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On Soft Almost πg-continuous functions

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Abstract

The object of this paper is to introduce a new class of functions called soft almost πg-continuous functions. This class turns out to be the natural tool for studying different class of soft compact spaces. Further soft almost πg-open and soft almost πg-closed functions are obtained as generalizations of soft open and soft closed functions respectively.

Keywords: soft πg-closed set, soft πg-open set, soft πg-Continuity, soft almost πg-continuity, soft almost open πg-continuity, soft almost closed πg-continuity

1. Introduction

Molodtsov [8] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Recently Muhammad Shabir and Munazza Naz [10] introduced soft topological spaces which are defined over an initial universe with a fixed set of parameters. Kharal et al. [5] introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [1] in 2013 studied and discussed the properties of Soft continuous mappings. In this paper, we give some characterizations of soft almost πg-continuous function and the relations of such function with other types of soft functions are obtained.

2. Preliminaries

Definition: 2.1[8]

Let U be the initial universe and P(U) denote the power set of U. Let E denote the set of all parameters. Let A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P (U).

Definition: 2.2[7]
A subset (A, E) of a topological space X is called soft generalized-closed **(**soft g-closed**)** if cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft open in X.

Definition: 2.3[2]
A subset (A, E) of a topological space X is called soft regular closed, if cl(int(A, E))=(A, E).The complement of soft regular closed set is soft regular open set.

Definition: 2.4[2]
The finite union of soft regular open sets is said to be soft π-open. The complement of soft π-open is said to be soft π-closed.

Definition: 2.5[2]
A subset (A, E) of a topological space X is called soft πg-closed in a soft topological space (X, τ, E), if cl(A, E) $\tilde{\subseteq}$ (U, E) whenever (A, E) $\tilde{\subseteq}$ (U, E) and (U, E) is soft π-open in X.

Definition: 2.6[1]
Let (F, E) be a soft set over X. The soft set (F, E) is called soft point, denoted by (x_e, E), if for element e∈E, F(e) = $\{x\}$ and F(e') = $\emptyset$ for all e'∈ E$-\{$e$\}$.

Definition: 2.7[12]
Let (X, τ, E) and (Y, τ', E) be two topological spaces. A function f: $(X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be Soft Semi continuous(Soft pre-continuous, Soft α-continuous, Soft β-continuous), if f^{-1}(G, E) is soft semi open(soft pre-open, soft α-open, soft β-open) in (X, τ, E) for every soft open set (G, E) of (Y, τ', E).

Definition: 2.8[3]
Let (X, τ, E) and (Y, τ', E) be two topological spaces. A function f: $(X, \tau, E) \rightarrow (Y, \tau', E)$ is said to be Soft regular continuous(Soft π-continuous, Soft g-continuous, Soft πg-continuous), if f^{-1}(G, E) is soft regular open(soft π-open, soft g-open, soft πg-open) in (X, τ, E) for every soft open set (G, E) of (Y, τ', E).

Definition: 2.9[3]
A function f: $(X, \tau, E) \rightarrow (Y, \tau', E)$ is soft πg-irresolute, if f^{-1}(G, E) is soft πg-open in (X, τ, E) for every soft πg-open set (G, E) of (Y, τ', E).

Definition: 2.10[2]
A space (X, τ, E) is called soft πg-$T_{1/2}$ [6], if every soft πg-closed set is soft closed, or equivalently every soft πg-open set is soft open.

Definition: 2.11[3]
A function f: (X, τ, E) → (Y, τ′, E) is called $\tilde{S}\pi$g-open, if image of each soft open set in X is $\tilde{S}\pi$g-open in Y.

Definition: 2.12[4]
A function f: (X, τ, E) → (Y, τ′, E) is called soft contra πg-continuous, if f^{-1}(F, E) is soft πg-closed in X for every soft open set (F, E) of Y.

Definition: 2.13[4]
A space (X, τ, E) is said to be soft πg-compact, if every soft πg-open cover of X has a finite sub cover.

Definition: 2.14[4]
A space (X, τ, E) is said to be soft countably πg-compact, if every soft πg-open countably cover of X has a finite subcover.

Definition: 2.15[4]
A space (X, τ, E) is said to be soft πg-Lindelöf, if every soft πg-open cover of X has a countable subcover.

Definition: 2.16[4]
A space (X, τ, E) is called soft πg-connected provided that X cannot be written as the union of two disjoint non-empty soft πg-open sets.

Definition: 2.17[4]
A space (X, τ, E) is said to be $\tilde{S}\pi$g-T_2 if for each pair of distinct soft points x and y in X, there exist (F, E) ∈ $\tilde{S}\pi$GO(X, x) and (G, E) ∈ $\tilde{S}\pi$GO(X, y) such that (F, E)∩ (G, E) = ∅.

Definition: 2.18[10]
A space (X, τ, E) is said to be soft Hausdorff, if for each pair of distinct points x and y in X, there exists soft open sets (A, E) and (B, E) containing x and y such that (A, E) ∩ (B, E) = ∅.

Definition: 2.19[4]
A function f: (X, τ, E) → (Y, τ′, E) is said to be soft R-Map, if f^{-1}(A, E) is soft regular closed in X for every soft regular closed (A, E) of Y.

Definition: 2.20[4]
A space (X, τ, E) is said to be soft submaximal, if each soft dense subset of X is soft open.

3. Soft Almost πg-continuous functions

Definition: 3.1
A function f: (X, τ, E) → (Y, τ′, E) is said to be soft almost πg-continuous, if f^{-1}(A, E) is soft πg-open in X for every soft regular open (A, E) of Y.

Theorem: 3.2
The following statements are equivalent for a function f: $(X, \tau, E) \to (Y, \tau', E)$

1. f is soft almost πg-continuous.
2. f^{-1}(F, E) ∈ $\tilde{s}\pi$GC(X) for every soft (F, E) ∈ $\tilde{s}$RC(Y).
3. For each x ∈ X and each soft regular closed set (F, E) in Y containing f(x), there exists soft πg-closed set (U, E) in X containing x such that f(U, E) $\tilde{\subset}$(F, E).
4. For each x ∈ X and each soft regular open set (F, E) in Y containing f(x), there exists soft πg-open set (K, E) in X not containing x such that f^{-1}(F, E) $\tilde{\subset}$(K, E).
5. f^{-1}(int(cl(G, E))) ∈ $\tilde{s}\pi$GO(X) for every soft open subset (G, E) of Y.
6. f^{-1}(cl(int(F, E))) ∈ $\tilde{s}\pi$GC(X) for every soft closed subset (F, E) Of Y.

Proof:
(1) ⟹ (2)
Let (F, E) ∈ $\tilde{s}$RC(Y).Then Y\ (F, E) ∈ $\tilde{s}$RO(Y) By (1) f^{-1}(Y\ (F, E)) = X\ f^{-1} (F, E) ∈ $\tilde{s}\pi$GO(X).Thus f^{-1}(A, E) ∈ $\tilde{s}\pi$GC(X).
(2)⟹ (3)
Let (F, E) be a soft regular closed set in Y containing f(x). Then f^{-1} (F, E) ∈ $\tilde{s}\pi$GC(X) and x ∈ f^{-1} (F, E) by (2). Take (U, E) = f^{-1} (F, E). Then f(U, E) $\tilde{\subset}$(F, E).
(3) ⟹ (2)
Let (F, E) ∈ $\tilde{s}$RC(Y) and x∈ f^{-1} (F, E). From (3) there exists a soft πg-closed set (U, E) in X containing x such that f(U, E) $\tilde{\subset}$(F, E).We have f^{-1} (F, E) = ∪{(U, E): x∈ f^{-1} (F, E)}. Thus f^{-1} (F, E) is soft πg-closed set.
(3) ⟹ (4)
Let (F, E) be a soft regular open set in Y not containing f(x). Then Y\ (F, E) is soft regular closed set containing f(x). By (3) there exists a soft πg-closed set (U, E) in X containing x such that f (U, E) $\tilde{\subset}$ Y\ (F, E). Hence (U, E) $\tilde{\subset}$ f^{-1} (Y\ (F, E)) $\tilde{\subset}$ X\ f^{-1} (F, E). Then f^{-1} (F, E) $\tilde{\subset}$ X\ (U, E). Take (K, E) = X\(U, E). Then we obtain a soft πg-open set (K, E) in X not containing x such that f^{-1}(F, E) $\tilde{\subset}$(K, E).
(4) ⟹ (3)
Let (F, E) be a soft regular closed set in Y containing f(x). then Y\(F, E) is a soft regular open set in Y not containing f(x). By (4) there exists a soft πg-open set (K, E) in X not containing x such that f^{-1}(Y\(F, E)) $\tilde{\subset}$(K, E). That is X\ f^{-1} (F, E) $\tilde{\subset}$ (K, E) implies X\ (K, E) $\tilde{\subset}$ f^{-1} (F, E). Hence f(X\ (K, E)) $\tilde{\subset}$ (F, E). Take (U, E) = X\ (K, E). Then (U, E) is soft πg-closed set in X containing x such that f(U, E) $\tilde{\subset}$(F, E).
(1) ⟹ (5)
Let (G, E) be a soft open subset of Y. Since int(cl(G, E)) is soft regular open then by (1) f^{-1}(int(cl(G, E))) ∈ $\tilde{s}\pi$GO(X).
(5) ⟹ (1)
Let (G, E) ∈ $\tilde{s}$RO(Y).Then (G, E) is open in Y. BY (5) f^{-1}(int(cl(G, E))) ∈ $\tilde{s}\pi$GO(X) implies f^{-1}(G, E) ∈ $\tilde{s}\pi$GO(X). Hence f is soft almost πg-continuous.
(2) ⟺ (6) is similar as **(1) ⟺ (5).**

Theorem: 3.3
If f: $(X, \tau, E) \to (Y, \tau', E)$ is a soft almost πg-continuous function then the following properties hold:
1. $\tilde{s}\pi$g-cl(f^{-1}(cl(int(cl(B, E)))) $\tilde{\subseteq}$ f^{-1}(cl(B, E)) for every soft subset (B, E) of Y.
2. $\tilde{s}\pi$g-cl(f^{-1}(cl(int(F, E)))) $\tilde{\subseteq}$ f^{-1}(F, E) for every soft closed set (F, E) of Y.
3. $\tilde{s}\pi$g-cl(f^{-1}(cl(V, E))) $\tilde{\subseteq}$ f^{-1}(cl(V, E)) for every soft open set (V, E) of Y.

Theorem: 3.4
Every restriction of a soft almost πg-continuous function is soft almost πg-continuous.

Proof:
Let f: $(X, \tau, E) \to (Y, \tau', E)$ be a soft almost πg-continuous function of X into Y and (A, E) be any soft open subset of X. For any soft regular open subset (F, E) of Y, $(f|(A, E))^{-1}$(F, E) = (A, E) ∩ f^{-1}(F, E). Since f is almost πg-continuous f^{-1}(F, E) ∈ $\tilde{s}\pi$GO(X).Hence (A, E) ∩ f^{-1}(F, E) relatively soft πg-open subset of (A, E). That is $(f|(A, E))^{-1}$(F, E) is soft πg-open subset of (A, E). Hence f|(A, E) is soft almost πg-continuous.

Theorem: 3.5
If f: $(X, \tau, E) \to (Y, \tau', E)$ is a soft function of X into Y and X = (A, E) ∪ (B, E) where (A, E) and (B, E) are soft πg-closed and f|(A, E) and f|(B, E) are soft almost πg-continuous, then f is soft almost πg-continuous.

Proof:
Let (F, E) be any soft regular closed set of Y. Since f|(A, E) and f|(B, E) are soft almost πg-continuous, $(f|(A, E))^{-1}$(F, E) and $(f|(B, E))^{-1}$(F, E) are soft πg-closed in (A, E) and (B, E) respectively. Since (A, E) and (B, E) are soft πg-closed subsets of X, $(f|(A, E))^{-1}$(F, E) and $(f|(B, E))^{-1}$(F, E) are soft πg-closed subsets of X. Also f^{-1}(F, E) = $(f|(A, E))^{-1}$(F, E) ∪ $(f|(B, E))^{-1}$(F, E) is soft πg-closed in X. Hence f is soft almost πg-continuous.

Theorem: 3.6
If f: $(X, \tau, E) \to (Y, \tau', E)$ is a soft function of X into Y and X = (A, E) ∪ (B, E) and f|(A, E) and f|(B, E) are both soft almost πg-continuous at a point *x* belonging to (A, E) ∩ (B, E), then f is soft almost πg-continuous at *x*.

Proof:
Let (U, E) be any soft regular open set containing f(*x*). Since *x* ∈ (A, E) ∩ (B, E) and f|(A, E), f|(B, E) are both soft almost πg-continuous at *x*, therefore there exist soft πg-open sets (F, E) and (G, E) such that *x* ∈ (A, E) ∩ (F, E) and f((A, E) ∩ (F, E)) $\tilde{\subseteq}$(U, E) and *x* ∈ (B, E) ∩ (G, E) and f((B, E) ∩ (G, E)) $\tilde{\subseteq}$(U, E).Since X = (A, E) ∪ (B, E), f((A, E) ∩ (B, E)) = f((A, E) ∩ (F, E) ∩ (G, E)) ∪ f(B, E) ∩ (F, E) ∩ (G, E)) $\tilde{\subseteq}$ f((A, E) ∩ (F, E)) ∪ f((B, E) ∩ (G, E)) $\tilde{\subseteq}$ (U, E).Thus (F, E) ∩ (G, E) = (K, E) is a soft πg-

open set containing x such that f(K, E) $\tilde{\subseteq}$ (U, E). Hence f is soft almost πg-continuous at x.

Theorem: 3.7
If a function f: X → $\prod Y_i$ is soft almost πg-continuous, then $P_i \circ$ f: X → Y_i is soft almost πg-continuous for each i ∈ I, where P_i is the projection of $\prod Y_i$ onto Y_i.

Proof:
Let (V_i, E) be any soft regular open set of Y_i. Since P_i is a soft continuous open, it is a soft R-map. Hence P_i^{-1}(V_i, E) is soft regular open in $\prod Y_i$. Thus $(P_i \circ f)^{-1}$(V_i, E) = $f^{-1}(P_i^{-1}$(V_i, E)) is soft πg-open in X. Therefore $P_i \circ$ f is soft almost πg-continuous.

Theorem: 3.8
If a function f: $\prod X_i \to \prod Y_i$ is soft almost soft πg-continuous, then f_i: $X_i \to Y_i$ is soft almost πg-continuous for each i ∈ I.

Proof:
Let k be an arbitrarily fixed index and (V_k, E) be any soft regular open set of Y_k. Then $\prod Y_k \times$ (V_k, E) is soft regular open in $\prod Y_i$ where $j \in I$ and $j \neq k$. Hence $f^{-1}(\prod Y_k \times$ (V_k, E)) = $\prod Y_k \times f_k^{-1}$($V_k$, E) is soft πg-open in $\prod X_i$. Thus f_k^{-1}(V_k, E) is soft πg-open in $\prod X_k$. Hence f_k is soft almost πg-continuous.

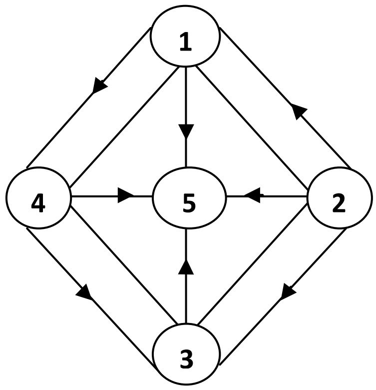
Definition: 3.9
A function f: (X, τ, E) → (Y, τ′, E) is called

1. Soft almost g-continuous, if f^{-1}(A, E) is soft g-closed in X for every soft regular closed (A, E) of Y.
2. Soft almost π-continuous, if f^{-1}(A, E) is soft π-closed in X for every soft regular closed (A, E) of Y.
3. Soft completely continuous, if f^{-1}(A, E) is soft regular closed in X for every soft closed set (A, E) of Y.

Theorem: 3.10

1. Every soft R-Map is soft almost π-continuous.
2. Every soft almost π-continuous is soft almost-continuous.
3. Every soft almost π-continuous is soft almost πg-continuous.
4. Every soft almost continuous is soft almost g-continuous.
5. Every soft almost g-continuous is soft almost πg-continuous.

Remark: 3.11
The following diagram holds for the above implications. Also none of the results are reversible as seen in the following examples.



1. Soft almost π-continuous
2. Soft R-map
3. Soft almost g-continuous
4. Soft almost continuous
5. Soft almost πg-continuous

Example 3.12

Let X = {a, b, c, d}, Y= {a, b, c}, E= {e_1, e_2}. Let F_1, F_2, F_3, F_4, F_5, F_6 and G_1, G_2, G_3, G_4, G_5, G_6 G_7 are functions from E to P(X) and E to P(Y) are defined as follows: $F_1(e_1)$ = {c}, $F_1(e_2)$ = { a}; $F_2(e_1)$ = {d }, $F_2(e_2)$ = {b }; $F_3(e_1)$ = {c, d }, $F_3(e_2)$ = { a, b}; $F_4(e_1)$ = {a, d }, $F_4(e_2)$ = {b, d } $F_5(e_1)$= {b, c, d }, $F_5(e_2)$ = {a, b, c }; $F_6(e_1)$ = {a, c, d }, $F_6(e_2)$ = {a, b, d } and $G_1(e_1)$ = {b }, $G_1(e_2)$ = {a }; $G_2(e_1)$ = {a, c }, $G_2(e_2)$ = {b, c }, $G_3(e_1)$ = {b }, $G_3(e_2)$ = X, $G_4(e_1)$ = ∅, G_4 (e_2) = {a }, $G_5(e_1)$ = {a, c }, $G_5(e_2)$ = X, $G_6(e_1)$ = ∅, $G_6(e_2)$ = {b, c }, $G_7(e_1)$ = ∅, $G_7(e_2)$ = X. Then τ = {$\tilde{\emptyset}, \tilde{X}$, ($F_1$, E) ($F_2$, E), ($F_3$, E), ($F_4$, E), ($F_5$, E), ($F_6$, E)} is a soft topological space over X and τ' = {$\tilde{\emptyset}, \tilde{Y}$, ($G_1$, E), ($G_2$, E), ($G_3$, E), ($G_4$, E), ($G_5$, E), ($G_6$, E), ($G_7$, E), } is a soft topological space over Y. If the function f: $(X, \tau, E) \to (Y, \tau', E)$ is defined as f (a) =b, f (b) =a, f(c) =c, f (d) =d, then f is soft almost continuous but not soft almost π-continuous.

Example 3.13

Let X = {a, b, c, d}, Y= {a, b, c}, E= {e_1, e_2}. Let F_1, F_2, F_3, F_4, F_5, F_6 and G_1, G_2, G_3, G_4, G_5, G_6 G_7 are functions from E to P(X) and E to P(Y) are defined as follows: $F_1(e_1)$ = {c}, $F_1(e_2)$ = { a}; $F_2(e_1)$ = {d }, $F_2(e_2)$ = {b }; $F_3(e_1)$ = {c, d }, $F_3(e_2)$ = { a, b}; $F_4(e_1)$ = {a, d }, $F_4(e_2)$ = {b, d } $F_5(e_1)$= {b, c, d }, $F_5(e_2)$ = {a, b, c }; $F_6(e_1)$ = {a, c, d }, $F_6(e_2)$ = {a, b, d } and $G_1(e_1)$ = {b }, $G_1(e_2)$ = {a }; $G_2(e_1)$ = {a, c }, $G_2(e_2)$ = {b, c }, $G_3(e_1)$ = {b }, $G_3(e_2)$ = X, $G_4(e_1)$ = ∅, G_4 (e_2) = {a }, $G_5(e_1)$ = {a, c }, $G_5(e_2)$ = X, $G_6(e_1)$ = ∅, $G_6(e_2)$ = {b, c }, $G_7(e_1)$ = ∅, $G_7(e_2)$ = X. Then τ = {$\tilde{\emptyset}, \tilde{X}$, ($F_1$, E) ($F_2$, E), ($F_3$, E), ($F_4$, E), ($F_5$, E), ($F_6$, E)} is a soft topological space over X and τ' = {$\tilde{\emptyset}, \tilde{Y}$, ($G_1$, E), ($G_2$, E), ($G_3$, E), ($G_4$, E), ($G_5$, E), ($G_6$, E), ($G_7$, E), } is a soft topological space over Y. If the function f: $(X, \tau, E) \to (Y, \tau', E)$ is defined as f(a) =b, f(b) =d, f(c) =c, f(d) =a, then f is soft almost πg-continuous but not soft almost π-continuous.

Example 3.14

Let X = {a, b, c, d}, Y= {a, b, c}, E= {e_1, e_2}. Let F_1, F_2, F_3, F_4 and G_1, G_2, G_3, G_4, G_5, G_6 G_7 are functions from E to P(X) and E to P(Y) are defined as follows:
$F_1(e_1)$ = {a}, $F_1(e_2)$ = { d}; $F_2(e_1)$ = {b }, $F_2(e_2)$ = {c }; $F_3(e_1)$ = {a, b }, $F_3(e_2)$ = {c, d}; $F_4(e_1)$ ={b, c, d }, $F_4(e_2)$ = {a, b, c} and $G_1(e_1)$ = {b }, $G_1(e_2)$ = {a }; $G_2(e_1)$ = {a, c }, $G_2(e_2)$ = {b, c }, $G_3(e_1)$ = {b }, $G_3(e_2)$ = X, $G_4(e_1)$ = $\emptyset$, G_4 (e_2) = {a }, $G_5(e_1)$ = {a, c }, $G_5(e_2)$ = X, $G_6(e_1)$ = $\emptyset$, $G_6(e_2)$ = {b, c }, $G_7(e_1)$ = $\emptyset$, $G_7(e_2)$ = X. Then τ = {$\tilde{\emptyset}$, $\tilde{X}$, (F_1, E) (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)} is a soft topological space over X and τ' = {$\tilde{\emptyset}$, $\tilde{Y}$, (G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E), (G_6, E), (G_7, E), } is a soft topological space over Y. If the function f: $(X, \tau, E) \to (Y, \tau', E)$ is defined as f (a) =b, f (b) =d, f(c) =c, f (d) =a then f is soft almost g-continuous but not soft almost continuous.

Example 3.15

In example: 3.13 we see that f is soft almost πg-continuous but not soft almost g-continuous, since f^{-1}(G_2, E) is not soft g-closed in X.

Lemma: 3.16

Let (X, τ, E) be a soft topological space. If (U, E), (V, E) ∈ $\tilde{S}$πGO(X) and X is a soft submaximal space then $(U \times V, E)$ ∈ $\tilde{S}$πGO(X).

Theorem: 3.17

Let f: $(X, \tau, E) \to (Y, \tau', E)$ be soft function and let g: $(X, \tau, E) \to$ (X × Y, τ×τ', E) be the soft graph function of f, defined by $g(x) = (x, f(x))$ for every x ∈ X. Suppose that X be soft submaximal space. Then g is soft almost πg-continuous, if and only if f is soft almost continuous.

Proof:

Let x ∈ X and (V, E) ∈ $\tilde{S}$RO(Y) containing f(x). Then we have g(x) = $(x, f(x))$ ∈ X × (V, E) ∈ $\tilde{S}$RO(X × Y). Since g is soft almost πg-continuous, g^{-1} (X × (V, E)) $= f^{-1}$ (V, E) ∈ $\tilde{S}$πGO(X). Thus f is soft almost πg-continuous.
Conversely let x ∈ X and (W, E) ∈ $\tilde{S}$RO(X×Y) containing g(x).Then there exists (U, E) ∈ $\tilde{S}$RO(X) and (V, E) ∈$\tilde{S}$RO(Y) such that $(x, f(x))$ ∈ $(U \times V, E)$ $\tilde{\subset}$ (W, E). Since f is soft almost πg-continuous, f^{-1} (V, E) ∈ $\tilde{S}$πGO(X).Say (A, E) = f^{-1} (V, E) and take (B, E) = (U, E)∩(A, E).By previous lemma (B, E) ∈ $\tilde{S}$πGO(X) and g(B, E) $\tilde{\subset}$ $(U \times V, E)$ $\tilde{\subset}$ (W, E).This shows that g is soft almost πg-continuous.

Theorem: 3.18

Let f: $(X, \tau, E) \to (Y, \tau', E)$and g: $(Y, \tau', E) \to (Z, \tau'', E)$be soft almost πg-continuous and Y is soft Hausdorff. If X is soft submaximal then the set {x ∈ X: f(x) = g(x)} is soft πg-closed in X.

Proof:

Let (A, E) = { x ∈ X: f(x) = g(x)} and x ∈ X\ (A, E).Then f(x) ≠ g(x). Since Y is soft Hausdorff, there exist soft open sets (U, E) and (V, E) of Y, such that f(x) ∈(U, E), g(x) ∈(U, E) and (U, E)∩(A, E) = ∅. since f and g are soft almost πg-continuous, (G, E) = f^{-1} (int(cl(U, E))) ∈ $\tilde{S}$πGO(X, x)and (H, E) = g^{-1} (int(cl(V, E))) ∈ $\tilde{S}$πGO(X, x).Take (W, E) = (G, E)∩(H, E) then (W, E) ∈ $\tilde{S}$πGO(X, x) and f(W, E)∩ g(W, E) $\tilde{\subseteq}$ int(cl(U, E)) ∩ int(cl(V, E)) = ∅. Therefore (W, E)∩(A, E) = ∅. Hence x ∈ X\$\tilde{S}$πg-cl (A, E). This shows that (A, E) is soft πg-closed in X.

Theorem: 3.19

Let f: $(X,\tau,E) \to (Y,\tau',E)$and g: $(Y,\tau',E) \to (Z,\tau'',E)$be functions. Then the following properties hold:

1. if f is soft almost πg-continuous and g is soft R-map, then g ∘ f: $(X,\tau,E) \to (Z,\tau'',E)$ is soft almost πg-continuous.
2. if f is soft πg-irresolute and g is soft πg-continuous, then g ∘ f: $(X,\tau,E) \to (Z,\tau'',E)$ is soft almost πg-continuous.
3. if f is soft almost πg-continuous and g is soft completely continuous, then g ∘ f: $(X,\tau,E) \to (Z,\tau'',E)$ is soft almost πg-continuous.
4. if f is soft almost πg-continuous and g is soft almost continuous, then g ∘ f: $(X,\tau,E) \to (Z,\tau'',E)$ is soft almost πg-continuous.

Definition: 3.20

A function f: $(X,\tau,E) \to (Y,\tau',E)$ is said to be soft weakly πg-continuous, if f^{-1}(cl(A, E)) is soft πg-open in X for every soft open set (A, E) of Y.

Theorem: 3.21

Let f: $(X,\tau,E) \to (Y,\tau',E)$ be a soft function. Suppose that X is soft πg-$T_{1/2}$ space and Y is soft regular space. Then the following properties equivalent:

1. f is soft πg-continuous
2. f is soft almost πg-continuous
3. f is soft weakly πg-continuous

Proof:

(1) ⟹ (2) ⟹(3). This is obvious.

Theorem: 3.22

If for each pair of distinct points x and y in a soft space X, there exists a function f of X into a soft Hausdorff space Y such that

1. f(x) ≠ f(y)
2. f is soft weakly πg-continuous at x and
3. f is soft almost πg-continuous at y, then X is soft πg-T_2.

Proof:

Since Y soft Hausdorff, there exists soft open sets (U, E) and (V, E) of Y such that f(x) ∈ (U, E) and f(y) ∈ (V, E) and (U, E)∩(V, E) = ∅. Hence cl(U, E)∩(int(cl(V, E)) = ∅. Since f is soft weakly πg-continuous at x, there exists (A, E) ∈ $\tilde{S}$πGO(X, x) such that f(A, E) $\tilde{\subseteq}$cl(U, E). Since f is soft almost πg-continuous at y, f^{-1} (int(cl(V, E)) =

(B, E) ∈ $\tilde{S}\pi GO(X, y)$. Therefore we obtain (A, E) ∩(B, E) =∅. This shows that X is soft πg-T_2.

Theorem: 3.23
If f: $(X, \tau, E) \rightarrow (Y, \tau', E)$ is soft almost πg-continuous surjective function and X is soft πg-connected space, then Y is soft connected.

Proof:
Suppose Y is not soft connected. Then there exist non-empty disjoint soft open subsets (U, E) and (V, E) of Y such that Y = (U, E) ∪ (V, E). Since f is soft almost πg-continuous, then f^{-1}(U, E) and f^{-1}(V, E) are non-empty disjoint soft πg-clopen sets in X. Then we have X = f^{-1}(U, E) ∪ f^{-1}(V, E) such that f^{-1}(U, E) and f^{-1}(V, E) are disjoint. This shows that X is not soft πg-connected which is a contradiction. Hence Y is soft connected.

Definition: 3.24
A function f: $(X, \tau, E) \rightarrow (Y, \tau', E)$has a soft (πg, r)-graph if for each (*x, y*) ∈ X×Y\ G(f), there exists (U, E) ∈ $\tilde{S}\pi GO(X, x)$ and a regular open set (V, E) of Y containing *y* such that (U×V, E)∩ G(f) = ∅.

Lemma: 3.25
A function f: $(X, \tau, E) \rightarrow (Y, \tau', E)$has a soft (πg, r)-graph if and only if for each (*x, y*) ∈ X×Y such that *y* ≠f(*x*), there exists a soft πg-open set (U, E) and a regular open set (V, E) containing *x* and *y* respectively such that f(U, E) ∩ (V, E) = ∅.

Theorem: 3.26
If f: $(X, \tau, E) \rightarrow (Y, \tau', E)$ is a soft almost πg-continuous function and Y is soft Hausdorff then f has a soft (πg, r)-graph.

Proof:
Let (*x, y*) ∈ X×Y such that *y* ≠f(*x*),. Then there exists a soft open sets (U, E) and (V, E) such that, *y* ∈ (U, E), f(*x*) ∈ (V, E) and (U, E) ∩ (V, E) = ∅. Hence int(cl(U, E)) ∩ int(cl(V, E)) = ∅. Since f is soft almost πg-continuous, f^{-1}(int(cl(U, E))) = (W, E) ∈ $\tilde{S}\pi GO(X, x)$. This implies that f(W, E) ∩ int(cl(U, E)) = ∅. Therefore f has a soft (πg, r)-graph.

Definition: 3.27
A space (X, τ, E) is said to be:

1. Soft nearly compact, if every soft regular open cover of X has a finite soft subcover.
2. soft nearly countably compact, if every countable soft cover of X by soft regular open sets has a finite soft subcover.
3. Soft nearly Lindelof, if every cover of X by soft regular open sets has a countable soft subcover.

Theorem: 3.28
Let f: $(X,\tau,E) \to (Y,\tau',E)$ be a soft almost πg-continuous surjection. Then the following statements hold:
1. If X is soft πg-compact, then Y is soft nearly compact
2. If X is soft πg-Lindelof, then Y is soft nearly Lindelof.
3. If X is soft countably πg-compact, then Y is soft nearly countably compact.

4. Soft almost πg-open function and soft almost πg-closed function

Definition: 4.1
A function f: $(X,\tau,E) \to (Y,\tau',E)$ is called soft almost open(Soft almost closed), if the image of every soft regular open subset of X is soft open(soft regular closed) subset of Y.

Definition: 4.2
A function f: $(X,\tau,E) \to (Y,\tau',E)$ is called soft almost πg-open(Soft almost πg-closed), if the image of every soft regular open subset of X is soft πg-open (soft πg-closed) subset of Y.

Remark: 4.3
A one to one soft function is soft almost πg-open if and if it is soft almost πg-closed.

Remark: 4.4
Every soft πg-open function is soft almost πg-open. But the converse is not true in general.

Example: 4.5
Let X ={a, b, c, d}, Y= {a, b, c, d}, E= {e_1, e_2}. Let F_1, F_2, F_3, $F_{4,}$ $F_{5,}$ F_6 and G_1, $G_{2,}$ $G_{3,}$ $G_{4,}$ are functions from E to P(X) and E to P(Y) are defined as follows:
$F_1(e_1)$ = {c}, $F_1(e_2)$ = { a}; $F_2(e_1)$ = {d }, $F_2(e_2)$ = {b }; $F_3(e_1)$ = {c, d }, $F_3(e_2)$ = { a, b}; $F_4(e_1)$ = {a, d }, $F_4(e_2)$ = {b, d } $F_5(e_1)$= {b, c, d }, $F_5(e_2)$ = {a, b, c }; $F_6(e_1)$ = {a, c, d }, $F_6(e_2)$ = {a, b, d } and $G_1(e_1)$ = {a }, $G_1(e_2)$ = {d }; $G_2(e_1)$ = {b}, $G_2(e_2)$ = {c }, $G_3(e_1)$ = {a, b }, $G_3(e_2)$ ={c, d}, $G_4(e_1)$ ={b, c, d}, G_4 (e_2) = {a, b, c }.Then $\tau = \{\widetilde{\emptyset}, \widetilde{X}$, ($F_1$, E) ($F_2$, E), ($F_3$, E), ($F_4$, E), ($F_5$, E), ($F_6$, E)} is a soft topological space over X and τ' = {$\widetilde{\emptyset}, \widetilde{Y}$, ($G_1$, E), ($G_2$, E), ($G_3$, E), ($G_4$, E)} is a soft topological space over Y. If the function f: $(X,\tau,E) \to (Y,\tau',E)$ is defined as f(a) =d, f(b) =a, f(c) =c, f(d) =b, then f is soft almost πg-open but not soft πg-open.

Theorem: 4.6
If f: $(X,\tau,E) \to (Y,\tau',E)$ is a soft almost closed function of X onto Y then for every soft regular open subset (G, E) of X and for every soft point $y \in$Y such that $f^{-1}(y)$ $\widetilde{\subseteq}$(G, E) we have $y \in$ int(f(G, E).

Proof:
Since (G, E) is soft regular open, X\(G, E) is soft regular closed. Since f is soft almost πg-continuous, f(X\(G, E)) is soft πg-closed. Since $f^{-1}(y) \tilde{\subseteq}$(G, E), $y \notin$ f(X\(G, E)). Hence there must exist a soft open set (U, E) containing *y* such that (U, E) ∩ f(X\(G, E)) = ∅. Then $y \in$ (U, E) $\tilde{\subseteq}$ (f(G, E). this shows that *y* is a soft interior point of f(G).

Theorem: 4.7
A surjection f: $(X, \tau, E) \to (Y, \tau', E)$ is soft almost πg-closed if and only if for each subset (A, E) of Y and each (U, E) ∈ $\tilde{S}$RO(X) containing f^{-1}(A, E) there exists a soft πg-open set (V, E) of Y such that (A, E) $\tilde{\subseteq}$(U, E) and f^{-1}(V, E) $\tilde{\subseteq}$(U, E).

Proof:
Suppose that f is soft almost πg-closed. Let (A, E) be a subset of Y and (U, E) ∈ $\tilde{S}$RO(X) containing f^{-1}(A, E). If (V, E) = Y\f(X\(U, E)) then (V, E) is soft πg-open set of Y such that (A, E) $\tilde{\subseteq}$(U, E) and f^{-1}(V, E) $\tilde{\subseteq}$(U, E).

Conversely let (F, E) be any soft regular closed set of X. Then f^{-1}(Y\f(F, E)) $\tilde{\subseteq}$ $X\backslash$(F, E) and $X\backslash$(F, E) ∈ $\tilde{S}$RO(X). Then there exists a soft πg-open set (V, E) of Y such that Y\f(F, E) $\tilde{\subseteq}$(V, E) and f^{-1}(V, E) $\tilde{\subseteq}$ $X\backslash$(F, E). Therefore Y\(V, E) $\tilde{\subseteq}$ f(F, E) $\tilde{\subseteq}$ $f(X\backslash f^{-1}$((V, E) $\tilde{\subseteq}$ $Y\backslash$(V, E). Hence we obtain f(F, E) = Y\(V, E) and f(F, E) is soft πg-closed in Y which shows that f is soft πg-closed.

Definition: 4.8
A space (X, τ, E) is said to be soft quasi-normal, if for any two disjoint soft π-closed sets (A, E) and (B, E) in (X, τ, E), there exists disjoint soft open sets (U, E) and (V, E) such that (A, E) $\tilde{\subseteq}$ (U, E) and (B, E) $\tilde{\subseteq}$ (V, E).

Definition: 4.9
A function f: $(X, \tau, E) \to (Y, \tau', E)$ is called
1. Soft π-closed injection, if f (A, E) is soft π-closed in Y for every soft π-closed set (A, E) of X.
2. Soft almost π-continuous, if f^{-1}(A, E) is soft π-closed in X for every soft regular closed set (A, E) of Y.
3. Soft π-irresolute, if f^{-1}(A, E) is soft is soft π-closed in X for every soft π-closed set (A, E) of Y.

Theorem: 4.10
If f: $(X, \tau, E) \to (Y, \tau', E)$ is soft almost πg-continuous, soft π-closed injection and Y is soft quasi-normal space then X is soft quasi-normal.

Proof:
Let (A, E) and (B, E) be any disjoint soft π-closed sets of X. Since f is a soft π-closed injection, f(A, E) and f(B, E) are disjoint soft π-closed sets of Y. Since Y is soft quasi-normal there exists disjoint soft open sets (U, E) and (V, E) of Y such that f(A, E) $\tilde{\subseteq}$ (U, E) and f(B, E) $\tilde{\subseteq}$ (V, E). Now if (G, E) = intcl(U, E) and (H, E) = intcl(V, E), then

(G, E) and (H, E) are disjoint soft regular open sets such that f(A, E) $\tilde{\subseteq}$ (G, E) and f(B, E) $\tilde{\subseteq}$ (H, E).Since f is soft almost πg-continuous, f^{-1}(G, E) and f^{-1}(H, E)are disjoint soft πg-open sets containing (A, E) and (B, E) which shows that X is soft quasi-normal.

Lemma: 4.11
A surjection f: $(X, \tau, E) \to (Y, \tau', E)$ is soft almost closed if and only if for each subset (A, E) of Y and each (U, E) (U, E) ∈ $\tilde{S}$RO(X) containing f^{-1}(A, E) there exists a soft open set (V, E) of Y such that (A, E) $\tilde{\subseteq}$ (V, E) and f^{-1} (V, E) $\tilde{\subseteq}$ (U, E).

Theorem: 4.12
If f: $(X, \tau, E) \to (Y, \tau', E)$ is soft almost π-continuous, soft almost closed surjection and X is soft quasi-normal space then Y is soft quasi-normal.

Proof:
Let (A, E) and (B, E) be any two disjoint soft closed sets of Y. Then f^{-1} (A, E) and f^{-1} (B, E) are disjoint soft π-closed sets of X. Since X is soft quasi-normal there exists a disjoint soft open sets (U, E) and (V, E) such that f^{-1} (A, E) $\tilde{\subseteq}$ (U, E) and f^{-1} (B, E) $\tilde{\subseteq}$ (V, E). Let (G, E) = intcl (U, E) and (H, E) = intcl(V, E). Then (G, E) and (H, E) are disjoint soft regular open sets of X such that f^{-1} (A, E) $\tilde{\subseteq}$ (G, E) and f^{-1} (B, E) $\tilde{\subseteq}$ (H, E). Take (K, E) = Y\f(X\ (G, E)) and (L, E) = Y\f(X\(H, E)). By previous lemma, (K, E) and (L, E) are soft open sets of Y such that (A, E) $\tilde{\subseteq}$ (K, E) and (B, E) $\tilde{\subseteq}$ (L, E), f^{-1} (K, E) $\tilde{\subseteq}$ (G, E) and f^{-1} (L, E) $\tilde{\subseteq}$ (H, E). Since (G, E) and (H, E) are disjoint, (K, E) and (L, E) are disjoint. Since (K, E) and (L, E) are soft open, we obtain (A, E) $\tilde{\subseteq}$ int(K, E), (B, E) $\tilde{\subseteq}$ int(L, E) and (A, E) $\tilde{\subseteq}$ int(K, E)∩ (B, E) $\tilde{\subseteq}$ int(L, E) $= \emptyset$. Therefore Y is soft quasi-normal.

Lemma: 4.13
A subset (A, E) of a space X is soft πg-open if and only if (F, E) $\tilde{\subseteq}$ int (A, E) whenever (F, E) is soft π-closed and (F, E) $\tilde{\subseteq}$ (A, E)

Theorem: 4.14
Let f: $(X, \tau, E) \to (Y, \tau', E)$ be a soft π-continuous and soft almost πg-closed surjection. If X is soft quasi-normal space then Y is soft quasi-normal.

Proof:
Let (A, E) and (B, E) be any two disjoint soft π-closed sets of Y. Since f is soft π-continuous, f^{-1} (A, E) and f^{-1} (B, E) are disjoint soft π-closed sets of X. Since X is soft quasi-normal there exists a disjoint soft open sets (U, E) and (V, E) of X such that f^{-1} (A, E) $\tilde{\subseteq}$ (U, E) and f^{-1} (B, E) $\tilde{\subseteq}$ (V, E). Let (G, E) = intcl (U, E) and (H, E) = intcl (V, E). Then (G, E) and (H, E) are disjoint soft regular open sets of X such that f^{-1} (A, E) $\tilde{\subseteq}$ (G, E) and f^{-1} (B, E) $\tilde{\subseteq}$ (H, E). Then by theorem: 4.10 there exists soft πg-open sets (K, E) and (L, E) of Y such that (A, E) $\tilde{\subseteq}$ (K, E) and (B, E) $\tilde{\subseteq}$ (L, E), f^{-1} (K, E) $\tilde{\subseteq}$ (G, E) and f^{-1} (L, E) $\tilde{\subseteq}$ (H, E). Since (G, E) and (H, E) are disjoint, (K, E)

and (L, E) are disjoint. By previous we obtain (A, E) $\tilde{\subseteq}$ int (K, E), (B, E) $\tilde{\subseteq}$ int(L, E) and (A, E) $\tilde{\subseteq}$ int(K, E)$\cap$ (B, E) $\tilde{\subseteq}$ int(L, E) $= \emptyset$. Therefore Y is soft quasi-normal.

Definition: 4.15

A function f: $(X, \tau, E) \to (Y, \tau', E)$ is said to be soft quasi πg-compact, if it is onto and if (A, E) is soft πg-open (soft πg-closed) whenever f^{-1} (A, E) is soft open (soft closed).

Definition: 4.16

A function f: $(X, \tau, E) \to (Y, \tau', E)$ is said to be soft almost quasi πg-compact, if it is onto and if (A, E) is soft πg-open (soft πg-closed) whenever f^{-1} (A, E) is soft regular open (soft regular closed).

Theorem: 4.17

A function f: $(X, \tau, E) \to (Y, \tau', E)$ of X onto Y is soft almost quasi πg-compact if and only if the image of every soft regular open inverse is soft πg-open.

Proof:

Let f be a soft almost quasi πg-compact. Let (A, E) be any soft regular open inverse set. Then since f^{-1} (f(A, E)) = (A, E) is soft regular open, f(A, E) is soft πg-open.
Conversely, if f^{-1} (F, E) be soft regular open, then f^{-1} (A, E) is soft regular inverse set. Therefore f(f^{-1} (A, E)) is soft πg-open. That is (F, E) is soft πg-open. Hence f is soft almost quasi πg-compact.

Corollory: 4.18

A function f: $(X, \tau, E) \to (Y, \tau', E)$ of X onto Y is soft almost quasi πg-compact if and only if the image of every soft regular closed inverse is soft πg-closed.

Theorem: 4.19

If f: $(X, \tau, E) \to (Y, \tau', E)$ is one to one functions of X onto Y then the following properties are equivalent:

1. f is soft almost πg-open
2. f is soft almost πg-closed.
3. f is soft almost quasi πg-compact
4. f^{-1} is soft almost πg-continuous

Theorem: 4.20

Suppose that f: $(X, \tau, E) \to (Y, \tau', E)$and g: $(Y, \tau', E) \to (Z, \tau'', E)$ be functions. Then the following properties:

1. if f is soft almost πg-continuous and if g ∘ f is soft πg-open then g is soft almost πg-open.
2. if f is soft almost πg-continuous and if g ∘ f is soft πg-closed then g is soft almost πg-closed.
3. if f is soft almost πg-continuous and if g ∘ f is soft quasi πg-compact then g is soft almost quasi πg-compact.

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