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New Structures on Upper Flexible Q-Fuzzy Group

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Abstract

In this paper, the concept of upper-fuzzy flexible set is introduced. We are generalize a new definition of upper flexible Q-fuzzy groups and using this definition some properties were derived.

Keywords: Flexible fuzzy set, upper flexible Q-fuzzy subset, flexible Q-fuzzy group, flexible Q-fuzzy normal subgroups.

INTRODUCTION

The fundamental concept of fuzzy sets was initiated by LoftiZadeh in 1965 and opened a new path of thinking to mathematicians, engineers, physicists, chemists and many other due to its diverse application is various fields. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, coding theory, group theory, real analyses, hectare theory etc. In 1971, Rosenfeld first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic structures. Thereafter the notion of different fuzzy algebraic structures such as fuzzy ideals in rings and semi rings etc, have seriously studied by mathematicians. In 1975, Zadeh introduced the concepts of interval-valued fuzzy sets, where the values of member instead of the real points. His definitions has been generalized by Anthony and Sherwood [1]. They introduced the concept of fuzzy normal subgroups also Mukherjee and Bhattacharya [4] studied the normal fuzzy groups and fuzzy cosets, on the other hand the notion of a fuzzy subgroup abelian group was introduced by Murali and Makamba [6]. A.Solairaju and R.Nagarajan introduced the concept of Q-fuzzy algebraic structures [8][9]. The purpose of this paper is to generalize new definition of upper flexible Q-fuzzy groups and using this definition to study some properties for this subject.

PRELIMINARIES

We present below basic results and definitions of a Q-fuzzy flexible group.

Definition 2.1: Let X be a set. Then a mapping that is $\mu : x \to [0,1]$ is called a fuzzy subset of x.

Definition 2.2: Let Q and G be a set and a group respectively. A mapping A: G X G $\to$ [0,1] is called Q-fuzzy set in G. For any Q-fuzzy set A in G and t€[0,1]. The set U(A:t ={x€G/ A(x,q)≥t, q€Q}) which is called an upper cut of A.

Definition 2.3: Let G be any group. A mapping μ: G$\to$[0,1] is called a upper fuzzy group of G if

i) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$

ii) $\mu(x^{-1}) \leq \mu(x)$ for all x, y €G

Definition 2.4: A Q-fuzzy set A is called Q-fuzzy group of G if

(QFG1) : $A(xy,q) \geq \min\{A(x,q),A(y,q)\}$

(QFG2): $A(x^{-1},q) = A(x,q)$

(QFG3) : $A(x,q) = 1$ for all x,y€G and q€G.

Proposition 2.1: If μ is a Q-fuzzy group of a group G having identity e, then

i) $\mu(x^{-1},q) = \mu(x,q)$

ii) $\mu(e,q) \leq \mu(x,q)$ $\forall$x€G

Definition 2.5: Let μ be a Q-fuzzy group of G. then μ is called a Q-fuzzy normal group if $\mu(xy , q) = \mu(yx,q)$ $\forall$ x, y €G.

Definition 2.6: Let X be a set then a mapping μ:X x Q$\to$ $M^*([0,1])$ is called flexible subset of X, where $M^*([0,1])$ denotes the set of all non empty subset of [0,1].

Definition 2.7: Let X be a non empty set and μ and λ be two flexible Q-fuzzy subset of X. Then the intersection of μ and λ denoted by $\mu\cap\lambda$ and defined by

$\mu\cap\lambda = \{\min\{a,b\}/ a\in\mu(x),b\in\lambda(x)\}$ for all $x\in X$.

The union of μ and λ and denoted by $\mu\cup\lambda$ and defined by

$\mu\cup\lambda = \{\max\{a,b\}/ a\in\mu(x),b\in\lambda(x)\}$ for all $x\in X$.

Definition 2.8: Let X be a groupoid that is a set which is closed under a binary relation denoted multiplicatively. A mapping is called upper flexible Q-fuzzy groupoid if for all $x,y\in$ x, following conditions hold:

(i) $\inf \mu(xy,q) \leq S\{\inf \mu(x,q), \inf \mu(y,q)\}$

(ii) $\sup \mu(xy,q) \leq S\{\sup \mu(x,q), \sup \mu(y,q)\}$

Definition2.9: Let G be a group. A mapping $\mu : G\to M^*([0,1]$ is called a upper flexible Q-fuzzy group of G if for all $x,y\in G$, following conditions hold:

(i) $\inf \mu(xy,q) \leq S\{ \inf \mu(x,q), \inf \mu(y,q)\}$
(ii) $\sup \mu(xy,q) \leq S\{ \sup \mu(x,q), \sup \mu(y,q)\}$
(iii) $\inf \mu(x^{-1},q) \leq \inf \mu(x,q)$
(iv) $\sup \mu(x^{-1},q) \leq \sup \mu(x,q)$

Note: In definition * if $\mu: G\rightarrow[0,1]$, then $\mu(x)\ \forall\ x\in G$ are real points in [0,1] and also $\inf(\mu(x)) = \sup \mu(x) = \mu(x),\ x\in G$. Thus definition * reduces to definition of Rosenfeld's upper fuzzy groups. So Upper flexible group is a generalization of Rosenfeld's fuzzy group.

PROPERTIES OF UPPER Q-FUZZY FLEXIBLE GROUP

Proposition 3.1: An upper flexible Q-fuzzy subset μ of a group G is a upper flexible Q-fuzzy group iff for all $x,y\in G$ followings are hold.

(i) $\inf \mu(xy^{-1},q) < S\ \{\inf \mu(x,q), \inf \mu(y,q)\}$
(ii) $\sup\mu(xy^{-1},q) < S\ \{\sup \mu(x,q), \sup \mu(y,q)\}$

Proof : At first let μ be upper flexible Q-fuzzy group of G and $x,y\in G$. Then

$\inf\mu(xy^{-1},q) < S\ \{\inf \mu(x,q), \inf \mu(y^{-1},q)\}$
$= S\ \{\inf \mu(x,q), \inf \mu(y,q)\}$ and
$\sup \mu(xy^{-1},q) < S\ \{\sup \mu(x,q), \sup \mu(y,q)\}$
$= S\ \{\sup \mu(x,q), \sup \mu(y,q)\}$

Conversely, let μ be upper flexible Q-fuzzy subset of G and given conditions hold. Then for all $x\in G$, we have

$\inf \mu(e,q) = \inf \mu(xx^{-1},q)\leq S\{ \inf \mu(x,q), \inf \mu(x,q)\}= \inf \mu(x,q)$ ______(1)

$\sup \mu(e,q) = \sup \mu(xx^{-1},q)\leq S\{ \sup \mu(x,q), \sup \mu(x,q)\}=\sup \mu(x,q)$ ______(2)

So,$\inf \mu(x^{-1},q) = \inf \mu(ex^{-1},q)\leq S\{ \inf \mu(e,q), \inf \mu(x,q)\}= \inf \mu(x,q)$ by (1)
andsup $\mu(x^{-1},q) = \sup \mu(ex^{-1},q)\leq S\{ \sup \mu(e,q), \sup \mu(x,q)\}= \sup \mu(x,q)$ by (2)
Again $\inf\mu(xy,q) \leq S\ \{\inf \mu(x,q), \inf \mu(y^{-1},q)\}$
$\leq S\ \{\inf \mu(x,q), \inf \mu(y,q)\}$
and $\sup \mu(xy,q) \leq S\ \{\sup \mu(x,q), \sup \mu(y,q)\}$
$\leq S\ \{\sup \mu(x,q), \sup \mu(y,q)\}$

Hence μ is a upper flexible Q-fuzzy group of G.

Proposition3.2: If μ is an upper flexible Q-fuzzy groupoid of a infinite group G, then μ is an upper flexible Q-fuzzy group of G.

Proof: Let $x\in G$. Since G is finite, x has finite order, say p. then $x^p=e$, where e is the identity of G. Thus $x^{-1} = \mu^{\ p-1}$ using the definition of Upper Flexible Q-fuzzy groupoid, we have

$\inf \mu(x^{-1},q) = \inf \mu(x^{p-1},q) = \inf \mu(x^{p-2},q)\leq S\{ \inf \mu(x^{p-2},q), \mu(x,q)\}$
Again $\inf \mu(x^{p-2},q) = \inf \mu(x^{p-3},x,q) \leq S\{ \inf \mu(x^{p-3},q), \mu(x,q)\}$
Then we have $\inf \mu(x^{-1},q) \leq S\{ \inf \mu(x^{p-3},q), \inf \mu(x,q)\}$

So applying the definition of Upper flexible Q-fuzzy groupoid repeatedly, we have that $\inf \mu(x^{-1},q) \leq \inf \mu(x,q)$
Similarly we have $\sup \mu(x^{-1},q) \leq \sup \mu(x,q)$
Therefore μ is a Upper flexible Q-fuzzy group.

Proposition 3.3: The Intersection of any two Upper flexible Q-fuzzy groups is also a Upper flexible Q-fuzzy group of G.

Proof : Let A and B be any two Upper flexible Q-fuzzy groups of G and $x,y \in G$ then $\inf (A \cap B)(xy^{-1},q) = S\{\inf A(xy^{-1},q), \inf B(xy^{-1},q)\}$
$\leq S\{S\{\{\inf A(x,q), \inf A(x,q)\}, \{\inf B(x,q), \inf B(y,q)\}\}\}$
$=S\{S\{\inf A(x,q), \inf B(x,q)\}, S\{\inf A(x,q), \inf B(y,q)\}\}\}$

$= S\{\inf A\cap B(x,q), \inf A\cap B(y,q)\}$ ____________(1)

Again $\sup (A\cap B)(xy^{-1},q) = S\{\sup A(xy^{-1},q), \sup B(xy^{-1},q)\}$ by definition
$\leq S\{S\{\{\sup A(x,q),\sup A(x,q)\}, \{\sup B(x,q), \sup B(y,q)\}\}\}$
$=S\{S\{\sup A(x,q),\sup B(x,q)\}, S\{\sup A(x,q), \sup B(y,q)\}\}\}$

$= S\{\sup A\cap B(x,q), \sup A\cap B(y,q)\}$ ____________(2)

Hence by (1) and (2) and using proposition we say
$A\cap B$ is Upper flexible Q-fuzzy group of G.

Proposition 3.4: If A is a upper Q-fuzzy group of a group G having identity e, then $\forall x \in X$ i) $\inf A(x^{-1},q)=\inf A(x,q)$ and $\sup A(x^{-1},q) = \sup A(x,q)$
ii) $\inf A(e,q) \leq \inf A(x,q)$ and $\sup A(e,q) = \sup A(x,q)$

Proof : (i) As A is a upper Q-fuzzy group of a group G,
Then $\inf A(x^{-1},q) \leq \inf A(x,q)$
Again $\inf A(x,q)=\inf A((x^{-1})^{-1},q) \leq \inf A(x^{-1},q)$
So $\inf A(x^{-1},q)=\inf A(x,q)$
Similarly we can prove that $\sup A(x^{-1},q)=\sup A(x,q)$
$\inf A(e,q) = \inf A(xx^{-1},q) \leq S\{ \inf A(x,q), \inf A(x^{-1},q)\}$
and $\sup A(e,q) = \sup A(xx^{-1},q) \leq S\{\sup A(x,q), \sup A(x^{-1},q)\}$

Proposition 3.5: Let μ and λ be two upper flexible Q-fuzzy group of G_1 , G_2 respectively and let Q be a homomorphism from G_1 to G_2. Then
(i) $Q(\mu,q)$ is a upper flexible Q-fuzzy group of G_2
(ii) $Q(\lambda,q)$ is a upper flexible Q-fuzzy group of G_1

Proof : It is trivial
Remark: If μ is upper flexible Q-fuzzy group of G and K is subgroup of G then the restriction of μ to $K(\mu/K)$ is upper flexible Q-fuzzy group of K.

UPPER NORMAL FLEXIBLE Q-FUZZY GROUP

Definition 4.1 : If μ is an upper flexible Q-fuzzy group of a group G then μ is called a upper normal flexible Q-fuzzy group of G if for all $x,y\in G$

$$\inf \mu(xy) = \inf \mu(yx) \text{ and}$$

$\sup\mu(xy) = \sup \mu(yx)\}$

Proposition 4.1: The Intersection of any two Upper normal flexible Q-fuzzy groups of G is also a Upper normal flexible Q-fuzzy group of G.

Proof : Let A and B be any two Upper normal flexible Q-fuzzy groups of G. By proposition 3.3 $A\cap B$ is an Upper flexible Q-fuzzy group of G.
Let $x,y\in G$ then by definition
$\inf (A\cap B)(xy,q) = S\{\inf A(xy,q), \inf B(xy,q)\}$ by definition
$= S\{\inf A(yx,q), \inf B(yx,q)\}$
$= \inf A\cap B(yx,q)$
Similarly $\sup (A\cap B)(xy,q) = \sup (A\cap B)(yx,q)$
This shows that $A\cap B$ is Upper normal flexible Q-fuzzy group of G.

Proposition 4.2: The Intersection of any arbitrary collection of Upper normal flexible Q-fuzzy groups of a group G is also a Upper normal flexible Q-fuzzy group of G.

Proof: Let $x,y\in G$ and $\alpha \in G$
$\inf A(xy^{-1},q) = \inf A(\alpha^{-1}xy^{-1}\alpha,q)$ by definition
$= \inf A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha,q)$
$= \inf (A(\alpha^{-1}x\alpha,q),A((\alpha^{-1}y\alpha)^{-1},q))$
$\leq S\{\inf (A(\alpha^{-1}x\alpha,q), \inf A((\alpha^{-1}y\alpha),q))\}$
$= S\{\inf (A(x,q),A((y,q))\}$
Again $\sup A(xy^{-1},q) = \sup A(\alpha^{-1}xy^{-1}\alpha,q)$ by definition
$= \sup A(\alpha^{-1}x\alpha\alpha^{-1}y^{-1}\alpha,q)$
$= \sup (A(\alpha^{-1}x\alpha,q),A((\alpha^{-1}y\alpha)^{-1},q))$
$\leq S\{ \sup (A(\alpha^{-1}x\alpha,q), \sup A((\alpha^{-1}y\alpha),q))\}$
$= S\{ \sup (A(x,q),A((y,q))\}$

Hence by proposition 3.2, A is Upper normal flexible Q-fuzzy group of G.

CONCLUSION

In this paper, the concept of upper fuzzy flexible set is introduced and thereafter we defined upper Q-fuzzy flexible group and a few if its properties are discussed.

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