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Some Results On Root Square Mean Graphs

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Abstract

A graph $G = (V, E)$ with p vertices and q edges is said to be a Root Square Mean graph if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1,2, \ldots, q + 1$ in such a way that when each edge $e = uv$ is labeled with $f(e = uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the resulting edge labels are distinct. In this case f is called a Root Square Mean labeling of G.In this paper we investigate some results on Root Square Mean labeling of graphs.

Key Words: Graph, Root Square Mean graph, Path, Cycle, Complete graph, complete bipartite graph.

1. Introduction

The graph considered here will be finite, undirected and simple. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$.For all detailed survey of graph labeling we refer to Gallian [1].For all other standard terminology and notations we follow Harary[2]. S.S.Sandhya, S.Somasundaram and S.Anusa introduced the concept of Root Square Mean labeling of graphs in [3].In this paper we investigate some results on Root Square Mean labeling of graphs. The definitions and other information's which are useful for the present investigation are given below.

Definition1.1: A walk in which $u_1 u_2 \ldots u_n$ are distinct is called a path. A path on n vertices is denoted by P_n.

Definition1.2: A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

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Definition1.3: The Corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by one copy of G_1 and $|G_1|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition1.4: Any cycle with a pendent edge attached at each vertex is called a crown. It is denoted by $C_n \odot K_1$.

Definition1.5: A Dragon is formed by joining the end point of a path to a cycle. It is denoted by $C_n @ P_m$.

Definition1.6: A graph G is said to be complete if every pair of its distinct vertices are adjacent. A Complete graph on n vertices is denoted by K_n.

Definition1.7: The Complement $\bar{G}$ of a graph G has $V(G)$ as its vertex set, but two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

Definition1.8: An (n, t)-kite graph consists of a cycle of length n with t edges path attached to one vertex of a cycle.

Definition1.9: A complete bipartite graph is a bipartite graph with bipartition (V_1, V_2) such that every vertex of V_1 is joined to all the vertices of V_2.It is denoted by $K_{m,n}$, where $|V_1| = m$ and $|V_2| = n$.

Theorem1.10: Any path is a Root Square Mean graph.

Theorem1.11: Any Cycle is a Root Square Mean graph.

Theorem1.12: Combs are Root Square Mean graphs.

Theorem1.13: Complete graph K_n is a Root Square Mean graph if and only if $n \leq 4$.

Theorem1.14*:* $K_{1,n}$ is a Root Square Mean graph if and only if $n \leq 6$.

2. Main Results

Theorem2.1: Let P_n be the path and G be the graph obtained from P_n by attaching C_3 in both the end edges of P_n.Then G is a Root Square Mean graph.

Proof: Let P_n be the path $u_1 u_2 \dots u_n$ and $v_1 u_1 u_2, v_2 u_{n-1} u_n$ be the triangles which are connected to the path at the end. Define a function $f: V(G) \to \{1,2, \dots, q+1\}$ by

$f(u_i) = i + 1, 1 \leq i \leq n - 1$

$f(u_n) = n + 3$

$f(v_1) = 1, f(v_2) = n + 2$

The edges are labeled as

$f(u_1v_1) = 1, f(u_2v_1) = 2$
$f(u_{n-1}v_2) = n + 1, f(u_nv_2) = n + 3$
$f(u_{n-1}u_n) = n + 2$
$f(u_iu_{i+1}) = i + 2, 1 \leq i \leq n - 1.$
Hence f is a Root Square Mean labeling.

Example2.2: Root Square Mean labeling of G obtained from P_7 is given below.

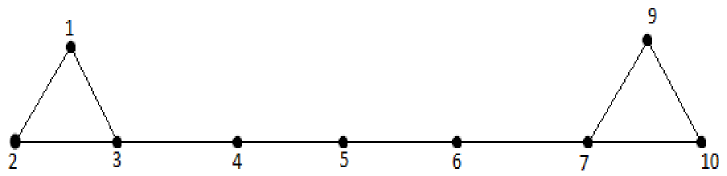

Figure1

Theorem2.3: The Crown $C_n \odot K_1$ is a Root Square Mean graph.

Proof: Let C_n be the cycle $u_1u_2 \ldots u_nu_1$ and v_i be the pendent vertices adjacent to $u_i, 1 \leq i \leq n$.
Define a function $f: V(C_n \odot K_1) \rightarrow \{1,2,3, \ldots, q + 1\}$ by
$f(u_i) = 2i - 1, 1 \leq i \leq n$
$f(v_i) = 2i, 1 \leq i \leq n$
Then we get distinct edge labels. Hence f is a Root Square Mean labeling.

Example2.4: The Root Square Mean labeling of $C_6 \odot K_1$ is given below.

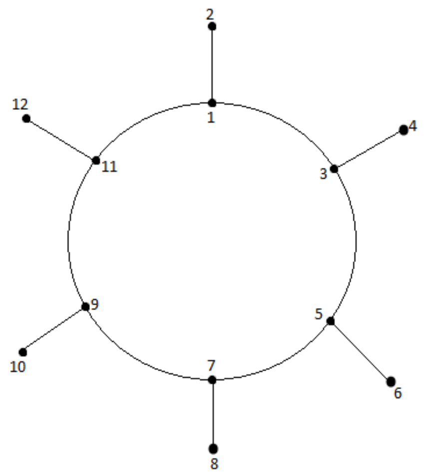

Figure2

Theorem2.5: Dragon$C_n@P_m$ is a Root Square Mean graph.

Proof: Let $v_1 v_2 \dots v_m$ be the path P_m be the cycleC_n and $v_1 v_2 \dots v_n$ be the path P_m. Here $u_n = v_1$.Define a function $f: V(C_n@P_m) \rightarrow \{1,2,3,\dots,q+1\}$ by
$f(u_i) = i, 1 \leq i \leq n,$
$f(v_{i+1}) = n + i, 1 \leq i \leq m.$
Then the edge labels are distinct. Hence f is a Root Square Mean labeling.

Example2.6: Root Square mean labeling of $C_6@P_6$ is given below.

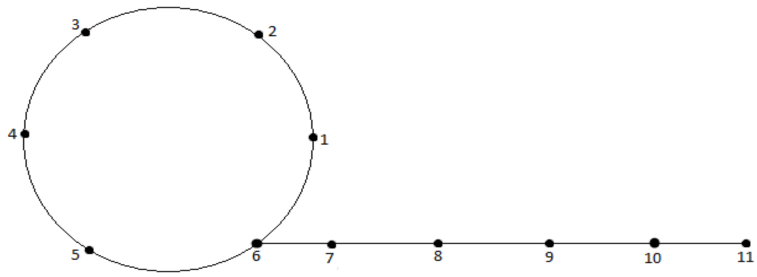

Figure3

Theorem2.7: Let G be a graph obtained by attaching a pendent edge to both sides of each vertex of a path P_n.Then G is a Root Square Mean graph.

Proof: Let G be the graph obtained by attaching pendent edges to both sides of each vertex of a path P_n.Let u_i, v_i and $w_i, 1 \leq i \leq n$ be the new vertices of G. Define a function $f: V(G) \rightarrow \{1,2,3,\dots,q+1\}$ by
$f(u_i) = 3i - 1, 1 \leq i \leq n$
$f(v_i) = 3i, 1 \leq i \leq n$
$f(w_i) = 3i - 2, 1 \leq i \leq n.$
Then the edges are labeled as
$f(u_i u_{i+1}) = 3i, 1 \leq i \leq n - 1,$
$f(u_i v_i) = 3i - 1, 1 \leq i \leq n$
$f(u_i w_i) = 3i - 2, 1 \leq i \leq n.$
Hence f is a Root Square Mean labeling Then the edge labels are distinct.

Example2.8: The graph obtained from P_6 is given below.

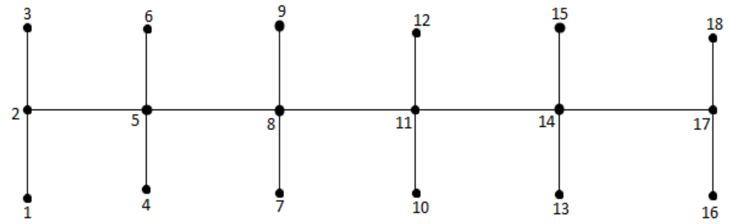

Figure4

Theorem2.9: $C_n \odot \overline{K_2}$ is a Root Square Mean graph for all $n \geq 3$.

Proof: Let C_n be the cycle $u_1u_2 \dots u_nu_1$ and v_i, w_i be the vertices adjacent to $u_i, 1 \leq i \leq n$.

Define a function $f: V(C_n \odot \overline{K_2}) \rightarrow \{1,2,3, \dots, q+1\}$ by

$f(u_i) = 3i - 1, 1 \leq i \leq n$

$f(v_i) = 3i - 2, 1 \leq i \leq n$

$f(w_i) = 3i, 1 \leq i \leq n$.

Then the edge labels are distinct. Hence f is a Root Square Mean labeling.

Example2.10: Here we display the Root Square Mean labeling of $C_6 \odot \overline{K_2}$

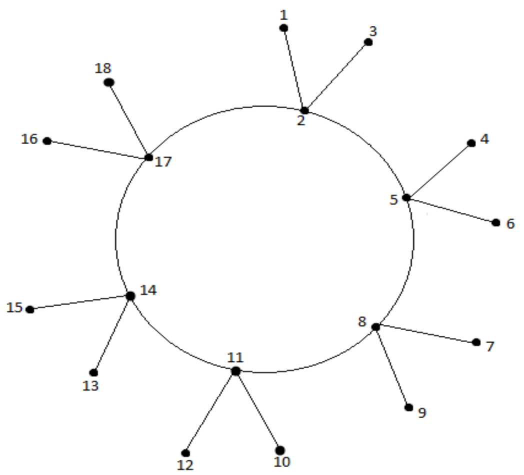

Figure 5

Theorem2.11: $C_n \odot \overline{K_3}$ is a Root Square Mean graph for all $n \geq 3$.

Proof: Let C_n be the cycle $u_1u_2 \dots u_nu_1$ and v_i, w_i and t_i be the vertices adjacent to $u_i, 1 \leq i \leq n$.

Define a function $f: V(C_n \odot \overline{K_3}) \rightarrow \{1,2,3, \dots, q+1\}$ by

$f(u_i) = 4i - 2, 1 \leq i \leq n$
$f(v_i) = 4i - 3, 1 \leq i \leq n$
$f(w_i) = 4i - 1, 1 \leq i \leq n$
$f(t_i) = 4i, 1 \leq i \leq n.$
Then the edge labels are distinct. Hence f is a Root Square Mean labeling.

Example2.12: Here we display the Root Square Mean labeling of $C_6 \odot \overline{K_3}$.

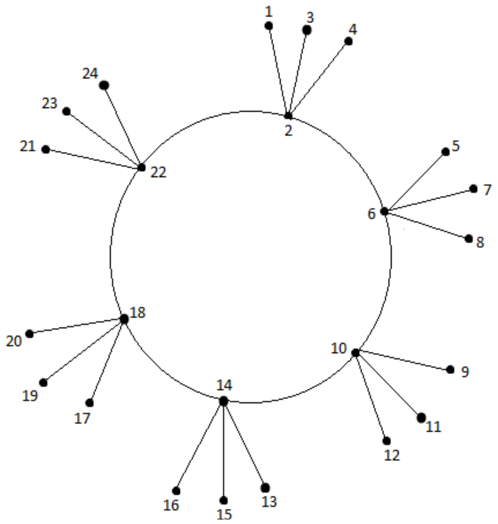

Figure 6

Theorem2.13: Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$.Then G is a Root Square Mean graph.

Proof: Let P_n be the path $u_1 u_2 \dots u_n$ and let v_i, w_i be the vertices of $K_{1,2}$ which are attached to the vertex u_i of P_n.
Define a function $f: V(G) \rightarrow \{1,2,3, \dots, q+1\}$ by
$f(u_i) = 3i - 2, 1 \leq i \leq n,$
$f(v_i) = 3i - 1, 1 \leq i \leq n,$
$f(w_i) = 3i, 1 \leq i \leq n.$
Then the edges are labeled as
$f(u_i u_{i+1}) = 3i, 1 \leq i \leq n - 1$
$f(u_i v_i) = 3i - 2, 1 \leq i \leq n$
$f(u_i w_i) = 3i - 1, 1 \leq i \leq n$
Hence f is a Root Square Mean labeling Then the edge labels are distinct.

Example2.14: Root Square Mean labeling of G obtained from P_4 is given below.

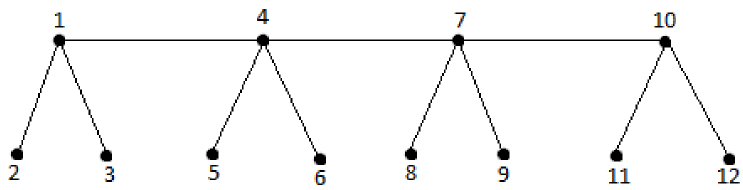

Figure 7

Theorem2.15: Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,3}$.Then G is a Root Square Mean graph.

Proof: Let P_n be the path $u_1u_2 \dots u_n$ and v_i, w_i and t_i be the vertices of $K_{1,3}$ which are attached to the vertices u_i of P_n.
Define a function $f: V(G) \to \{1,2,3, \dots, q+1\}$ by
$f(u_i) = 4i - 3, 1 \le i \le n$
$f(v_i) = 4i - 2, 1 \le i \le n$
$f(w_i) = 4i - 1, 1 \le i \le n$
$f(t_i) = 4i, 1 \le i \le n$.
Then the edges are labeled as
$f(u_iu_{i+1}) = 4i, 1 \le i \le n-1$
$f(u_iv_i) = 4i - 3, 1 \le i \le n$
$f(u_iw_i) = 4i - 2, 1 \le i \le n$
$f(u_it_i) = 4i - 1, 1 \le i \le n$
Then f is a Root Square Mean labeling.

Example2.16: The Root Square mean labeling of $P_4 \odot K_{1,3}$ is given below.

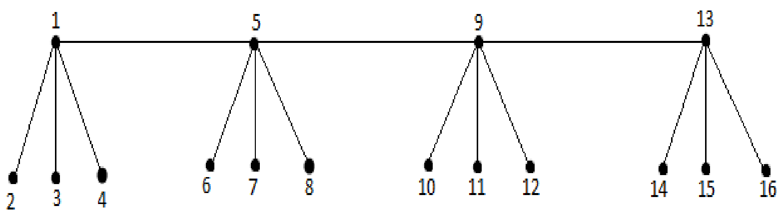

Figure 8

Theorem2.17: Let G be a graph obtained by joining a pendent vertex with a vertex of degree two of a comb graph. Then G is a Root Square Mean graph.

Proof: Let the vertex set of the comb be $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$.Let P_n be the path $u_1u_2 \dots u_n$.Join a vertex v_i to $u_{i,}1 \le i \le n$.Let G be a graph obtained by joining a pendent vertex w to u_n.Define a function $f: V(G) \to \{1,2,3, \dots, q+1\}$ by
$f(u_i) = 2i - 1, 1 \le i \le n$
$f(v_i) = 2i, 1 \le i \le n$
$f(w) = 2n + 1.$
Then the edges are labeled as
$f(u_iu_{i+1}) = 2i, 1 \le i \le n - 1$
$f(u_iv_i) = 2i - 1, 1 \le i \le n$
$f(u_nw) = 2n.$
This gives a Root Square Mean labeling of G.

Example2.18: Root Square Mean labeling of G with 11 vertices and 10 edges is given below

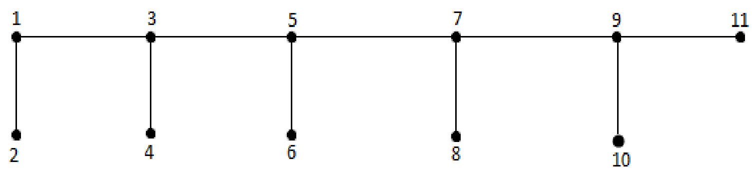

Figure 9

In the similar manner, we can see the Root Square Mean labeling of G obtained by joining a pendent vertex with a vertex of degree two on both ends of a comb graph. Root Square Mean labeling of G with 10 vertices and 9 edges is shown below.

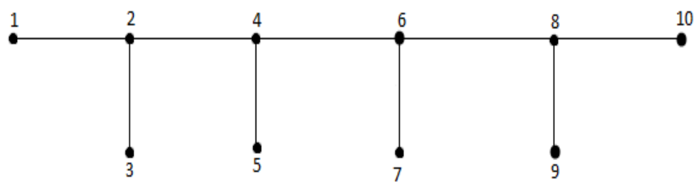

Figure10

Theorem2.19: A $(m, n) -$ Kite graph is a Root Square Mean graph.

Proof: Let $u_1u_2 \dots u_mu_1$ be the given cycle of length m and $v_1v_2 \dots v_n$ be the given path of length n. Define a function $f: V(G) \to \{1,2,3, \dots, q+1\}$ by
$f(u_i) = i, 1 \le i \le m$
$f(v_i) = m + i, 1 \le i \le n$

Then the edge labels are distinct. Hence $(m, n) -$ Kite graph is a Root Square Mean graph.

Example2.20: The Root Square Mean labeling of $(5,6) -$ Kite graph is shown below.

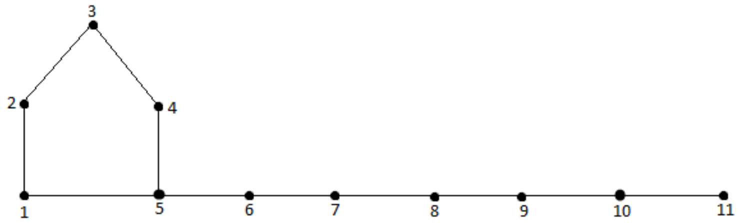

Figure 11

Theorem2.21: $K_{n,n}$ is a Root Square Mean graph if and only if $n \leq 2$.

Proof: Clearly $K_{1,1}$ is a Root Square Mean graph.The labeling pattern of $K_{1,1}$ and $K_{2,2}$ are shown below.

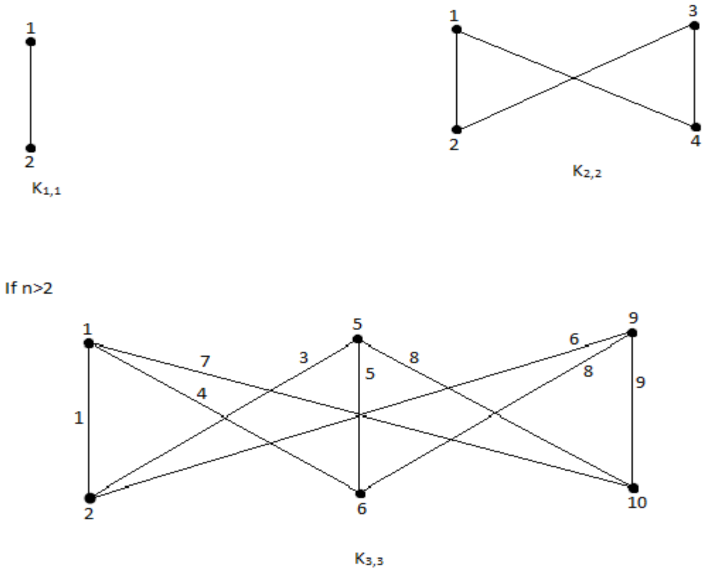

Figure12

Here the edge labels of $(5,10), (9,6)$ are repeated. Hence $K_{n,n}$ is a Root Square Mean graph if and only if $n \leq 2$.

Conclusion

All graphs are not Root Square Mean graphs. It is very interesting to investigate graphs which admit Root Square Mean labeling. In this paper, we proved that Dragon, Crown, Carona of some graphs are Root Square graphs. It is possible to investigate similar results for several other graphs.

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