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Super Geometric Mean Labeling Of Some Disconnected Graphs

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ABSTRACT

Let f: V(G) $\rightarrow$ {1,2,...,p+q} be an injective function. For a vertex labeling " f ", the induced edge labeling f*(e=uv) is defined by, f*(e)=$\left\lceil\sqrt{f(u)f(v)}\right\rceil$ or $\left\lfloor\sqrt{f(u)f(v)}\right\rfloor$.Then " f " is called a **"Super Geometric mean labeling"** if {f(V(G))}$\cup${f(e):e$\in$E(G)}={1,2,…,p+q}. A graph which admits Super Geometric mean labeling is called **"Super Geometric mean graph".** In this paper we prove that some disconnected graphs are Super Geometric mean graphs.

Key words: Graph, Super Geometric mean graph, Path, Comb and Ladder.

1. Introduction

The graphs considered here are simple, finite and undirected graphs. Let V(G) denote the vertex set of G and E(G) denote the edge set of G. For a detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of **"Geometric mean labeling"** has been introduced by S.Somasundaram, R.Ponraj and P.Vidhyarani in [4]. S.S.Sandhya, E. Ebin Raja Merly and B.Shiny introduced **"Super Geometric mean labeling"** in [5].

In this paper, we investigate "Super Geometric mean labeling" behavior of some disconnected graphs.

Now we will give the following definitions which are necessary for our present investigation.

Definition: 1.1

A graph G=(V,E) with p vertices and q edges is called a "**Geometric mean graph"** if it is possible to label the vertices $x \in V$ with distinct labels f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labeled with

f(e=uv)=$\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ then the edge labels are distinct. In this case, "f" is called a **"Geometric mean labeling"** of G.

Definition: 1.2

Let f: V(G) $\rightarrow$ {1,2,…,p+q} be an injective function. For a vertex labeling " f ", the induced edge labeling f* (e=uv) is defined by,

f*(e)=$\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$. Then " f " is called a **"Super Geometric mean labeling"** if $\{f(V(G))\} \cup \{f(e): e \in E(G)\} = \{1,2,\ldots,p+q\}$.A graph which admits Super Geometric mean labeling is called "**Super Geometric mean graph".**

Definition: 1.3

The **union** of two graphs $G_1=(V_1E_1)$ and $G_2=(V_2, E_2)$ is a graph $G=G_1 \cup G_2$ with vertex set $V=V_1 \cup V_2$ and the edge set $E=E_1 \cup E_2$.

Definition: 1.4

A **path** P_n is a walk in which all the vertices are distinct.

Definition: 1.5

A graph obtained by joining a single pendant edge to each vertex of a path is called a **Comb** ($P_n A K_1$).

Definition: 1.6

The **Ladder** L_n, n≥2 is the product graph $P_n x P_2$ and contains 2n vertices and 3n-2 edges.

Definition: 1.7

The graph $\mathbf{P_n A K_{1,2}}$ is obtained by attaching $K_{1,2}$ to each vertex of P_n.

Definition: 1.8
The graph $\mathbf{P_n A K_{1,3}}$ is obtained by attaching $K_{1,3}$ to each vertex of P_n.

2. Main Results

Theorem: 2.1
$P_m \cup P_n$ is a Super Geometric mean graph.

Proof:
Let $P_m = u_1\, u_2 \ldots u_m$ be a path on "m" vertices.
Let $P_n = t_1 t_2 \ldots t_n$ be another one path on "n" vertices.
Let $G = P_m \cup P_n$
Define a function f: $V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,
$f(u_i)=2i-1,\ 1 \le i \le m$
$f(t_i)=2m+2i-2,\ 1 \le i \le n$
Edge labels are given by,
$f(u_i\, u_{i+1})=2i,\ 1 \le i \le m-1$
$f(t_i t_{i+1})=2m+2i-1,\ 1 \le i \le n-1$
$\therefore$ The edge labels are distinct.
Thus "f" provides a Super Geometric mean labeling.
Hence $P_m \cup P_n$ is a Super Geometric mean graph.

Example: 2.2
A Super Geometric mean labeling of $P_7 \cup P_8$ is shown below.

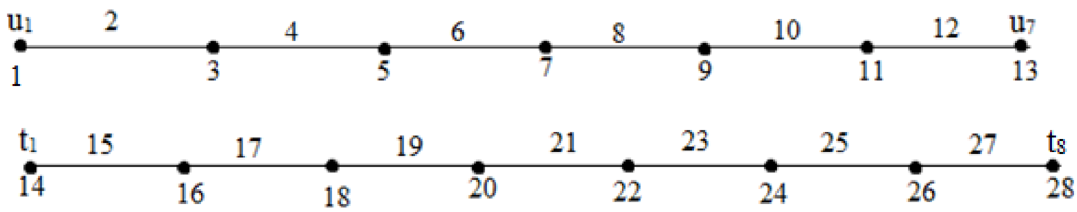

Figure: 1

Theorem: 2.3
$(P_m A K_1) \cup P_n$ is a Super Geometric mean graph.

Proof:
Let $(P_m A K_1)$ be a Comb graph obtained from a path $P_m = v_1 v_2 \ldots v_m$ by joining a vertex u_i to v_i, $1 \le i \le m$. Let $P_n = w_1 w_2 \ldots w_n$ be a path.
Let $G=(P_m A K_1) \cup P_n$
Define a function $f: V(G) \rightarrow \{1,2,...,p+q\}$ by,
$f(v_i)=4i-1,\ 1 \le i \le m$

$f(u_i)=4i-3,\ 1\leq i\leq m$

$f(w_i)=4m+2i-2,\ 1\leq i\leq n$

Edges are labeled with,

$f(v_iv_{i+1})=4i,\ 1\leq i\leq m-1$

$f(u_iv_i)=4i-2,\ 1\leq i\leq m$

$f(w_iw_{i+1})=4m+2i-1,\ 1\leq i\leq n-1$

Thus we get distinct edge labels.

Hence $(P_m \mathbb{A} K_1)\cup P_n$ is a Super Geometric mean graph.

Example: 2.4

Super Geometric mean labeling of $(P_6 \mathbb{A} K_1)\cup P_5$ is given below.

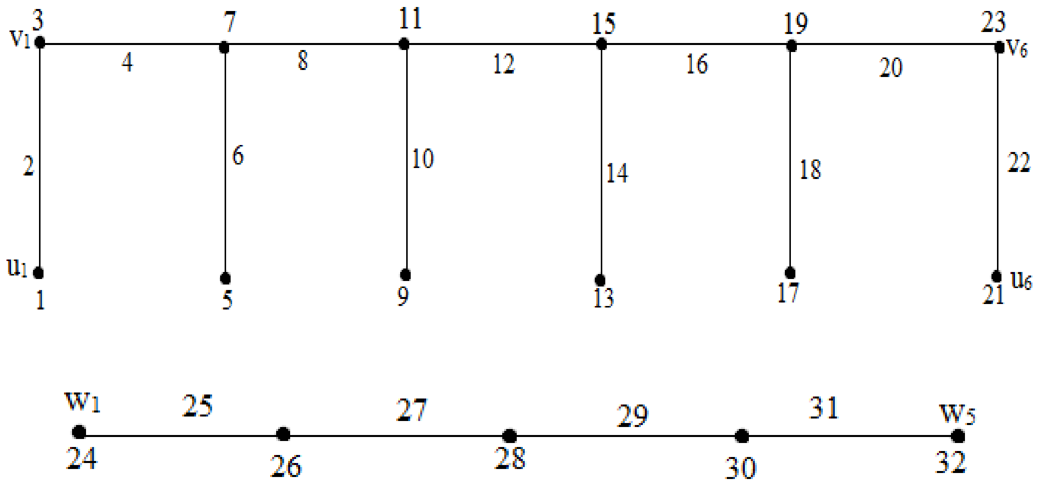

Figure: 2

Theorem: 2.5

$L_m\cup P_n$ is a Super Geometric mean graph.

Proof:

Let $L_m=P_m\times P_2$ be a ladder, $P_m=v_1v_2\ldots v_m$ Let $P_n=w_1w_2\ldots w_n$ be a path

Let $G=L_m\cup P_n$.

Define a function $f: V(G)\rightarrow\{1,2,\ldots,p+q\}$ by,

$f(v_1)=1$

$f(v_i)=5i-2,\ 2\leq i\leq m.$

$f(u_1)=4$

$f(u_i)=5i-4,\ 2\leq i\leq m.$

$f(w_i)=5m+2i-3,\ 1\leq i\leq n.$

Edges are labeled with,

$f(v_1v_2)=3$

$f(v_iv_{i+1})=5i,\ 2\leq i\leq m-1$

$f(u_1u_2)=5,$

$f(u_iu_{i+1})=5i-1,\ 2\le i\le m-1$
$f(v_iu_i)=5i-3,\ 1\le i\le m$
$f(w_iw_{i+1})=5m+2i-2,\ 1\le i\le n-1$
$\therefore$ We get distinct edge labels.
Hence " f " provides a Super Geometric mean labeling.
$\therefore L_m\cup P_n$ is a Super Geometric mean graph.

Example: 2.6
Super Geometric mean labeling of $L_5\cup P_6$ is displayed below.

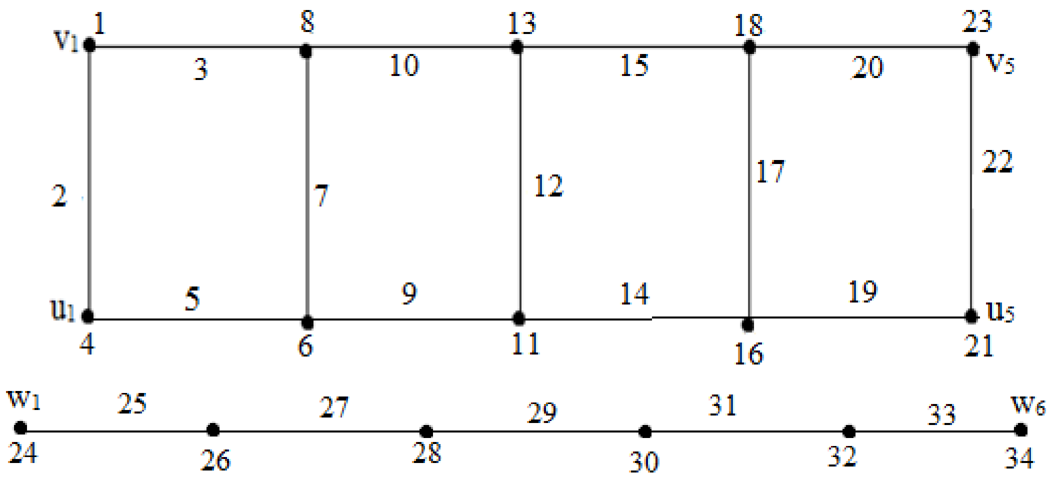

Figure: 3

Theorem: 2.7
$(P_m\mathring{A}K_{1,2})\cup P_n$ is a Super Geometric mean graph.

Proof:
Let $(P_m\mathring{A}K_{1,2})$ be a graph obtained by attaching each vertex of a path P_m to the central vertex of $K_{1,2}$ where $P_m=u_1u_2...u_m$.
Let v_i and w_i be the vertices of $K_{1,2}$ which are attached with the vertex u_i of P_m, $1\le i\le m$.
Let $P_n=z_1z_2...z_n$ be a path.
Let $G=(P_m\mathring{A}K_{1,2})\cup P_n$.
Define a function $f: V(G)\rightarrow\{1,2,\ldots,p+q\}$ by
$f(u_i)=6i-3,\ 1\le i\le m$
$f(v_i)=6i-5,\ 1\le i\le m$
$f(w_i)=6i-1,\ 1\le i\le m$
$f(z_i)=6m+2i-2,\ 1\le i\le n$
Edges are labeled with,
$f(u_iu_{i+1})=6i,\ 1\le i\le m-1$
$f(u_iv_i)=6i-4,\ 1\le i\le m$

$f(u_iw_i)=6i-2$ $1\leq i\leq m$
$f(z_iz_{i+1})=6m+2i-1,\ 1\leq i\leq n-1$
$\therefore$ The edge labels are distinct.
Hence G admits a Super Geometric mean labeling.
Hence $(P_m \mathbb{A} K_{1,2}) \cup P_n$ is a Super Geometric mean graph.

Example: 2.8
Super Geometric mean labeling of $(P_4 \mathbb{A} K_{1,2}) \cup P_5$ is shown below.

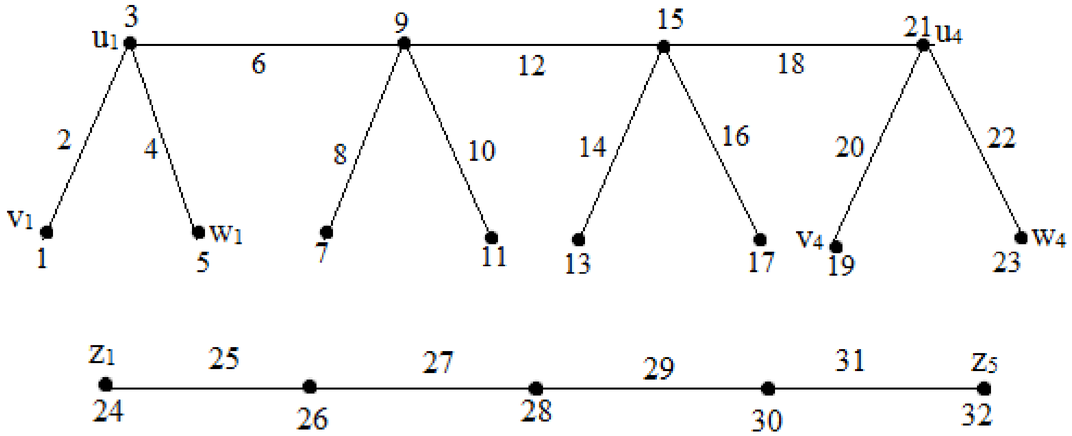

Figure: 4

Theorem: 2.9
$(P_m \mathbb{A} K_{1,3})$ is a Super Geometric mean graph.

Proof:
Let $(P_m \mathbb{A} K_{1,3})$ be a graph obtained by attaching each vertex of a path $P_m=u_1u_2\ldots u_m$ to the central vertex of $K_{1,3}$.
Let v_i, w_i and z_i be the vertices of $K_{1,3}$ which are attached with the vertex u_i of P_m, $1\leq i\leq m$.
Let $P_n=t_1t_2\ldots t_n$ be a path.
Let $G=(P_m \mathbb{A} K_{1,3}) \cup P_n$
Define a function $f: V(G)\rightarrow\{1,2,...,p+q\}$ by,
$f(u_i)=8i-3,\ 1\leq i\leq m$
$f(v_i)=8i-7,\ 1\leq i\leq m$
$f(w_i)=8i-5,\ 1\leq i\leq m$
$f(z_i)=8i-1,\ 1\leq i\leq m$
$f(t_i)=8m+2i-2,\ 1\leq i\leq n$
Edges are labeled with,
$f(u_iu_{i+1})=8i,\ 1\leq i\leq m-1$
$f(u_iv_i)=8i-6,\ 1\leq i\leq m$

$f(u_iw_i)=8i-4$, $1\leq i\leq m$
$f(u_iz_i)=8i-2$, $1\leq i\leq m$
$f(t_it_{i+1})=8m+2i-1$, $1\leq i\leq n-1$
From the above labeling pattern, both vertices and edges together get distinct labels from $\{1,2,\ldots,p+q\}$.
Hence $(P_m A K_{1,3})\cup P_n$ is a Super Geometric mean graph.

Example: 2.10
Super Geometric mean labeling of $(P_5 A K_{1,3})\cup P_4$ is given below.

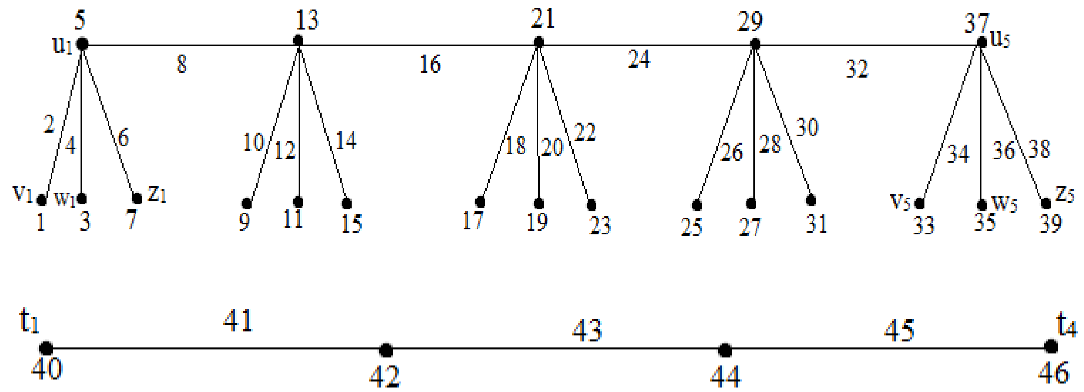

Figure: 5

Theorem: 2.11
Let G_1 be a graph obtained from a path $P_m=v_1v_2\ldots v_m$ by joining pendant vertices with the vertices of the path P_m alternatively. Let $P_n=w_1w_2\ldots w_n$ be another path. Let $G=G_1\cup P_n$. Then G is a Super Geometric mean graph.

Proof:
Let G_1 be a graph obtained from a path $P_m=v_1v_2\ldots v_m$ by joining pendant vertices with the vertices of the path P_m, alternatively.
Let $P_n=w_1w_2\ldots w_n$ be another one path.
Let $G=G_1\cup P_n$
Define a function $f: V(G)\rightarrow\{1,2,\ldots,p+q\}$ by,
$f(v_i)=3i$, $i=1,3,5,\ldots.m$
$f(v_{2i})=6i-1$, $1\leq i\leq\left(\frac{m-1}{2}\right)$
$f(u_i)=3i-2$, $i=1,3,5,\ldots m$
$f(w_i)=3m+2i-1$, $1\leq i\leq n$
Edges are labeled with,
$f(v_iv_{i+1})=3i+1$, $i=1,3,5,\ldots.,m-2$
$f(v_{2i}\, v_{2i+1})=6i$, $1\leq i\leq\left(\frac{m-1}{2}\right)$
$f(v_iu_i)=3i-1$, $i=1,3,5,\ldots,m$

$f(w_iw_{i+1})=3m+2i$, $1\le i\le n-1$

$\therefore$ We get distinct edge labels.

Hence $\{f(V(G))\}\cup\{f(e):e\in E(G)\}=\{1,2,\ldots,p+q\}$.

Hence G is a Super Geometric mean graph.

Example: 2.12

Let G_1 be a graph obtained from a path P_9 by joining pendant vertices with the vertices of P_9 alternatively. A Super Geometric mean labeling of $G=G_1\cup P_5$ is displayed below.

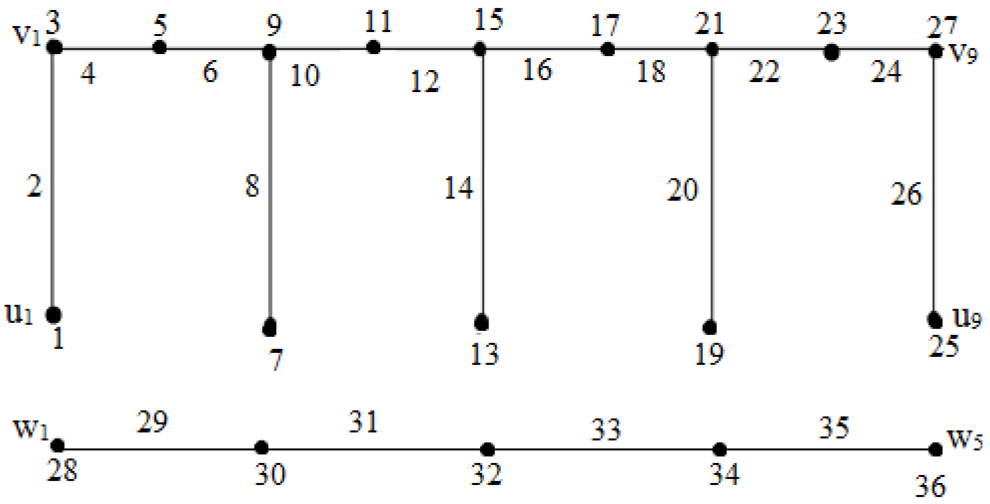

Figure: 6

Theorem: 2.13

Let G_1 be a graph obtained from a Ladder L_m, $m\ge 2$ by joining a pendant vertex with a vertex of degree two on both sides of upper and lower path of the ladder. Let $P_n=t_1t_2...t_n$ be another path. Let $G=G_1\cup P_n$. Then G is a Super Geometric mean graph.

Proof:

Let $L_m=P_m x P_2$ be a Ladder graph.

Let G_1 be a graph obtained from a Ladder by joining pendant vertices u,w,x,z with v_1, v_n, u_1, u_n (vertices of degree 2) respectively on both sides of upper and lower path of the ladder.

Let $P_n=t_1t_2\ldots t_n$ be another one path.

Let $G=G_1\cup P_n$

Define a function $f: V(G)\to\{1,2,\ldots,p+q\}$ by,

$f(u)=1$

$f(v_1)=5$

$f(v_i)=5i-1$, $2\le i\le m$

$f(w)=5m+5$

$f(x)=3$
$f(u_i)=5i+3,\ 1\le i\le m$
$f(z)=5m+6$
$f(t_i)=5m+2i+5,\ 1\le i\le n$
Edges are labeled with,
$f(v_iv_{i+1})=5i+2,\ 1\le i\le m-1$
$f(uv_1)=2$
$f(v_mw)=5m+2$
$f(xu_1)=4$
$f(u_iu_{i+1})=5i+5,\ 1\le i\le m-1$
$f(u_mz)=5m+4$
$f(v_iu_i)=5i+1,\ 1\le i\le m$
$f(t_it_{i+1})=5m+2i+6,\ 1\le i\le n-1$
In view of the above labeling pattern, " f " provides a Super Geometric mean labeling of G.
Hence G is Super Geometric mean graph.

Example: 2.14
A super Geometric mean labeling of G when m=5 and n=6 is shown below.

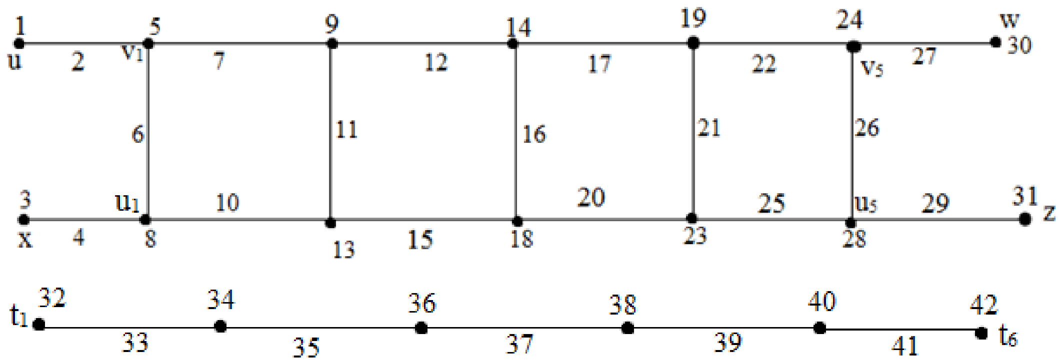

Figure: 7

Theorem: 2.15
Let G_1 be a graph obtained by joining a pendant vertex with a vertex of degree two on both sides of a Comb graph. Let $P_n=w_1w_2\ldots w_n$ be another path. Let $G=G_1\cup P_n$. Then G is a Super Geometric mean graph.

Proof:
Comb ($P_m \mathbb{A} K_1$) is a graph obtained from a path $P_m=v_1v_2\ldots v_m$ by joining a vertex u_i to v_i, $1\le i\le m$.
Let G_1 be a graph obtained by joining pendant vertices w and z to v_1 and v_m respectively.
Let $P_n=w_1w_2\ldots w_n$ be another one path.

Let $G=G_1 \cup P_n$.
Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,
$f(w)=1$
$f(v_1)=3$
$f(v_i)=4i+1,\ 2 \le i \le m$
$f(z)=4m+3$
$f(u_1)=5$
$f(u_i)=4i-1,\ 2 \le i \le m$
$f(w_i)=4m+2i+2,\ 1 \le i \le n$
Edges are labeled with
$f(wv_1)=2$
$f(v_iv_{i+1})=4i+2,\ 1 \le i \le m-1$
$f(v_nz)=4m+2$
$f(v_iu_i)=4i,\ 1 \le i \le m$
$f(w_iw_{i+1})=4m+2i+3,\ 1 \le i \le n-1$
$\therefore$ The edge labels are distinct.
Hence G is a Super Geometric mean graph.

Example: 2.16
A Super Geometric mean labeling of G when m=5, and n=4 is displayed below.

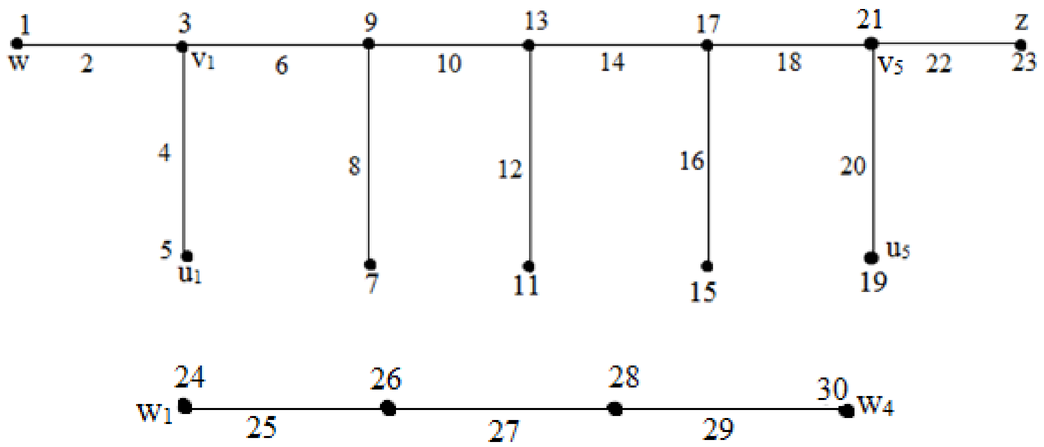

Figure: 8

Theorem 2.17
Let P_m be a path and G_1 be the graph obtained from P_m, by attaching C_3 in both end edges of P_m. Let $P_n=w_1w_2\ldots w_n$ be another path. Let $G=G_1 \cup P_n$. Then G is a Super Geometric mean graph.

Proof:
Let P_m be a path $u_1u_2\ldots u_n$ and $v_1u_1u_2$, $v_2u_{n-1}u_n$ be the triangles at the end edges of P_m. The resulting graph is G_1.

Let $P_n=w_1w_2...w_n$ be another one path.
Let $G=G_1\cup P_n$.
Define a function $f: V(G) \rightarrow \{1,2,\ldots,p+q\}$ by,
$f(v_1)=4$
$f(u_1)=1$
$f(u_i)=2i+2,\ 2\le i\le m-1$
$f(u_m)=2m+5$
$f(v_2)=2m+2$
$f(w_i)=2m+2i+4,\ 1\le i\le n$
Edges are labeled with
$f(v_1u_1)=2$
$f(v_1u_2)=5$
$f(u_1u_2)=3$
$f(u_iu_{i+1})=2i+3,\ 2\le i\le m-2$
$f(u_{m-1}\ u_m)=2m+3$
$f(v_2u_{m-1})=2m+1$
$f(v_2u_m)=2m+4$
$f(w_iw_{i+1})=2m+2i+5,\ 1\le i\le n-1$
$\therefore$ We get distinct edge labels.
Thus both vertices and edges together get distinct labels from $\{1,2,...,p+q\}$.
Hence G is a Super Geometric mean graph

Example: 2.18
A Super Geometric mean labeling of G when m=8 and n=5, is shown below.

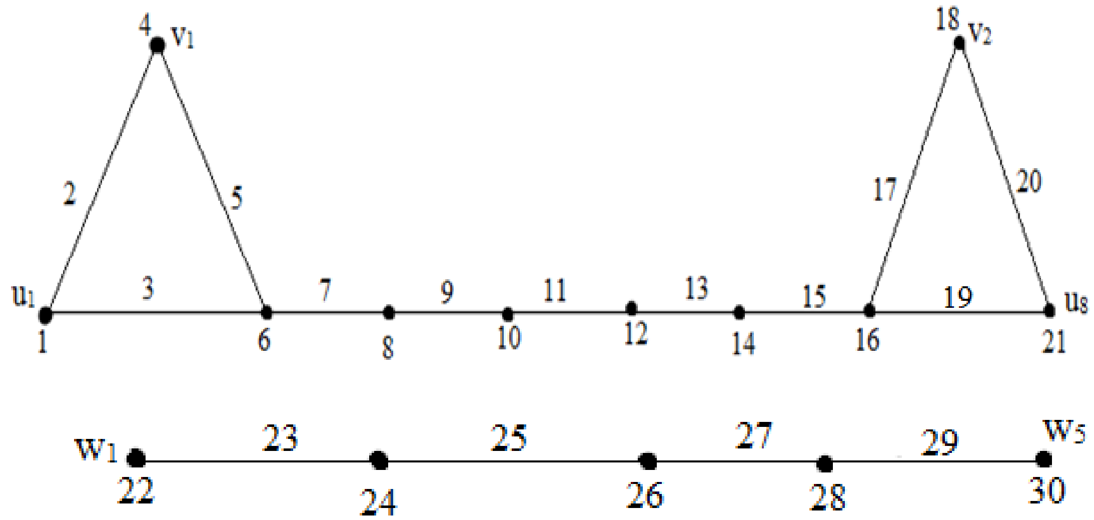

Figure: 9

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