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Heronian Mean Labeling of Some More Disconnected Graphs

S.S. Sandhya
Department of Mathematics, Sree Ayyappa College for Women, Chunkankadai– 629003,Tamilnadu, India.

E. Ebin Raja Merly
Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165,Tamilnadu, India.

S.D. Deepa
Research Scholar, Nesamony Memorial Christian College, Marthandam – 629165, Tamilnadu, India.

Abstract

In this paper, we contribute some more results for Heronian Mean labeling of disconnected graphs. We have already proved that disconnected Heronian Mean Graphs are again Heronian Mean Graphs. We use some more standard graphs to derive the results for Heronian Mean labeling for disconnected graphs.

Keywords: Graph, Heronian Mean Graph, Path, Cycle, Comb, Crown.

AMS Subject Classification: 05C78

1. INTRODUCTION

By a graph we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A. Gallian [1].For all other standard terminology and notations we follow Harary [2]. The concept of Mean labeling has been introduced by S. Somasundaram and R. Ponraj [3] in 2004. S.Somasundaram and S.S.Sandhya introduced Harmonic mean labeling [4] in 2012. Motivated by the above works we introduced a new type of labeling called **Heronian Mean Labeling in [5].** In this paper we investigate the Heronian Mean Labeling of some more disconnected graphs.We will provide brief summary of definitions and other information which are necessary for our present investigation.

A Path $\mathbf{P_n}$is a walk in which all the vertices are distinct. A **Cycle**$\mathbf{C_n}$is a Closed Path. The graph obtained by joining a single pendant edge to each vertex of a Path is called a **Comb**. The corona $G_1 \odot G_2$ is defined as the graph G obtained by taking one copy of G_1 (which has P_1vertices) and P_1 copies of $G_2$and then joining the i^th^ vertex of $G_1$to every vertices in the i^th^ copy of G_2 . The graph $C_n \odot K_1$ is called crown.

Definition 1.1:

A graph **G=(V,E)** with p vertices and q edges is said to be a **Heronian Mean graph** if it is possible to label the vertices $\mathbf{x \in V}$ with distinct labels $\mathbf{f(x)}$ from **1,2,…,q+1** in such a way that when each edge $\mathbf{e = uv}$ is labeled with,

$$\mathbf{f(e = uv)} = \left\lceil \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rceil (\mathbf{OR}) \left\lfloor \frac{\mathbf{f(u)} + \sqrt{\mathbf{f(u)f(v)}} + \mathbf{f(v)}}{\mathbf{3}} \right\rfloor$$

then the edge labels are distinct. In this case **f** is called a **HeronianMean labeling** of G.

Theorem 1.2: Any Path $P_n$is a Heronian mean graph.

Theorem 1.3: Any Comb$P_n \odot K_1$is a Heronian mean graph.

Theorem 1.4: Any Cycle $C_n$is a Heronian mean graph.

Theorem 1.5: Crown, $\boldsymbol{C_n \odot K_1}$ is a Heronian mean graph for all $\boldsymbol{n \geq 3}$.

2. MAIN RESULTS

Theorem: 2.1

$\mathbf{m}\boldsymbol{K_3}$ is a Heronian mean graph.

Proof:

Let the vertex set of $\mathbf{m}\boldsymbol{K_3}$be $V = V_1 \cup V_2 \cup \ldots \cup V_m$, Where $V_i = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}\}$.

Define a function $f: V(\boldsymbol{mK_3}) \to \{1,2,3, \ldots\ldots, q+1\}$ by

$$f\left(V_i^{\,j}\right) = 3(i-1) + j, \ \forall\, 1 \le i \le m, 1 \le j \le 3.$$

Edge labels of $\boldsymbol{mK_3}$ **are** $\{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \ldots, \boldsymbol{3m}\}$.
Hence $\boldsymbol{mK_3}$ is a Heronian mean graph.

Example 2.2: A Heronian mean labeling of $\boldsymbol{6K_3}$ is given below.

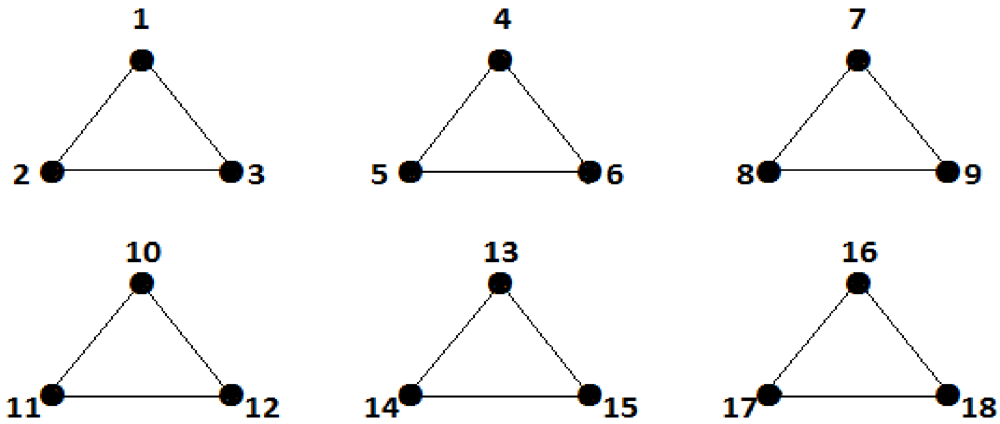

Figure:1

Theorem: 2.3
$\boldsymbol{mK_3} \cup \mathbf{P_n}$ is a Heronian mean graph for $n > 1$.

Proof:
Let the vertex set of $\mathbf{m}\boldsymbol{K_3}$ be $V = V_1 \cup V_2 \cup \ldots \cup V_m$, Where $V_i = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}\}$.
Let P_n be a path $u_1u_2u_3\ldots\ldots u_n$
Define a function $f: V(\boldsymbol{mK_3} \cup \boldsymbol{P_n}) \to \{1,2,3, \ldots, q+1\}$ by

$$f\left(V_i^{\,j}\right) = 3(i-1) + j, \qquad \forall\, 1 \le i \le m, 1 \le j \le 3.$$

$$\text{And } f(u_i) = 3m + i, \ 1 \le i \le n.$$

Edge labels of $\boldsymbol{mK_3}$ **are** $\{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \ldots, \boldsymbol{3m}\}$.
Edges labels of $\mathbf{P_n}\{3m+1, 3m+2, \ldots\ldots 3m+n-1\}$.
Hence $\boldsymbol{mK_3} \cup \boldsymbol{P_n}$ is a Heronian mean graph for $n > 1$.

Example 2.4: A Heronian mean labeling of $\boldsymbol{4K_3} \cup \boldsymbol{P_6}$ is given below.

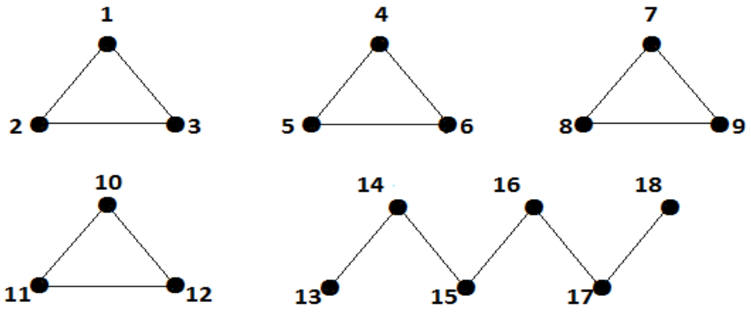

Figure: 2

Theorem: 2.5

$\boldsymbol{mK_3} \cup \mathbf{C_n}$ is a Heronian mean graph for $\boldsymbol{n} \geq \mathbf{3}$.

Proof:

Let the vertex set of mK_3 be $V = V_1 \cup V_2 \cup \ldots \cup V_m$, Where $V_i = \{V_i^{\,1}, V_i^{\,2}, V_i^{\,3}\}$.

Let C_n be a cycle $u_1u_2u_3\ldots\ldots u_n\, u_1$.

Define a function f: $V(\boldsymbol{mK_3} \cup \boldsymbol{C_n}) \rightarrow \{1,2,3,\ldots,q+1\}$ by

$$f(V_i^{\,j}) = 3(i-1) + j, \qquad \forall\, 1 \leq i \leq m, 1 \leq j \leq 3.$$

$$\text{And } f(u_i) = 3m + i,\ \ 1 \leq i \leq n.$$

Edge labels of $\boldsymbol{mK_3}$ are $\{1,2,3,\ldots,3m\}$.

Edges labels of $\mathbf{C_n}\{3m+1, 3m+2, \ldots\ldots 3m+n\}$.

Hence $\boldsymbol{mK_3} \cup \boldsymbol{C_n}$ is a Heronian mean graph for $\boldsymbol{n} \geq \mathbf{3}$.

Example 2.6: A Heronian mean labeling of $\boldsymbol{4K_3} \cup \boldsymbol{C_5}$ is given below.

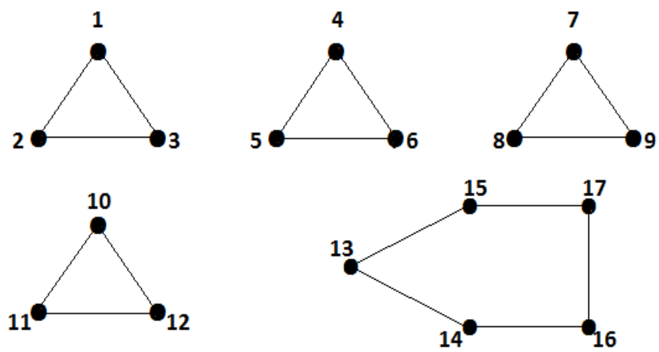

Figure:3

Theorem: 2.7

$\boldsymbol{mK_3} \cup (\mathbf{P_n} \odot \mathbf{K_1})$is a Heronian mean graph for $n > 1$.

Proof:

Let the vertex set of mK_3 be $V = V_1 \cup V_2 \cup \ldots \cup V_m$, Where $V_i = \{V_i{}^1, V_i{}^2, V_i{}^3\}$.
Let $P_n \odot K_1$ be a graph obtained from a path$x_1x_2 \ldots . x_n$by joining the vertexx_i to pendant vertexy_i.
Define a function,$f: V(\boldsymbol{mK_3} \cup (\mathbf{P_n} \odot \mathbf{K_1})) \to \{1,2,3, \ldots \ldots . q+1\}$ by

$$f(V_i{}^j) = 3(i-1) + j, \qquad \forall\, 1 \le i \le m, 1 \le j \le 3.$$

And $f(x_i) = 3\mathrm{m} + (2\mathrm{i} - 1)$, $1 \le i \le n$
$f(y_i) = 3\mathrm{m} + 2\mathrm{i}$, $1 \le i \le n$.
Edge labels of $\boldsymbol{mK_3}$ are $\{1,2,3, \ldots ,3m\}$.
Edges labels of $\mathbf{P_n} \odot \mathbf{K_1}\{3m+1, 3m+2, \ldots \ldots 3m+2n-1\}$.
Hence $\boldsymbol{mK_3} \cup (\mathbf{P_n} \odot \mathbf{K_1})$ is a Heronian mean graph for $n > 1$.

Example2.8:A Heronian mean labeling of $\boldsymbol{4K_3} \cup (\mathbf{P_5} \odot \mathbf{K_1})$is givenbelow.

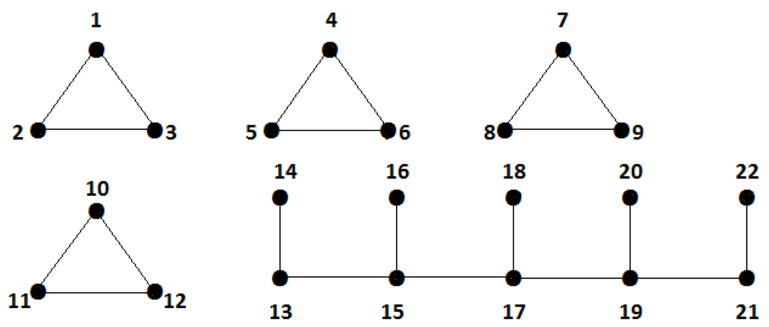

Figure:4

Theorem: 2.9

$\boldsymbol{mK_3} \cup (\mathbf{C_n} \odot \mathbf{K_1})$is a Heronian mean graph for $\boldsymbol{n} \ge \mathbf{3}$.

Proof:

Let the vertex set of $\mathbf{m}\boldsymbol{K_3}$ be $V = V_1 \cup V_2 \cup \ldots \cup V_m$, Where $V_i = \{V_i{}^1, V_i{}^2, V_i{}^3\}$.
Let $C_n \odot K_1$ be a graph obtained from a cycle $x_1x_2 \ldots . x_n x_1$ by joining the vertex x_i to pendant vertexy_i.
Define a function, $f: V(\boldsymbol{mK_3} \cup (\mathbf{C_n} \odot \mathbf{K_1})) \to \{1,2,3, \ldots \ldots . q+1\}$ by

$$f(V_i{}^j) = 3(i-1) + j, \qquad \forall\, 1 \le i \le m, 1 \le j \le 3.$$

And $f(x_i) = 3\mathrm{m} + 2\mathrm{i}$, $1 \le i \le n$
$f(y_i) = 3\mathrm{m} + (2\mathrm{i} - 1)$, $1 \le i \le n$.
Edge labels of $\boldsymbol{mK_3}$ are $\{1,2,3, \ldots ,3m\}$.
Edges labels of $\mathbf{C_n} \odot \mathbf{K_1}\{3m + 1,3m + 2, \ldots \ldots 3m + 2n - 1\}$.
Hence $\boldsymbol{mK_3} \cup (\mathbf{C_n} \odot \mathbf{K_1})$ is a Heronian mean graph for $\boldsymbol{n} \ge \mathbf{3}$.

Example 2.10: A Heronian mean labeling of $\mathbf{4}\boldsymbol{K_3} \cup (\mathbf{C_3} \odot \mathbf{K_1})$is given below.

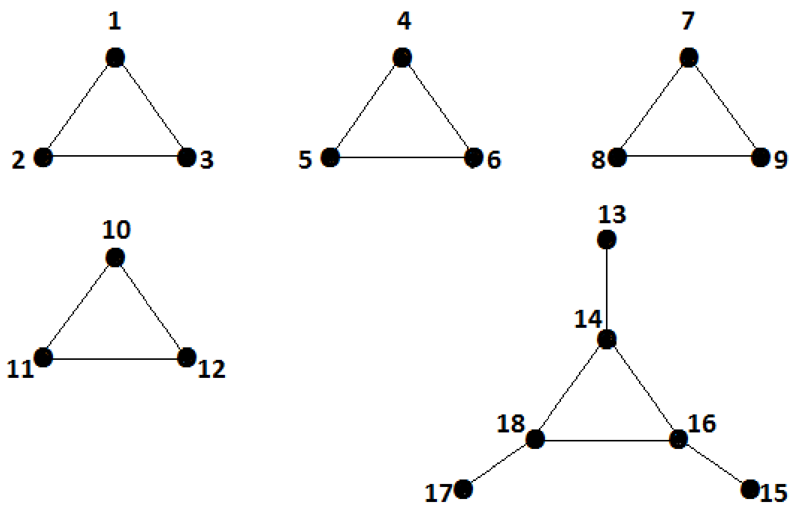

Figure:5

Theorem: 2.11
$\mathbf{m}\boldsymbol{C_n}$is a Heronian mean graph for $\boldsymbol{n} \ge \mathbf{3}$.

Proof:
Let the vertex set of $\mathbf{m}\boldsymbol{C_n}$be $V = V_1 \cup V_2 \cup \ldots \cup V_m$, Where $V_i = \{{V_i}^1, {V_i}^2, \ldots., {V_i}^m\}$ and the edge set of $\mathbf{m}\boldsymbol{C_n}$be $E = E_1 \cup E_2 \cup \ldots \cup E_m$, Where $E_i = \{{e_i}^1, {e_i}^2, \ldots., {e_i}^n\}$
Define a function f: $\mathrm{V}(\boldsymbol{mC_n}) \to \{1,2,3, \ldots\ldots, \mathrm{q} + 1\}$ by

$$f({V_i}^j) = n(i - 1) + j,\ \forall\, 1 \le i \le m,\ 1 \le j \le n.$$

Edge labels of $\boldsymbol{mC_n}$are $\{1,2,3, \ldots, mn\}$.
Hence $\boldsymbol{mC_n}$ is a Heronian mean graphfor $\boldsymbol{n} \ge \mathbf{3}$.

Example 2.12: A Heronian mean labeling of $\boldsymbol{6C_5}$is given below.

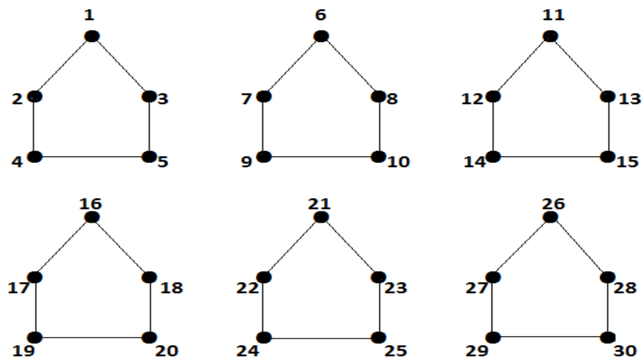

Figure:6

Theorem: 2.13
$\mathbf{m}\boldsymbol{C_n} \cup \boldsymbol{P_k}$is a Heronian mean graph for $\boldsymbol{m, k \geq 1}$ ***and*** $\boldsymbol{n \geq 3}$.

Proof:
Let $\mathbf{m}\boldsymbol{C_n}$be the m copies of $\boldsymbol{C_n}$and $\boldsymbol{P_k}$be the path of length k. The vertex set of m$C_n$be $V = V_1 \cup V_2 \cup \ldots \cup V_m$, Where $V_i = \{V_i{}^1, V_i{}^2, \ldots, V_i{}^m\}$ and the edge set of m$C_n$be $E = E_1 \cup E_2 \cup \ldots \cup E_m$, Where $E_i = \{e_i{}^1, e_i{}^2, \ldots, e_i{}^n\}$. Let $w_1, w_2, \ldots, w_k$ be the vertices of $\boldsymbol{P_k}$.
Define a function f: V$(\boldsymbol{mC_n} \cup \boldsymbol{P_k}) \rightarrow \{1,2,3, \ldots, q+1\}$ by

$$f(V_i{}^j) = n(i-1) + j, \ \forall\, 1 \leq i \leq m, \ 1 \leq j \leq n$$

$$\text{And } f(w_i) = \mathrm{mn} + \mathrm{i}, \quad 1 \leq i \leq k$$

Edge labels of $\boldsymbol{mC_n}$are $\{1,2,3, \ldots, mn\}$.
Edges labels of $\mathbf{P_k}$are $\{mn+1, mn+2, \ldots \ldots mn+k-1\}$.
Hence $\boldsymbol{mC_n} \cup \boldsymbol{P_k}$ is a Heronian mean graph for $\boldsymbol{m, k \geq 1}$ ***and*** $\boldsymbol{n \geq 3}$.

Example 2.14: A Heronian mean labeling of $\boldsymbol{3C_4} \cup \boldsymbol{P_7}$is given below.

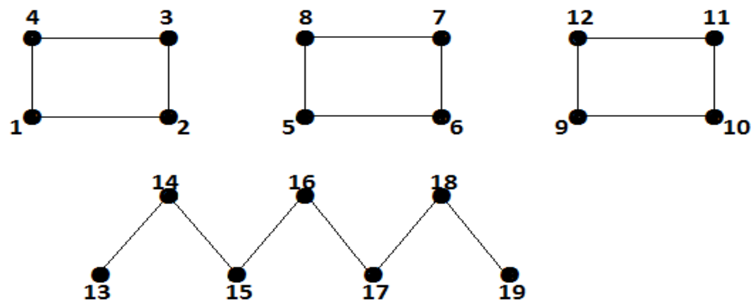

Figure:7

Theorem: 2.15

$\mathbf{m}\boldsymbol{C_n} \cup \boldsymbol{C_k}$ is a Heronian mean graph for $\boldsymbol{m} \geq \mathbf{1}$ ***and*** $\boldsymbol{n}, \boldsymbol{k} \geq \mathbf{3}$.

Proof:

Let m$C_n$be the m copies of C_n and $C_k$be the cycle. The vertex set of m$C_n$be $V = V_1 \cup V_2 \cup \dots \cup V_m$, Where $V_i = \{V_i^{\,1}, V_i^{\,2}, \dots, V_i^{\,m}\}$ and the edge set of m$C_n$be $E = E_1 \cup E_2 \cup \dots \cup E_m$, Where $E_i = \{e_i^{\,1}, e_i^{\,2}, \dots, e_i^{\,n}\}$. Let $w_1, w_2, \dots, w_k w_1$ be a cycle.

Define a function f: V$(\boldsymbol{mC_n} \cup \boldsymbol{C_k}) \rightarrow \{1,2,3, \dots, \mathrm{q}+1\}$ by

$$f(V_i^{\,j}) = n(i-1)+j, \ \forall\, 1 \leq i \leq m, \ 1 \leq j \leq n$$

$$\text{And } f(w_i) = \mathrm{mn} + \mathrm{i}, \quad 1 \leq i \leq k$$

Edge labels of $mC_n$are $\{1,2,3, \dots, mn\}$.

Edges labels of P_k are $\{mn+1, mn+2, \dots \dots mn+k\}$.

Hence $\boldsymbol{mC_n} \cup \boldsymbol{C_k}$ is a Heronian mean graph for $\boldsymbol{m} \geq \mathbf{1}$ ***and*** $\boldsymbol{n}, \boldsymbol{k} \geq \mathbf{3}$.

Example 2.16: AHeronian mean labeling of$\mathbf{3}\boldsymbol{C_4} \cup \boldsymbol{C_5}$is given below.

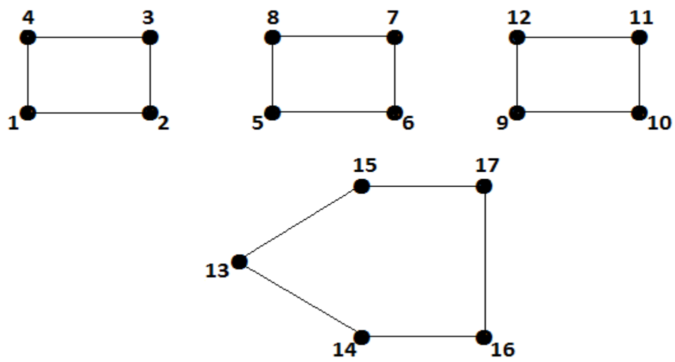

Figure:8

Theorem: 2.17

$\boldsymbol{mC_n} \cup (\boldsymbol{P_k} \odot \boldsymbol{K_1})$ is a Heronian mean graph for $\boldsymbol{m}, \boldsymbol{k} \geq \mathbf{1}$ ***and*** $\boldsymbol{n} \geq \mathbf{3}$.

Proof:

Let m$C_n$be the m copies of C_n and $P_k \odot K_1$ be a graph obtained from a path $x_1 x_2 \dots x_k$ by joining the vertex x_i to pendant vertex y_i.The vertex set of m$C_n$be $V = V_1 \cup V_2 \cup \dots \cup V_m$,Where $V_i = \{V_i^{\,1}, V_i^{\,2}, \dots, V_i^{\,m}\}$ and the edge set of m$C_n$be $E = E_1 \cup E_2 \cup \dots \cup E_m$, Where $E_i = \{e_i^{\,1}, e_i^{\,2}, \dots, e_i^{\,n}\}$.

Define a function f: V$(\boldsymbol{mC_n} \cup (\boldsymbol{P_k} \odot \boldsymbol{K_1})) \rightarrow \{1,2,3, \dots, \mathrm{q}+1\}$ by

$$f(V_i^{\,j}) = n(i-1)+j, \ \forall\, 1 \leq i \leq m, \ 1 \leq j \leq n$$

$$\text{And } f(x_i) = \mathrm{mn} + (2\mathrm{i}-1), \quad 1 \leq i \leq k$$

$$f(y_i) = \mathrm{mn} + 2\mathrm{i}\,, \quad 1 \leq i \leq k.$$

Edge labels of mC_n are $\{1,2,3, \dots, mn\}$.

Edges labels of $\boldsymbol{P_k} \odot \boldsymbol{K_1}$ are $\{mn+1, mn+2, \dots \dots mn+2k-1\}$.

Hence $\boldsymbol{mC_n} \cup (\boldsymbol{P_k} \odot \boldsymbol{K_1})$ is a Heronian mean graph for $m, k \geq 1$ *and* $n \geq 3$.

Example 2.18: A Heronian mean labeling of $\boldsymbol{3C_4} \cup (\boldsymbol{P_5} \odot \boldsymbol{K_1})$ is given below**.**

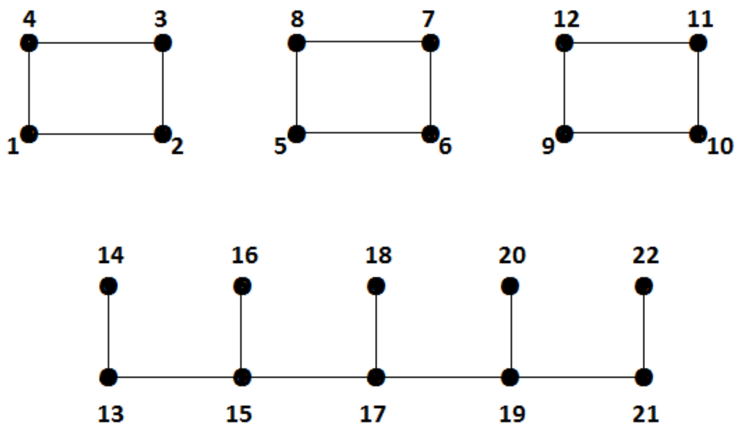

Figure:9

Theorem: 2.19

$\mathbf{m}\boldsymbol{C_n} \cup (\boldsymbol{C_k} \odot \boldsymbol{K_1})$ is a Heronian mean graph for $\boldsymbol{m \geq 1}$ $\boldsymbol{and}$ $\boldsymbol{n, k \geq 3}$.

Proof:

Let $\mathrm{m}C_n$ be the m copies of C_n and $\mathrm{C_k} \odot \mathrm{K_1}$ be a graph obtained from a cycle $\mathrm{x_1 x_2} \dots . \mathrm{x_k} x_1$ by joining the vertex $\mathrm{x_i}$ to pendant vertex y_i.The vertex set of $\mathrm{m}C_n$ be $V = V_1 \cup V_2 \cup \dots \cup V_m$, Where $V_i = \{{V_i}^1, {V_i}^2, \dots., {V_i}^m\}$ and the edge set of $\mathrm{m}C_n$ be $E = E_1 \cup E_2 \cup \dots \cup E_m$, Where $E_i = \{{e_i}^1, {e_i}^2, \dots., {e_i}^n\}$.

Define a function $\mathrm{f}: \mathrm{V}(mC_n \cup (C_k \odot K_1)) \to \{1,2,3, \dots.., \mathrm{q}+1\}$ by

$$f({V_i}^j) = n(i-1) + j, \ \forall\, 1 \leq i \leq m, \ 1 \leq j \leq n$$

$$\text{And} \quad f(x_i) = \mathrm{mn} + 2\mathrm{i}\,, \quad 1 \leq i \leq k$$

$$f(y_i) = \mathrm{mn} + (2\mathrm{i} - 1)\,, \quad 1 \leq i \leq k.$$

Edge labels of mC_n are $\{1,2,3, \dots, mn\}$.

Edges labels of $C_k \odot K_1$ are $\{mn+1, mn+2, \dots \dots mn+2k\}$.

Hence $mC_n \cup (C_k \odot K_1)$ is a Heronian mean graph for $m, k \geq 1$ *and* $n \geq 3$.

Example 2.20: A Heronian mean labeling of $3C_4 \cup (C_3 \odot K_1)$ is given below.

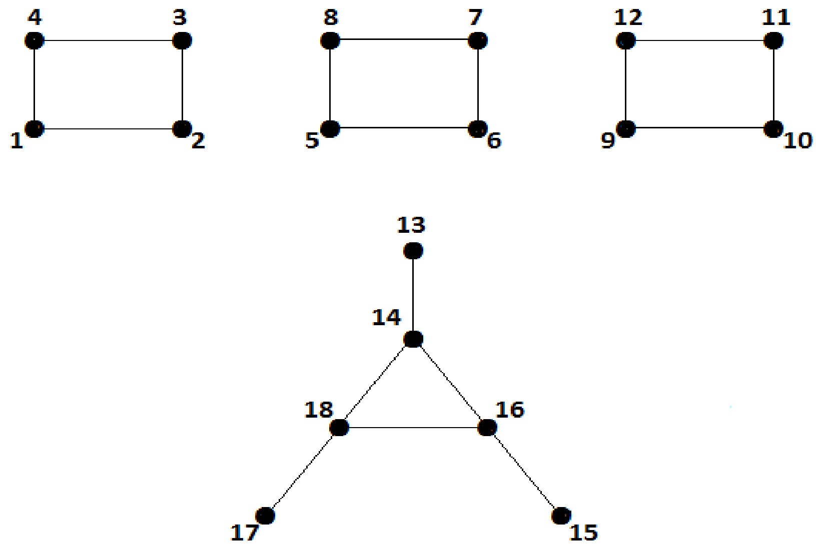

Figure:10

3. CONCLUSION:

The Study of labeled graph is important due to its diversified applications. It is very interesting to investigate disconnected graphs which admit Heronian Mean Labeling. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

4. ACKNOWLEDGEMENT:

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