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F^*- Bi Near Subtraction Semigroups

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Abstract

In this paper we introduce the notion of F^*- bi-near subtraction semigroup. Also we give characterizations of F^*- bi-near subtraction semigroup.

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Key words: F^*- bi-near subtraction semigroup, strong S_1-near subtraction semigroup, strong S_2- -near subtraction semigroup, Mate function, Boolean.

1.Introduction

In 2007, Dheena[1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz[4]. Zekiye Ciloglu, Yilmaz Ceven [5] gave the notation of Fuzzy Near Subtraction semigroups. Seydali Fathima et.al[2,3] introduced the notation of S_1-near subtraction semigroup and S_2-near subtraction semigroup. Recently Firthous et.al[2] introduced the notation of F- Bi near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of F^*- Bi near subtraction semigroup .

2. Preliminaries :

A non-empty subset X together with two binary operations "−" and "." is said to be ***subtraction semigroup*** If (i) (X,−) is a subtraction algebra (ii) (X, .) is a semi group (iii) $x(y-z)=xy-xz$ and $(x-y)z= xz-yz$ for every x, y, $z\in X$.A non-empty subset X together with two binary operations "−" and "." is said to be ***near subtraction semigroup*** if (i) (X,−) is a subtraction algebra (ii) (X,.) is a semi group and (iii) $(x-y)z= xz-yz$ for every x, y, $z\in X$.. A non-empty subset $X=X_1\cup X_2$ together with two binary operations"-" and "." Is said to be ***bi-near subtraction semigroup***(right). If (i) $(X_1,-,.)$ is a near-subtraction semigroup (ii) $(X_2,-,.)$ is a subtraction semigroup. A non-empty subset X is said to be ***S_1-near subtraction semi group*** if for every $a\in X$ there exists $x\in X^*$ such that axa=xa.. A non-empty subset X is said to be ***S_2-near subtraction semi group*** if for every $a\in X$ there exists $x\in X^*$ such that axa=ax.. A non-empty subset X is said to be ***strong S_1-near subtraction semi group*** if aba=ba for all a, $b\in X$.. A non-empty subset X is said to be ***strong S_2-near subtraction semi group*** if aba=ab for all a, $b\in X$. If there exists a map $f:X\rightarrow Y$ such that a = a f(a) a for all a in X then f is called a ***mate function*** for X. An element $a\in X$ is said to be ***Boolean*** if a^2 =a. A non-empty subset $X=X_1\cup X_2$ together with two binary operations"-" and "." Is said to be *F*- ***bi near subtraction semigroups***. If (i) for every $a\in X_1$ there exists $x\in X_1^*$ such that axa=xa. (ii) for every $a\in X_2$ there exists $x\in X_2^*$ such that axa=ax.

3..F^*-Bi near Subtraction Semigroup

Definition 3.1

A non-empty subset $X=X_1\cup X_2$ together with two binary operations"-" and "." Is said to be F^*- **bi near subtraction semigroups (strong F-*Bi near Subtraction Semigroups)***. If (i) if aba=ba for all a, $b\in X_1$. (ii) if aba=ab for all a, $b\in X_2$.

Example 3.2

Let X_1={0, a,b,1} in which "–" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Thus X_1 is a strong s_1-near subtraction semi group

Let X_2={0,a,b,1} in which "–" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	0	a	0
b	0	0	b	b
1	0	a	b	1

Thus X_2 is a strong S_2-near subtraction semigroup. Hence, X= $X_1\cup X_2$is a strong F- bi-near subtraction semi group (F*-bi-near subtraction semi group).

Result 3.3

Every F^*-bi near Subtraction Semigroup is a F- bi near Subtraction Semigroup

<u>Proof</u>: Let X= $X_1\cup X_2$ be a F^*-*bi near Subtraction Semigroup where* X_1 is a strong S_1-

near subtraction semigroup and X_2 is a strong S_2-near subtraction semigroupBy [3], Every Strong S_1-near subtraction semigroup is a S_1-near subtraction semigroup By [4], Every Strong S_2-near subtraction semigroup is a S_2-near subtraction semigroupHere, X_1 is an S_1-near subtraction semigroup and X_2 is an S_2-near subtraction semigroup.Thus $X= X_1 \cup X_2$ is a F- bi near Subtraction Semigroup.

Remark 3.4: The converse of the above result need not be true.

Example 3.4

Let $X_1=\{0, a,b,c\}$ in which "–" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	0	0	0	a
b	0	0	0	b
c	0	0	0	c

Thus $(X_{1,}$ -, .) is not a strong s_1-near subtraction semi group (since $aba \neq a$)

Let $X_2=\{0,a,b,c\}$ in which "–" and "." be defined by

-	0	a	b	c
0	0	0	0	0
a	a	0	a	b
b	b	b	0	b
c	c	c	c	0

.	0	a	b	c
0	0	0	0	0
a	a	a	b	a
b	0	0	0	0
c	0	0	0	c

Thus (X_2 , -, .) is not a strong S_2-near subtraction semigroup (since $bab \neq ab$). Hence, $X= X_1 \cup X_2$is not a strong F- bi-near subtraction semi group (F*-bi-near subtraction semi group).

4.Results on *F^*-bi near Subtraction Semigroup.*

Proposition 4.1

The intersection of strong S_1-near subtraction semigroup and strong S_2-near subtraction semigroup is sub commutative near subtraction semigroup.

Proof

Let X_1 is a strong S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that axa=xa. ---(1) (by [3], Every Strong S_1-near subtraction semigroup is a S_1-near subtraction semigroup)Let X_2 is an Strong S_2-near subtraction semigroup, there exists $x \in X_2^*$ such that axa=ax -----(2) (by [3], Every Strong S_2-near subtraction semigroup is a S_2-near subtraction semigroup)From (1) and (2), we get xa=axThus, X is a sub commutative near subtraction semigroup.

Proposition 4.2

Let X be a F^**-bi near Subtraction Semigroup Then X has a mate function if and only if*

X is Boolean.

Proof: Let $X= X_1 \cup X_2$ be a F^*-*bi near Subtraction Semigroup where* X_1 is a strong S_1-near subtraction semigroup and X_2 is a strong S_2-near subtraction semigroup Assume that f is a mate function for X_1.If $a \in X_1$ then a f(a)=a. Since X_1 is a strong S_1-near subtraction semigroup, a f(a) a = f(a) a (Ie.,) a = f(a) a. Now, a^2 = a a= a (f(a) a) = a(Ie.,) a^2 = a Thus *X_1 is Boolean.*Assume that f is a mate function for X_2.Ifa$\in X_2$ then a =a f(a) a .Since X_2 is a strong S_2-near subtraction semigroup,a f(a) a = a f(a)(Ie.,) a = a f(a). Now, a^2 = a a= (a f(a)) a = a(Ie.,) a^2 =a Thus *X_2 is Boolean.*Therefore $X= X_1 \cup X_2$ where *X_1 is Boolean and X_2 is Boolean.Hence, X is Boolean.*

Proposition 4.3
Let X be a F^*-*bi near Subtraction Semigroup Then aXbX* $\cup$ XaXb=abX$\cup$Xab for all a,b$\in$ X.

Proof : Let $X= X_1 \cup X_2$ be a F^*-*bi near Subtraction Semigroup where* X_1 is a strong S_1-near subtraction semigroup and X_2 is a strong S_2-near subtraction semigroup *Since* X_1 is a strong S_1-near subtraction semigroup, a b a=b a for all a,b $\in X_1$.Let $x_1 \in$ a X_1b X_1. Then there exists n,$n^1 \in X_1$ such that X_1=a n b n^1 =a (nb) n^1= a (bnb) n^1= (ab) nb$n^1 \in$ abX_1 (Ie.,) $X_1 \in$ abX_1; a X_1b $X_1 \subseteq$ abX_1 ---(1)Let $y_1 \in$ abX_1Then there exists m $\in X_1$suchthat Y_1 = abm = a(bm) =a(mbm)$\in$a X_1b X_1. (Ie.,) $Y_1 \in$a X_1b X_1.ab$X_1 \subseteq$ a X_1b X_1 -------(2) from (1) and (2) we have a X_1b X_1 = abX_1 for all a,b $\in X_1$.*Given* X_2 is a strong S_2-near subtraction semigroup, a b a=b a for all a,b $\in X_2$.Let $x_2 \in X_2$ a X_2b. Then there exists n,$n^1 \in X_2$ such that X_2= n a n^1b =n (a n^1 a) n^1= (n a n^1) ab$\in X_2$ab (Ie.,) $X_2 \in X_2$ab X_2 a X_2b $\subseteq X_2$ ab --------(3) Let $y_2 \in X_2$ab Then there exists m $\in X_2$ such that Y_2= m ab = (ma)b =(mam)b =mamb$\in X_2$ a X_2b. (Ie.,) $Y_2 \in X_2$ a X_2b. X_2ab $\subseteq X_2$ a X_2b -------(4)from (3) and (4) we have X_2 a X_2b= X_2 ab for all a,b $\in X_2$.*Since* $X= X_1 \cup X_2$ is a F^*-*bi near Subtraction Semigroup* Hence, *aXbX* $\cup$ XaXb=abX$\cup$Xab for all a,b$\in$ X.

Proposition 4.4
Let X be a F^*-*bi near Subtraction Semigroup Then ab and* ba$\in$E for all a, b $\in$ X.

Proof: Let $X= X_1 \cup X_2$ be a F^*-*bi near Subtraction Semigroup where* X_1 is a strong S_1-near subtraction semigroup and X_2 is a strong S_2-near subtraction semigroup *Since* X_1 is a strong S_1-near subtraction semigroup, a x a=x a for all a,x $\in X_1$. Let a,b$\in X_1$. Now, ab=bab=(ba)b=(aba)b=ab^2. (Ie.,) ab= ab^2. Which implies ab $\in$E In similar way we have ba $\in$E *Since* X_2 is a strong S_2-near subtraction semigroup, x y x =x y for all x,y $\in X_2$. Let a,b$\in X_2$. Now, $(ab)^2$=abab= a(bab)= a(ba)=aba=ab. (Ie.,) ab^2= ab. Which implies ab $\in$E In similar way we have ba $\in$E for all a,b $\in X_2$. Therefore $X= X_1 \cup X_2$ is a F^*-*bi near Subtraction Semigroup* Thus *ab and* ba$\in$E for all a, b $\in$ X.

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