance and equivalence principles. The field equation in Bimetric theory of gravitation are given by Rosen's (1973) as

$$N_j^i - \frac{1}{2}\, N\, \delta_j^i = -8\pi K T_j^i \qquad (1)$$

where

$N_j^i = \frac{1}{2}\gamma^{ab}\left(g^{hi}\, g_{hi}|a\right)|b$ and $K = \left(\frac{g}{\gamma}\right)^{\frac{1}{2}}$

A vertical bar '|' denotes the covariant differentiation with respect to $\gamma_{ij}$ and $T_j^i$ is the energy momentum tensor of the matter.

$g = \det g_{ij}$ , $\gamma = \det \gamma_{ij}$

Rosen [1], Yilmaz (3), Karade (4), Mohanty and Sahoo (5), Reddy, Venkateswaralu (6) and many other have discussed various cosmological models in Bimetric theory of gravitation.

Cosmological models with fluids plays a significant role in the study of evolution of universe. Many homogeneous and inhomogeneous models filled with perfect fluids have obtained by a number of authors Viz. Mazumber (7), Tabu (8) and others.

Cosmic strings have received considerable attention in cosmology as they are belived to give rise density perturbations leading to the formation of galaxies. Letelier (9), Pradhan and Mathur (10) and many other authors have studied about string cosmologies in the theory of relativity.

Magnetic field also plays an important role in the description of the energy distribution in the universe as it contains highly ionized matter and it directly affects the rate of expansion of the universe. The authors Panigrahy (11), Mohanty (12) and others have done extensive works in this field.

In this paper we obtained radiating model $(\rho = 3P)$ as a coupling effect of perfect fluid cloud massive string and magnetic field and studied some physical and geometrical properties.

**2. Metric and field equation:**

We have considered the spherically symmetric line element of the form

$$dS^2 = -e^{\lambda}\, dr^2 - r^2\, d\theta^2 - r^2\, sin^2\theta\, d\phi^2 + e^{\mu}\, dt^2 \quad (2)$$

Where '$\lambda$' and '$\mu$' are function of '$t$' only.

The energy momentum tensor of perfect fluid is given by

$$T_{ij\,(P)} = (P+\rho)\, V_i\, V_j - P\, g_{ij} \quad (3)$$

Together with $g_{ij}\, V^i\, V^j = 1$

Where $V^i, P, \rho$ are the four velocity vector, proper pressure and energy density of the perfect fluid.

Using co-moving coordinate system for metric (2), using the energy momentum tensor (3), we get the following existing components.

$$T^1_{1\,(P)} = T^2_{2\,(P)} = T^3_{3\,(P)} = -P \text{ and } T^4_{4\,(P)} = \rho \quad (4)$$

The energy momentum tensor for cosmic string along X-axis is

$$T^i_{j\,(String)} = \rho\, u^i\, u_j - \lambda\, x^i\, x_j \quad (5)$$

Where $x^i$ is the direction of anisotropy.

Together with $u^i\, u_j = 1 = -x_i\, x^j$ and $u^i\, x_j = 0$.

The existing components of equation (5) for metric (2) for co-moving co-ordinate system are

$$T^1_{1\,(String)} = \lambda, T^4_{4\,(String)} = \rho, T^2_{2\,(String)} = T^3_{3\,(String)} = 0 \quad (6)$$

The energy momentum tensor for electromagnetic field is

$$E^i_{j\,(mag)} = -F^{ir}\,F_{jr} + \frac{1}{4}\,F^{ab}\,F_{ab}\,g^i_j \quad (7)$$

Where $F_{ij}$ is the electromagnetic field tensor.

The existing components of equation (7) for metric (2) for co-moving co-ordinate system are

$$E^1_{1\,(mag)} = \eta, E^2_{2\,(mag)} = E^3_{3\,(mag)} = 3\eta \; and \; E^4_{4\,(mag)} = -\eta \quad (8)$$

where $\eta = \frac{A^2\,cosec^2\theta}{2r^4}$, (9) $F_{23}$ = constant = A and $F^{23} = g^{22}\,g^{33}\,F_{23} = \frac{A\,cosec^2\theta}{r^4}$.

The energy momentum tensor as a coupling effect of perfect fluid, cloud massive string and magnetic field is given by

$$T^i_j = T^i_{j\,(P)} + T^i_{j\,(String)} + T^i_{j\,(mag)} \quad (10)$$

Using equations (4), (6) and (8) in equation (10) we get the following existing components.

$$T^1_1 = P - \lambda + \eta, T^2_2 = T^3_3 = P + 3\eta \; and \; T^4_4 = -(2\rho + \eta)\,. \quad (11)$$

Using equation (10) in the field equation (1) of Bimetric theory we get the following equations.

$$\lambda_{44} - \mu_{44} = -32\pi K(P - \lambda + \eta) \quad (12)$$

$$\lambda_{44} + \mu_{44} = 32\pi K(P + 3\eta) \quad (13)$$

$$\mu_{44} - \lambda_{44} = -32\pi K(2\rho + \eta) \quad (14)$$

## 3. Cosmological solution:

Adding equation (13) and (14) we get,

$$\lambda_{44} = 16\pi K(P + 2\eta - 2\rho) \quad (15)$$

Adding equation (12) and (13) we get,

$$\lambda_{44} = 16\pi K(2\eta + \lambda) \quad (16)$$

## 4. Using radiating model $\boldsymbol{\rho = 3P}$ and $\boldsymbol{\lambda = \eta}$:

We get, $\mu_{44} = 16\pi K(2\eta - 5P)$

$$= -48\pi K\eta \; (if\; P = \eta) \quad (17)$$

Similarly, $\lambda_{44} = 16\pi K(2\eta + \lambda) = 48\pi K\eta \; (\because \lambda = \eta)$ (18)

The value of '$K$' for metric (2) is

$$K = \left(\frac{g}{r}\right)^{\frac{1}{2}} = e^{\frac{\lambda+\mu}{2}}\,r^2\,sin\theta\,. \quad (19)$$

Using equation (9) and (19) in equation (17) we get,

$$\lambda_{44} = \frac{24\pi\,A^2}{r^2\,sin\theta}\,e^{\frac{\lambda+\mu}{2}}\,. \quad (20)$$

To avoid the complicacy in integration we have chosen

$$\lambda = -\mu \quad (21)$$

for which we have taken $P = \eta$ in equation (17)

Now we get the value of '$\lambda$' after integration as

$$\lambda = \frac{12\pi\,A^2}{r^2\,sin\theta}\,t^2 + c_1 t + c_2 \quad (22)$$

Where $c_1$ and $c_2$ are constants of integration. Ultimately equation (21) yields

$\mu = -\left(\frac{12\pi A^2}{r^2 \, sin\theta} \, t^2 + c_1 t + c_2\right)$ (23)

Using the values of $\lambda$ and $\mu$ in metric (2) we get the new form of metric as

$dS^2 = -e^{\left(\frac{12\pi A^2}{r^2 \, sin\theta} t^2 + c_1 t + c_2\right)} dr^2 - r^2 \, d\theta^2 - r^2 \, sin^2\theta \, d\phi^2 + e^{-\left(\frac{12\pi A^2}{r^2 \, sin\theta} t^2 + c_1 t + c_2\right)} dt^2$ (24)

## 5. Physical and geometrical properties:

- Volume $(V) = (-g)^{\frac{1}{2}} = r^2 \, sin\theta$, which is independent of time but expansion of the model is proportional to '$r^2$'.
- The model is geodesic in nature as $\dot{V_i} = 0$.
- Since $\sigma^2 = \frac{1}{4}\lambda^2 \neq 0$, the model shows anisotropy throughout the evolution.
- The vertices tensor, $\omega_{ij} = 0$. Hence the model of the universe is non-rotating by nature.
- The scalar expansion $(\theta)$ = $V^i_{;i}$ = 0, i.e. the model has no scalar expansion.
- Deceleration parameter $(q)$ is given by Einstein's theory of relativity as

$$q = -3\theta^2 \left[\theta_{;\alpha} V^\alpha + \frac{1}{3} \, \theta^2\right]$$

Since $\theta = 0$, so $q = 0$, i.e. the model is neither accelerating nor decelerating but the volume expansion is independent of time.

- The condition $P = \eta$ to solve the integration implies that,

$P = \eta = \frac{A^2 \, coses^2\theta}{2r^4}$ [by equation (9)]

$= \lambda = \frac{\rho}{3}$ [by radiating model]

As '$A$' is constant and for some particular values of $r$ and taking $\theta = 90^0$, we see that

$$[(P) \; or \; (\eta) \; or \; (\lambda) \; or \; (\rho)] \; \propto \; \frac{1}{r^4}$$

That means the cosmic string wavelength the proper pressure and density of perfect fluid and the magnetic field decrease sharply as the fourth power of the parameter '$r$'. And all the three fields are absent when '$r \to \infty$' but are independent of cosmic time.

## 6. Conclusion:

In this paper we have obtained a radiating model as a coupling effect of perfect fluid, could massive string and magnetic field on a homogeneous spherically symmetric metric in Bimetric relativity. The model is found to be non-accelerating but the volume expansion of the universe is dependent on parameter '$r$'. The effect of all the three fields are found to be zero when '$r \to \infty$' and the model shows anisotropy throughout the evolution.

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