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Geometric Mean Labelings of trees in $\Gamma(Z_n)$

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Abstract

Let R be a commutative ring and let Z(R) be its set of zero-divisors. We associate a graph Γ(R) to R with vertices Z(R)* = Z(R)–{0}, the set of non-zero divisors of R and for distinct u, v ∈ Z(R)*, the vertices u and v are adjacent if and only if uv = 0. In this paper, we evaluate the geometric mean labelings in a graph obtained by attaching the central vertex of $\Gamma(Z_{2p})$, where $p > 2$ is any prime number.

Subject Classification: 05C69

Keywords: Geometric Mean Labelings, Zero Divisor Graphs.

1 Introduction

Let R be a commutative ring and let Z(R) be its set of zero-divisors. We associate a graph Γ(R) to R with vertices Z(R)* = Z(R)–{0}, the set of non-zero divisors of R and for distinct u, v ∈ Z(R)*, the vertices u and v are adjacent if and only if uv = 0. The zero divisor graph is very useful to find the algebraic structures and properties of rings. The idea of a zero divisor graph of a commutative ring was introduced by I. Beck in [2]. The first simplication of Beck's zero divisor graph was introduced by D. F. Anderson and P. S. Livingston [1]. Their motivation was to give a better illustration of the zero divisor structure of the ring. D.F.Andersonand P.S.Livingston, and others, e.g., [3, 4, 5, 6], investigate the interplay between the graph theoretic properties of $\Gamma(R)$ and the ring theoretic properties of R. Throughout this paper, we consider the commutative ring R by Z_n and zero divisor graph Γ(R) by $\Gamma(Z_n)$.

S. Somasundaram and P. Vidhyarani [7] introduced the concept of Geometric mean labelings. Let a graph G be simple with p vertices and q edges. G is said to be a Geometric mean graph if it possible to label vertices $x \in V$ with distinct labels $f(x)$ from 1, 2,..., q+1 in such a way that when edge e = uv is labeled with $\left\lceil \sqrt{f(u)f(v)} \right\rceil$ or $\left\lfloor \sqrt{f(u)f(v)} \right\rfloor$ then the resulting edge labels are distinct. In this case, f is called Geometric mean labeling of G.

2 Geometric Mean Labelings of Trees in $\Gamma(Z_n)$

In this section, we evaluate the geometric mean labelings of trees in $\Gamma(Z_n)$. Also, we evaluate the geometric mean labelings in a graph obtained by attaching the central vertex of $\Gamma(Z_{2p})$, where p>2 is any prime number.

Theorem 2.1 If $p > 7$ is any prime number, $\Gamma(Z_{2p})$ is not a geometric mean graph.

Proof: We know that the vertex set of $\Gamma(Z_{2p})$ is { 2, 4, 6, …, 2(p–1), p} and the order of $\Gamma(Z_{2p})$ is p. Let us assure that $\Gamma(Z_{2p})$ is geometric mean graph. Clearly, the vertex 'p' is adjacent with all other remaining vertices in $\Gamma(Z_{2p})$. Let the central vertex of $\Gamma(Z_{2p})$ is p. The other vertices be v_1, v_2, ….v_{p-1}, respectively. Then define function
l: $V(\Gamma(Z_{2p})) \to \{1, 2, .., p\}$

Case (i): Let $l(p) \neq 1$.
The central vertex cannot be labeled 1. Otherwise the edge joining 1 and 4 should be labeled 2 and the edge joining (1, 2) and (1, 3) get the same label 1. Clearly, get the edge label 1 the central vertex 'p' has to be either 2 or more than 2.

Case(ii): $l(p) = 2$.
Let the central vertex be labeled 2 and all the other vertices be labeled 1, 3, 4,...., p, respectively. Edge joining the vertices (2, 5) and (2, 6) receives the same label namely 4. So, we can't assign the label 2 for the central vertex 'p'.

Case(iii): $l(p) = 3$.
Let the central vertex p labeled 3 and remaining vertices be labeled 1, 2, 4, ….., (p) respectively. Clearly, the edge joining the vertices (3, 6) and (3, 7) receives the same label namely 5. From the above cases, it can be seen that there is a contradiction with the assumption. Hence, when the prime number $P > 7$, $\Gamma(Z_{2p})$ is not a geometric mean graph.

Theorem 2.2 Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $\Gamma(Z_{2p})$, where p=3. Then, G admits a geometric mean labeling.

Proof: Let $\Gamma(Z_{2p})$ be a star graph namely, $K_{1,2}$, where p = 3. The vertex set of $\Gamma(Z_6)$ is {2, 3, 4}. Let P_n be the path namely v_1, v_2, v_3, …., v_n and u_1, u_2 be the vertices of $K_{1,2}$ which the attached to vertex v_i of P_n. The number of vertices in G is equal to the sum

of the vertices in the path and every vertex in P_n adjacent with two end vertices. That is $n(V(G)) = n(V(P_n)) + 2n(V(P_n))$ = n + 2n = 3n. Clearly, the graph is tree with 3n vertices and 3n–1 edges.
Define a function l: V(G) → {1, 2,….,3n} by $l(v_1) = 3$, $l(v_i) = 3i - 2 \;\; 2 \le i \le n$. Clearly, the vertices in $\Gamma(Z_6)$ is {2, 3, 4}. Remaining vertices in $\Gamma(Z_6)$ as u_1, w_1 and p, respectively. Then $l(u_1) = 1$, $l(u_i) = 3i–1$, $2 \le i \le n$ and $l(w_1) = 2$, $l(w_i) = 3i \;\; 2 \le i \le n$. The remaining one vertex 'p' in $\Gamma(Z_6)$ is central vertex which is attaching to every vertex v_i in P_n. Then the label of the edge $v_iv_{i+1} = 3i$, $1 \le i \le n-1$, the label of the edge $v_iu_i = 3i–2$, $1 \le i \le n$, the label of the edge $v_iw_i = 3i–2$, $1 \le i \le n$ and the label of the edge $u_iw_i = 3i–1$, $1 \le i \le n$. Then, the function l is a geometric mean labeling of G.

Theorem 2.3 Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $\Gamma(Z_{2p})$, where n = 2001 and p=5. Then G admits a geometric mean labeling.

Proof: Let P_{2001} be the path $v_1v_2….v_{2001}$ and p_i, q_i, r_i, s_i be the end vertices of $\Gamma(Z_{10})$ which are attached to the vertices v_i of P_{2001}. Clearly, the graph G has 10005 vertices and 10004 edges. Define a function l: V(G) → {1, 2,….,10005} as $l(v_i) = 5i - 2$ $1 \le i \le 2001$, $l(p_i) = 5i - 4$, $1 \le i \le 2001$, $l(q_i) = 5i - 3$, $1 \le i \le 2001$, $l(r_i) = 5i - 4$, $1 \le i \le 2001$, and $l(s_i) = 5i$, $1 \le i \le 2001$. Then the label of the edge $v_iv_{i+1} = 5i$, $1 \le i \le n-1$, the label of the edge $v_ip_i = 5i–4$, $1 \le i \le n-1$, the label of the edge $v_iq_i = 5i–3$, $1 \le i \le n-1$, the label of the edge $v_ir_i = 5i–2$, $1 \le i \le n-1$, the label of the edge $v_is_i = 5i–1$, $1 \le i \le n-1$. Clearly, the function l is a geometric mean graph.

Theorem 2.4 Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $\Gamma(Z_{2p})$, where n > 2001 and p=5. Then G is not a geometric mean labeling.

Proof: Let P_n be the path $v_1v_2….v_n$ and p_i, q_i, r_i, s_i be the end vertices of $\Gamma(Z_{10})$ which are attached to the vertices v_i of P_n. Clearly, the graph G has 5n vertices and 5n–1 edges. Define a function l: V(G) → {1, 2,….,5n} as $l(v_i) = 5i–2$, $1 \le i \le n$, $l(p_i) = 5i - 4$, $1 \le i \le n$, $l(q_i) = 5i - 3$, $1 \le i \le n$, $l(r_i) = 5i - 4$, $1 \le i \le n$, and $l(s_i) = 5i$, $1 \le i \le n$. Then the label of the edge $v_iv_{i+1} = 5i$, $1 \le i \le n-1$, the label of the edge $v_ip_i = 5i–4$, $1 \le i \le n-1$, the label of the edge $v_iq_i = 5i–3$, $1 \le i \le n-1$, the label of the edge $v_ir_i = 5i–2$, $1 \le i \le n-1$, the label of the edge $v_is_i = 5i–1$, $1 \le i \le n-1$. Using above theorem (2.3), if n=2001 then the graph G admits a geometric mean labeling. But n=2002, then the last five vertices are {10001, 10002, 10003, 10004, 10005}. Clearly, the label of the edge (10001)(10003) is 10002 and the label of the edge (10002)(10003) is also 10002 which is a contradiction for the definition of geometric mean that is all the edges have different labels. Hence, n is large, the graph G obtained by attaching each vertex of P_n to the central vertex of $\Gamma(Z_{10})$ is not a geometric mean graph.

Theorem 2.5 Let G be a graph obtained by attaching the central vertex of $\Gamma(Z_6)$ to the vertices of P_n whose degree is 2. Then G admits a geometric mean labeling.

Proof: The graph G has 3n–4 vertices and 3n–5 edges. Let P_n be the path $v_1v_2\ldots.v_n$ and u_i, w_i be the end vertices of $\Gamma(Z_6)$ which are attached to the vertices v_i of P_n. Define a function V(G) → {1, 2,….,3n–4} as $l(v_1)=1$, $l(v_i)=3(i-1)$, $2\le i\le n-1$, $l(v_n)=l(v_{n-1})+2$, $l(u_2)=2$, $l(u_i)=l(u_{i-1})+3$, $3\le i\le n-1$, $l(w_i)=l(v_i)+1$, $2\le i\le n-1$. Then the label of the edge v_iv_{i+1} = $3i$–2, $1\le i\le n-1$, the label of the edge v_iu_i = $3i$–4, $2\le i\le n-1$, the label of the edge v_iw_i = $3i$–3, $2\le i\le n-1$. Hence the graph G admits a geometric mean labeling.

Theorem 2.6 Let G be a graph obtained by attaching the central vertex of $\Gamma(Z_{10})$ to the vertices of P_n whose degree is 2. Then G does not admits a geometric mean labeling.

Proof: The graph G has 5n–8 vertices and 5n–9 edges. Let P_n be the path $v_1v_2\ldots.v_n$ and p_i, q_i, r_i, w_i, be the end vertices of $\Gamma(Z_{10})$ which are attached to the vertices v_i of P_n. Define a function l: V(G) → {1, 2,….,5n–8}. The proof is by the method of induction on vertices in G.
Let n=3, then G be a graph obtained by attaching the central vertex of $\Gamma(Z_{10})$ to the vertices P_3 whose degree is 2. Clearly, in P_3 only one vertex has degree 2. So, attached the central vertex of $\Gamma(Z_{10})$ to the middle vertex of P_3. Now G has 7 vertices and 6 edges.
Let us take the vertex set of V(G) is { v_1, v_2, v_3, u_1, u_2, u_3, u_4 }, where v_1, v_2 and v_3 are vertices in P_3 and u_1, u_2, u_3, u_4 are end vertices in $\Gamma(Z_{10})$. Now Define a function l : V(G)→{1, 2,….,7} as $l(v_1) = 1$, $l(v_2) = 3$, $l(v_3) = 7$, $l(u_1) = 2$, $l(u_2) = 4$, $l(u_3) = 5$ and $l(u_4) = 6$. Then, the label of the edge v_1v_2 by 1, the label of the edge v_2u_1 by 2, the label of the edge v_2u_2 by 3, the label of the edge v_2u_3 by 4, the label of the edge v_2u_4 by 5, the label of the edge v_2v_3 by 5. Clearly, we get a contradiction that two edges has the same labeling, namely 5. Hence, any graph G, attaching the central vertex of $\Gamma(Z_{10})$) to the vertices of P_n whose degree is 2, does not admits a geometric mean labeling.

Theorem 2.7 Let G be a graph obtained by attaching the central vertex of $\Gamma(Z_{2p})$ to the vertices of P_n whose degree is 2, where $p\ge 5$ is any prime number. Then G does not admits a geometric mean labeling.

Proof: Using above theorems (2.5) and (2.6), the graph G contains a induced subgraph, namely $K_{1,5}$. Since, we know that $K_{1,5}$ is not a geometric mean graph. Hence, the theorem is true.

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