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Primitive Idempotents of Abelian Codes of Length $4p^nq^m$

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Abstract

Let p, q and l be distinct odd primes (l is of the type 4k+1) and $R_{4p^nq^m} = GF(l)[x]/(x^{4p^nq^m}-1)$.If $o(l)_{2p^n} = \phi(2p^n)$, (n ≥ 1) and $o(l)_{2q^m} = \phi(2q^m), (m \geq 1)$ with gcd ($\phi(2p^n)/2$, $\phi(2q^m)/2$) =1, then the explicit expressions for the complete set of 8mn+4n+4m+4 primitive idempotents in the ring $R_{4p^nq^m} = GF(l)[x]/(x^{4p^nq^m}-1)$ are obtained.

Keywords: primitive idempotents; primitive root; cyclotomic cosets.
MSC: 94B05, 20C05, 16S34, 12E20.

1. INTRODUCTION

Let F = GF(l) be a field of odd prime order l. Let $\eta \geq 1$ be an integer with gcd(l, η)=1.Let $R_\eta = GF(l)[x]/(x^\eta - 1)$ .The minimal cyclic codes of length η over GF(l) are the ideals of the ring R_η generated by the primitive idempotents.For $\eta = 2$, 4, p^n, $2p^n$, p an odd prime and l *is* primitive root mod(η) the primitive idempotent in R_η have been obtained by Arora and Pruthi [1,2] .When $m = p^n q$ where p, q are distinct odd primes and l is a primitive root mod p^n and q both with gcd

$(\phi(p^n)/2,\ \phi(q)/2) = 1$, the primitive idempotent in R_η have been obtained by, G.K.Bakshi and Madhu Raka [4]. When $\eta = 2p^nq^m$, where p,q are distinct odd primes and $o(l)_{2p^n} = \frac{\phi(2p^n)}{2}$, $o(l)_{q^m} = \frac{\phi(q^m)}{2}$, gcd $(\frac{\phi(2p^n)}{2}, \frac{\phi(q^m)}{2})$=1. Then the complete set of 8mn+4n+4m+2 Cyclotomic Cosets modulo $2p^nq^m$ are obtained by, Ranjeet Singh. In this paper, we consider the case when $\eta = 4p^nq^m$ where p, q are distinct odd primes $o(l)_{2p^n} = \phi(2p^n)$, $(n \geq 1)$ and $o(l)_{2q^m} = \phi(2q^m), (m \geq 1)$ with gcd $(\phi(2p^n)/2, \phi(2q^m)/2)$=1.We obtain explicit expressions for all the 8mn+4n+4m+4 primitive idempotents in $R_{4p^nq^m}$ (see theorem 2.3).

2. PRIMITIVE IDEMPOTENTS IN $R_{4p^nq^m} = GF(l)[x]/(x^{4p^nq^m} - 1)$

2.1. For $0 \leq s \leq \eta - 1$, let $C_s = \{s, sl, sl^2, \ldots, sl^{t_s - 1}\}$, where t_s is the least positive integer such that $sl^{t_s} \equiv s \pmod{\eta}$ be the cyclotomic coset containing s, if α denotes a primitive ηth root of unity in some extension field of GF(l) then the polynomial $M^s(x) = \prod_{i \in C_s}(x - \alpha^i)$ is the minimal polynomial of α^s over GF(l).Let M_s be the minimal ideal in R_η generated by $\frac{x^\eta - 1}{M^s(x)}$ and θ_s be the primitive idempotent of M_s then we know by (Theorem1, [4]) the primitive idempotent θ_s corresponding to the cyclotomic coset C_s containing s in $R_{4p^nq^m}$ is given by $\theta_s = \sum_{i=0}^{4p^nq^m - 1} \varepsilon_i x^i$, where $\varepsilon_i = \frac{1}{4p^nq^m} \sum_{j \in C_s} \alpha^{-ij} \quad \forall i \geq 0$.Thus to describe θ_s it becomes necessary to compute ε_i.To compute ε_i numerically, we consider the case when $-C_1 = C_3$ and we get that $\varepsilon_i = \frac{1}{4p^nq^m}$

$$\sum_{j \in C_s} \alpha^{-ij} = \frac{1}{4p^nq^m} \sum_{j \in C_{3s}} \alpha^{ij} \quad \forall i \geq 0 \cdot$$

Lemma 2.2. Let p, q, l be distinct odd primes (l is of the type 4k+1) and n ≥ 1, m ≥ 1 are integers, $o(l)_{2p^n} = \phi(2p^n)$, $o(l)_{2q^m} = \phi(2q^m)$ and gcd$\left(\frac{\phi(2p^n)}{2}, \frac{\phi(2q^m)}{2}\right)$ =1.Then

$$0(l)_{4p^{n-j}q^{m-k}} = \frac{\phi(4p^{n-j}q^{m-k})}{4},$$

for all j, k, $0 \le j \le n-1,\ 0 \le k \le m-1$.

Theorem 2.3. The 8mn+4n+4m+4 primitive idempotents corresponding to cyclotomic cosets $C_0, C_{p^nq^m}, C_{2p^nq^m}, C_{3p^nq^m}\ C_{p^nq^k}, C_{2p^nq^k}, C_{3p^nq^k}, C_{4p^nq^k}\ C_{p^jq^m}, C_{2p^jq^m}, C_{3p^jq^m}$ and $C_{4p^jq^m}\ C_{p^jq^k}, C_{2p^jq^k}, C_{3p^jq^k}, C_{4p^jq^k}, C_{ap^jq^k}, C_{2ap^jq^k}, C_{3ap^jq^k}$, $C_{4ap^jq^k}, o \le j \le n-1, o \le k \le m-1$ in $R_{4p^nq^m}$ are

(i) $\theta_0(x) = \frac{1}{4p^nq^m}(1 + x + x^2 + \ldots + x^{4p^nq^m - 1})$.

(ii) $$\theta_{p^nq^m}(x) = \frac{1}{4p^nq^m}\{\sum_{(i,r)=(0,0)}^{(n,m)} \sigma_{4(i,r)}(x) + \sigma_{4a(i,r)}(x) - \sigma_{2(i,r)}(x) - \sigma_{2a(i,r)}(x) + i(\sigma_{3(i,r)}(x) + \sigma_{3a(i,r)}(x) - \sigma_{(i,r)}(x) - \sigma_{a(i,r)}(x))\}$$

(iii) $$\theta_{2p^nq^m}(x) = \frac{1}{4p^nq^m}\{\sum_{(i,r)=(0,0)}^{(n,m)} \sigma_{4(i,r)}(x) + \sigma_{4a(i,r)}(x) + \sigma_{2(i,r)}(x) + \sigma_{2a(i,r)}(x) - \sigma_{3(i,r)}(x) - \sigma_{3a(i,r)}(x) - \sigma_{(i,r)}(x) - \sigma_{a(i,r)}(x)\}$$

(iv) $$\theta_{3p^nq^m}(x) = \frac{1}{4p^nq^m}\{\sum_{(i,r)=(0,0)}^{(n,m)} \sigma_{4(i,r)}(x) + \sigma_{4a(i,r)}(x) - \sigma_{2(i,r)}(x) - \sigma_{2a(i,r)}(x) + i(\sigma_{a(i,r)}(x) + \sigma_{(i,r)}(x) - \sigma_{3(i,r)}(x) - \sigma_{3a(i,r)}(x))\}$$

(v) For $0 \le k \le m-1,$

$$\theta_{p^n q^k}(x) = \frac{\phi(q^{m-k})}{4p^n q^m}\{\sum_{(i,r)=(0,m-k)}^{(n-1,m-1)} [\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + i(\sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x))]$$

$$+ \sum_{(i,r)=(0,m-k)}^{(n-1,m-1)} [\sigma_{4a(i,r)}(x) - \sigma_{2a(i,r)}(x) + i(\sigma_{3a(i,r)}(x) - \sigma_{a(i,r)}(x))]$$

$$+ \sum_{(i,r)=(n,m-k)}^{(n,m-1)} [\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + i(\sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x))]$$

$$+ \sum_{(i,r)=(0,m)}^{(n-1,m)} [\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + i(\sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x))]$$

$$+ (\sigma_{4(n,m)}(x) - \sigma_{2(n,m)}(x)) + i(\sigma_{3(n,m)}(x) - \sigma_{(n,m)}(x))\}$$

(vi) $$\theta_{2p^n q^k}(x) = \frac{\phi(q^{m-k})}{4p^n q^m}\{\sum_{(i,r)=(0,m-k)}^{(n-1,m-1)} [\sigma_{4(i,r)}(x) + \sigma_{2(i,r)}(x) - \sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x)]$$

$$+ \sum_{(i,r)=(0,m-k)}^{(n-1,m-1)} [\sigma_{4a(i,r)}(x) + \sigma_{2a(i,r)}(x) - \sigma_{3a(i,r)}(x) - \sigma_{a(i,r)}(x)]$$

$$+ \sum_{(i,r)=(n,m-k)}^{(n,m-1)} [\sigma_{4(i,r)}(x) + \sigma_{2(i,r)}(x) - \sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x)]$$

$$+ \sum_{(i,r)=(0,m)}^{(n-1,m)} [\sigma_{4(i,r)}(x) + \sigma_{2(i,r)}(x) - \sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x)]$$

$$+(\sigma_{4(n,m)}(x) + \sigma_{2(n,m)}(x) - \sigma_{3(n,m)}(x) - \sigma_{(n,m)}(x))\}$$

(vii) $$\theta_{3p^n q^k}(x) = \frac{\phi(q^{m-k})}{4p^n q^m}\{\sum_{(i,r)=(0,m-k)}^{(n-1,m-1)} [\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + i(\sigma_{(i,r)}(x) - \sigma_{3(i,r)}(x))]$$

$$+ \sum_{(i,r)=(0,m-k)}^{(n-1,m-1)} [\sigma_{4a(i,r)}(x) - \sigma_{2a(i,r)}(x) + i(\sigma_{a(i,r)}(x) - \sigma_{3a(i,r)}(x))]$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x))]$$

$$+\sum_{(i,r)=(0,m)}^{(n-1,m)}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x))]$$

$$+(\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x))+i(\sigma_{(n,m)}(x)-\sigma_{3(n,m)}(x))\}$$

(viii) $$\theta_{4p^nq^k}(x)=\frac{\phi(q^{m-k})}{4p^nq^m}\{\sum_{(i,r)=(0,m-k)}^{(n-1,m-1)}[\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x)]$$

$$+\sum_{(i,r)=(0,m-k)}^{(n-1,m-1)}[\sigma_{4a(i,r)}(x)+\sigma_{2a(i,r)}(x)+\sigma_{3a(i,r)}(x)+\sigma_{a(i,r)}(x)]$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}[\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x)]$$

$$+\sum_{(i,r)=(0,m)}^{(n-1,m)}[\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x)]$$

$$+(\sigma_{4(n,m)}(x)+\sigma_{2(n,m)}(x)+\sigma_{3(n,m)}(x)+\sigma_{(n,m)}(x))\}$$

(ix) For $0\le j\le n-1$

$$\theta_{p^jq^m}(x)=\frac{\phi(p^{n-j})}{4p^nq^m}\{\sum_{(i,r)=(n-j,0)}^{(n-1,m-1)}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))]$$

$$+\sum_{(i,r)=(n-j,0)}^{(n-1,m-1)}[\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+i(\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x))]$$

$$+\sum_{r=0}^{m-1}[\sigma_{4(n,r)}(x)-\sigma_{2(n,r)}(x)+i(\sigma_{3(n,r)}(x)-\sigma_{(n,r)}(x))]$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}[\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))]$$

$$+(\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x)+i(\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x))\}$$

(x) For $0 \le j \le n-1$

$$\theta_{2p^j q^m}(x) = \frac{\phi(p^{n-j})}{4p^n q^m}\{\sum_{(i,r)=(n-j,0)}^{(n-1,m-1)} [\sigma_{4(i,r)}(x) + \sigma_{2(i,r)}(x) - \sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x)]$$

$$+ \sum_{(i,r)=(n-j,0)}^{(n-1,m-1)} [\sigma_{4a(i,r)}(x) + \sigma_{2a(i,r)}(x) - \sigma_{3a(i,r)}(x) - \sigma_{a(i,r)}(x)]$$

$$+ \sum_{r=0}^{m-1} [\sigma_{4(n,r)}(x) + \sigma_{2(n,r)}(x) - \sigma_{3(n,r)}(x) - \sigma_{(n,r)}(x)]$$

$$+ \sum_{(i,r)=(n-j,m)}^{(n-1,m)} [\sigma_{4(i,r)}(x) + \sigma_{2(i,r)}(x) - \sigma_{3(i,r)}(x) - \sigma_{(i,r)}(x)]$$

$$+ \sigma_{4(n,m)}(x) + \sigma_{2(n,m)}(x) - \sigma_{3(n,m)}(x) - \sigma_{(n,m)}(x)\}$$

(xi) For $0 \le j \le n-1$

$$\theta_{3p^j q^m}(x) = \frac{\phi(p^{n-j})}{4p^n q^m}\{\sum_{(i,r)=(n-j,0)}^{(n-1,m-1)} [\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + i(\sigma_{(i,r)}(x) - \sigma_{3(i,r)}(x))]$$

$$+ \sum_{(i,r)=(n-j,0)}^{(n-1,m-1)} [\sigma_{4a(i,r)}(x) - \sigma_{2a(i,r)}(x) + i(\sigma_{a(i,r)}(x) - \sigma_{3a(i,r)}(x))]$$

$$+ \sum_{r=0}^{m-1} [\sigma_{4(n,r)}(x) - \sigma_{2(n,r)}(x) + i(\sigma_{(n,r)}(x) - \sigma_{3(n,r)}(x))]$$

$$+ \sum_{(i,r)=(n-j,m)}^{(n-1,m)} [\sigma_{4(i,r)}(x) - \sigma_{2(i,r)}(x) + i(\sigma_{(i,r)}(x) - \sigma_{3(i,r)}(x))]$$

$$+(\sigma_{4(n,m)}(x) - \sigma_{2(n,m)}(x) + i(\sigma_{(n,m)}(x) - \sigma_{3(n,m)}(x))\}$$

(xii) For $0 \le j \le n-1$

$$\theta_{4p^j q^m}(x) = \frac{\phi(p^{n-j})}{4p^n q^m}\{\sum_{(i,r)=(n-j,0)}^{(n-1,m-1)} [\sigma_{4(i,r)}(x) + \sigma_{2(i,r)}(x) + \sigma_{3(i,r)}(x) + \sigma_{(i,r)}(x)]$$

$$+\sum_{(i,r)=(n-j,0)}^{(n-1,m-1)}[\sigma_{4a(i,r)}(x)+\sigma_{2a(i,r)}(x)+\sigma_{3a(i,r)}(x)+\sigma_{a(i,r)}(x)]$$

$$+\sum_{r=0}^{m-1}[\sigma_{4(n,r)}(x)+\sigma_{2(n,r)}(x)+\sigma_{3(n,r)}(x)+\sigma_{(n,r)}(x)]$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}[\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x)]$$

$$+\sigma_{4(n,m)}(x)+\sigma_{2(n,m)}(x)+\sigma_{3(n,m)}(x)+\sigma_{(n,m)}(x)\}$$

(xiii) For $0\le j\le n-1,\ 0\le k\le m-1$

$$\theta_{p^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[C_{(i+j,r+k)}\sigma_{(i,r)}(x)+B^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$A_{(i+j,r+k)}\sigma_{3(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[D_{(i+j,r+k)}\sigma_{a(i,r)}(x)+A^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$B_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+i(\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x)))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x)+i(\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x))]\}$$

(xiv) $$\theta_{2p^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[A^*_{(i+j,r+k)}\sigma_{(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$B^*_{(i+j,r+k)}\sigma_{3(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[B^*_{(i+j,r+k)}\sigma_{a(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$A^*_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)-\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)+\sigma_{2a(i,r)}(x)-\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)-\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)-\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))$$

$$+\sigma_{4(n,m)}(x)+\sigma_{2(n,m)}(x)-\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x)]\}$$

(xv) $$\theta_{3p^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[A_{(i+j,r+k)}\sigma_{(i,r)}(x)+A^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$C_{(i+j,r+k)}\sigma_{3(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[B_{(i+j,r+k)}\sigma_{a(i,r)}(x)+B^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$D_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+i(\sigma_{a(i,r)}(x)-\sigma_{3a(i,r)}(x)))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x)))$$

$$+\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x)+i(\sigma_{(n,m)}(x)-\sigma_{3(n,m)}(x))]\}$$

(xvi) $$\theta_{4p^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[C^*_{(i+j,r+k)}\sigma_{(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$D^*_{(i+j,r+k)}\sigma_{3(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[D^*_{(i+j,r+k)}\sigma_{a(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$C^*_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)+\sigma_{2a(i,r)}(x)+\sigma_{3a(i,r)}(x)+\sigma_{a(i,r)}(x))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x))$$

$$+\sigma_{4(n,m)}(x)+\sigma_{2(n,m)}(x)+\sigma_{3(n,m)}(x)+\sigma_{(n,m)}(x)]\}$$

(xvii) $$\theta_{ap^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[D_{(i+j,r+k)}\sigma_{(i,r)}(x)+A^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$B_{(i+j,r+k)}\sigma_{3(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[C_{(i+j,r+k)}\sigma_{a(i,r)}(x)+B^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$A_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+i(\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x)))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x)))$$

$$+\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x)+i(\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x))]\}$$

(xviii)
$$\theta_{2ap^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[B^*_{(i+j,r+k)}\sigma_{(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$A^*_{(i+j,r+k)}\sigma_{3(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[A^*_{(i+j,r+k)}\sigma_{a(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$B^*_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)-\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)+\sigma_{2a(i,r)}(x)-\sigma_{3a(i,r)}(x)-\sigma_{a(i,r)}(x))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)-\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)-\sigma_{3(i,r)}(x)-\sigma_{(i,r)}(x))$$

$$+\sigma_{4(n,m)}(x)+\sigma_{2(n,m)}(x)-\sigma_{3(n,m)}(x)-\sigma_{(n,m)}(x)]\}$$

(xix)

$$\theta_{3ap^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[B_{(i+j,r+k)}\sigma_{(i,r)}(x)+B^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$D_{(i+j,r+k)}\sigma_{3(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[A_{(i+j,r+k)}\sigma_{a(i,r)}(x)+A^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$C_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)-\sigma_{2a(i,r)}(x)+i(\sigma_{a(i,r)}(x)-\sigma_{3a(i,r)}(x)))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x)))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)-\sigma_{2(i,r)}(x)+i(\sigma_{(i,r)}(x)-\sigma_{3(i,r)}(x)))$$

$$+\sigma_{4(n,m)}(x)-\sigma_{2(n,m)}(x)+i(\sigma_{(n,m)}(x)-\sigma_{3(n,m)}(x))]\}$$

(xx)

$$\theta_{4ap^jq^k}(x)=\frac{1}{4p^nq^m}\{\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[D^*_{(i+j,r+k)}\sigma_{(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{2(i,r)}(x)+$$

$$C^*_{(i+j,r+k)}\sigma_{3(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{4(i,r)}(x)]$$

$$+\frac{1}{p^jq^k}\sum_{(i,r)=(0,0)}^{(n-j-1,m-k-1)}[C^*_{(i+j,r+k)}\sigma_{a(i,r)}(x)+D^*_{(i+j,r+k)}\sigma_{2a(i,r)}(x)+$$

$$D^*_{(i+j,r+k)}\sigma_{3a(i,r)}(x)+C^*_{(i+j,r+k)}\sigma_{4a(i,r)}(x)]$$

$$+\frac{\phi(p^{n-j})q^{m-k-1}}{2}\{\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{(i,m-k-1)}(x)+\sigma_{2(i,m-k-1)}(x)+\sigma_{3(i,m-k-1)}(x)+\sigma_{4(i,m-k-1)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k-1)}^{(n-1,m-k-1)}(\sigma_{a(i,m-k-1)}(x)+\sigma_{2a(i,m-k-1)}(x)+\sigma_{3a(i,m-k-1)}(x)+\sigma_{4a(i,m-k-1)}(x))\}$$

$$+p^{n-j-1}\frac{\phi(q^{m-k})}{2}\{\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{(n-j-1,r)}(x)+\sigma_{2(n-j-1,r)}(x)+\sigma_{3(n-j-1,r)}(x)+\sigma_{4(n-j-1,r)}(x)]$$

$$+\sum_{(i,r)=(n-j-1,m-k)}^{(n-j-1,m-1)}[\sigma_{a(n-j-1,r)}(x)+\sigma_{2a(n-j-1,r)}(x)+\sigma_{3a(n-j-1,r)}(x)+\sigma_{4a(n-j-1,r)}(x)]\}$$

$$+\frac{\phi(4p^{n-j}q^{m-k})}{4}[\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m-k)}^{(n-1,m-1)}(\sigma_{4a(i,r)}(x)+\sigma_{2a(i,r)}(x)+\sigma_{3a(i,r)}(x)+\sigma_{a(i,r)}(x))$$

$$+\sum_{(i,r)=(n,m-k)}^{(n,m-1)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x))$$

$$+\sum_{(i,r)=(n-j,m)}^{(n-1,m)}(\sigma_{4(i,r)}(x)+\sigma_{2(i,r)}(x)+\sigma_{3(i,r)}(x)+\sigma_{(i,r)}(x))$$

$$+\sigma_{4(n,m)}(x)+\sigma_{2(n,m)}(x)+\sigma_{3(n,m)}(x)+\sigma_{(n,m)}(x)]\}$$

where $A_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m-1}(\frac{-1+r+\gamma+\delta}{4})$, $B_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m-1}(\frac{-1+r-\delta-\gamma}{4})$

$C_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m-1}(\frac{-1-r-\gamma+\delta}{4})$, $D_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m-1}(\frac{-1-r+\gamma-\delta}{4})$

$A^*_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m-1}(\frac{1+r+\gamma+\delta}{4})$, $B^*_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m-1}(\frac{1+r-\delta-\gamma}{4})$

$C^*_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m-1}(\frac{1-r-\gamma+\delta}{4})$, $D^*_{(n-1,m-1)}=\mathbf{p}^{n-1}q^{m1}(\frac{1-r+\gamma-\delta}{4})$

where $r^2=-q$, $\gamma^2=-p$, $\delta^2=pq$.

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