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Some More Results on Super Heronian Mean Labeling

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Abstract

Here we look into some more results on Super Heronian Mean Labeling for some standard graphs.In this paper we prove that $P_n \odot K_{1,2}$, $P_n \odot K_{1,3}$, $P_n \odot K_3$, $(P_n \odot K_1) \odot K_{1,2}$, $(P_n \odot K_1) \odot K_{1,3}$ are Super Heronian Mean graphs.

Keywords: Graph, Super Heronian mean graph, $P_n \odot K_{1,2}$, $P_n \odot K_{1,3}$. $P_n \odot K_3$,($P_n \odot K_1) \odot K_{1,2}$, $(P_n \odot K_1) \odot K_{1,3}$.

1. INTRODUCTION

We start with simple,finite and undirected graph and have p vertices and q edges.For a detailed survey of graph labeling, we refer to J.A Gallian [1]. For standard terminology and notation we follow Harary [2].The concept of Super Heronian Mean Labeling was introduced by S.S.Sandhya,E.Ebin Raja Merly and G.D.Jemi in [7] . In this paper, we discuss some more results on Super Heronian Mean Labeling for some special graphs.

Definition: 1.1

Let f :V(G)→{1,2,------,p+q} be an injective function. For a vertex labeling "f" the induced edge labeling f*(e=uv) is defined by, f*(e)= $\left\lceil \frac{f(u)+ \sqrt{f(u)f(v)} +f(v)}{3} \right\rceil$ [OR] $\left\lfloor \frac{f(u)+ \sqrt{f(u)f(v)} +f(v)}{3} \right\rfloor$ Then "f" is called a **Super Heronian Mean Labeling** if {f(V(G)} U {f(e): e ϵ E(G)={1,2, ...,p+q}}. A graph which admits Super Heronian Mean Labeling is called **Super Heronian Mean Graph.**

Theorem :1.2 Any Path P_n is a Super Heronian Mean Graph.

Theorem :1.3 Any Cycle C_n is a Super Heronian Mean Graph.

Theorem :1.4 Any Comb ($P_n \odot K_1$) is a Super Heronian Mean Graph.

2. MAIN RESULTS

Theorem: 2.1

Let G be a graph obtained by joining a pendant vertex with a vertex of degree two on both sides of a **Comb graph**. Then G is a Super Heronian mean graph.

Proof:

Comb ($P_n \odot K_1$) is a graph obtained from a path $P_n=v_1v_2 \dots v_n$ by joining a vertex v_i to u_i, $1\leq i\leq n$.Let G be a graph obtained by joining pendant vertices w and z respectively.

Define a function f: $V(G)\rightarrow\{1,2, \dots ,p+q\}$ by,

$f(w)=1$,

$f(v_1)=3$,

$f(v_i)=4i+1$; $2\leq i\leq n$

$f(z)=4n+3$

$f(u_1)=5$,

$f(u_i)=4i-1$; $2\leq i\leq n$

Edges are labeled with,

$f(wv_1)=2$

$f(v_iv_{i+1})=4i+2$; $1\leq i\leq n-1$

$f(v_nz)=4n+2$

$f(v_iu_i)=4i$; $1\leq i\leq n$

:.$f(V(G))\cup\{f(e):e \in E(G)\}=\{1,2, \dots ,p+q\}$

Thus f provides Super Heronian mean labeling of G.

Hence G is a Super Heronian mean Graph.

Example:.2.2 A Super Heronian mean labeling of G when n=5 is given below

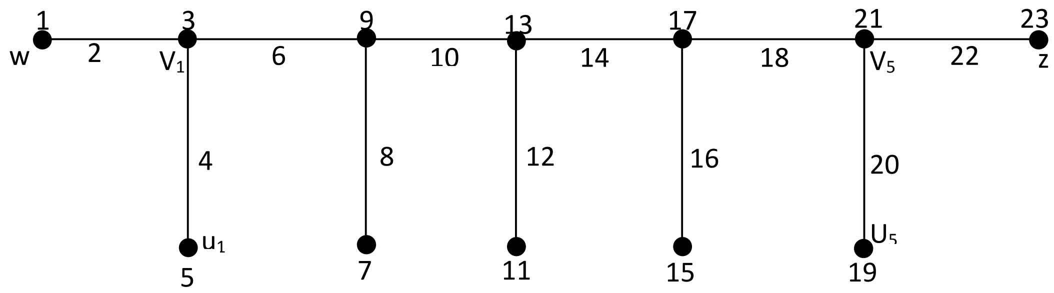

Figure :1

Theorem: 2.3

Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$. Then G is a Super Heronian mean graph.

Proof:

Let P_n be the path $u_1u_2 \ldots u_n$ and v_i,w_i be the vertices of $K_{1,2}$ which are attached to vertex u_i of P_n.The graph contain 3n vertices and 3n-1 edges.

Define a function f: V(G)→{1,2, . . . ,p+q} by,

$f(u_i)=6i-3$; $1\leq i\leq n$

$f(v_i)=6i-5$; $1\leq i\leq n$

$f(w_i)=6i-1$; $1\leq i\leq n$

Edges are labeled with,

$f(u_iu_{i+1})=6i$; $1\leq i\leq n-1$

$f(u_iv_i)=6i-4$; $1\leq i\leq n$

$f(u_iw_i)=6i-2$; $1\leq i\leq n$

This gives a Super Heronian mean labeling of G.

Example:2.4 A Super Heronian mean labeling of $P_4\odot K_{1,2}$ is given below.

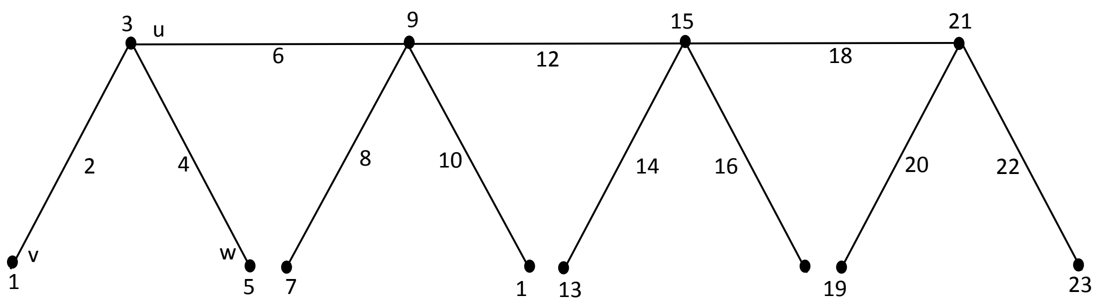

Figure :2

Theorem: 2.5

Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,3}$. Then G is a Super Heronian mean graph.

Proof:

Let P_n be the path $u_1,u_2 \ldots u_n$ and v_i,w_i,z_i be the vertices of $K_{1,2}$ which are attached to the vertex u_i of P_n .

Define a function $f: V(G) \rightarrow \{1,2, \ldots, p+q\}$ by,

$f(u_i)=8i-3$; $1\leq i\leq n$

$f(v_i)=8i-7$; $1\leq i\leq n$

$f(w_i)=8i-5$; $1\leq i\leq n$

$f(z_i)=8i-1$; $1\leq i\leq n$

Edges are labeled with,

$f(u_iu_{i+1})=8\,i$; $1\leq i\leq n-1$

$f(u_iv_i)=8i-6$; $1\leq i\leq n$

$f(u_iw_i)=8i-4$; $1\leq i\leq n$

$f(u_iz_i)=8i-2$; $1\leq i\leq n$

This gives a Super Heronian mean labeling of G.

Example:2.6 A Super Heronian mean labeling of $P_4 \odot K_{1,3}$ is given below.

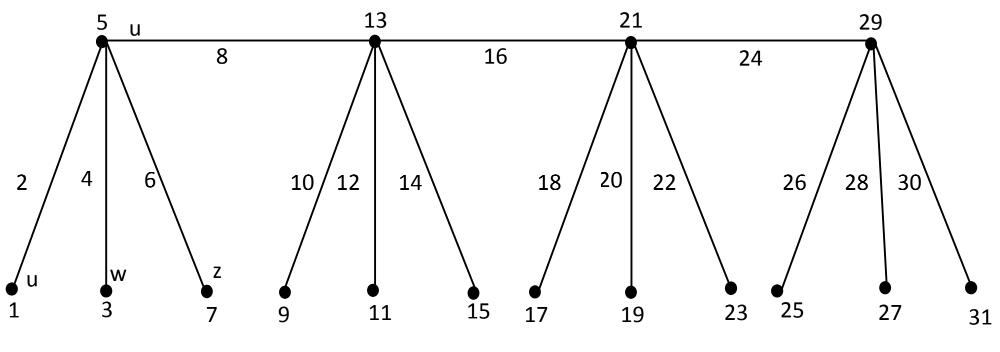

Figure :3

Theorem: 2.7

Let $G=P_n \odot C_3$ be a graph obtained by attaching C_3 to each vertex of a path P_n. Then G is a Super Heronian mean graphs.

Proof:

Consider a graph G is obtained by attaching C_3 to each vertex of a Path P_n.Let P_n be a path $u_1,u_2 \dots u_n$.Let u_i, v_i, w_i, $1\leq i\leq n$ be the vertices of C_3.

Define a function f: V(G)→{1,2, . . . ,p+q} by,

$f(u_i)=7i-3$; $1\leq i\leq n$

$f(v_i)=7i-6$; $1\leq i\leq n$

$f(w_i)=7i-1$; $1\leq i\leq n$

Edges are labeled with,

$f(u_iu_{i+1})=7i$; $1\leq i\leq n-1$

$f(u_iv_i)=7i-5$; $1\leq i\leq n$

$f(v_iw_i)=7i-4$; $1\leq i\leq n$

$f(u_iw_i)=7i-2$; $1\leq i\leq n$

f (V(G))U{f(e)=e ϵ E(G)}={1,2, . . . , p+q}

Hence G is a Super Heronian mean graph.

Example:2.8 A Super Heronian mean labeling of $P_4 \odot C_3$ is displayed below.

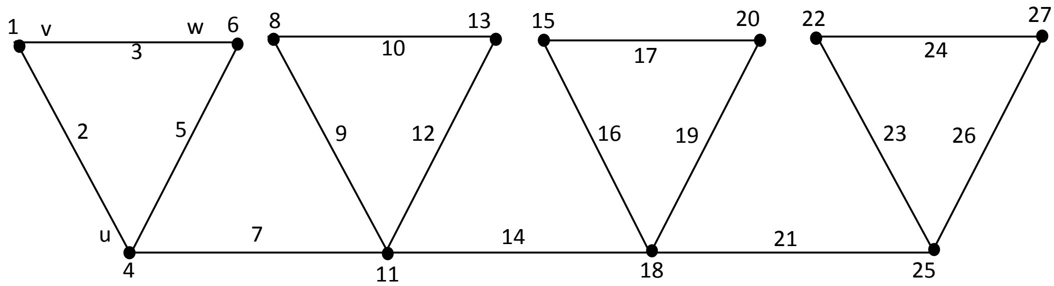

Figure :4

Theorem: 2.9

A graph obtained by attaching $K_{1,2}$ at each pendant vertex of a Comb is a Super Heronian mean graph.

Proof:

Let G_1be a comb and G be the graph obtained by attaching $K_{1,2}$ at each pendant vertex of G_1.Let its vertices be u_i, v_i, w_i, x_i, $1\leq i\leq n$.

Define a function f: V(G)→{1,2, . . . ,p+q} by,

$f(u_i)=8i-3$; $1\leq i\leq n$

$f(v_i)=8i-1;\ 1\leq i\leq n$

$f(w_1)=1$

$f(w_i)=8i-8\ ;\ 2\leq i\leq n$

$f(x_i)=8i-6\ ;\ 1\leq i\leq n$

Edges are labeled with,

$f(u_iu_{i+1})=8i+1\ ;\ 1\leq i\leq n-1$

$f(u_iv_i)=8i-2\ ;\ 1\leq i\leq n$

$f(v_iw_i)=8i-5\ ;\ 1\leq i\leq n$

$f(v_ix_i)=8i-4\ ;\ 1\leq i\leq n$

Thus both vertices and edges together get distinct labels from{1,2, . . . , p+q}.

Hence G is a Super Heronian mean graphs.

Example:2.10 A Super Heronian mean labelling of $G=(P_5 \odot K_{1,2})$ is given below.

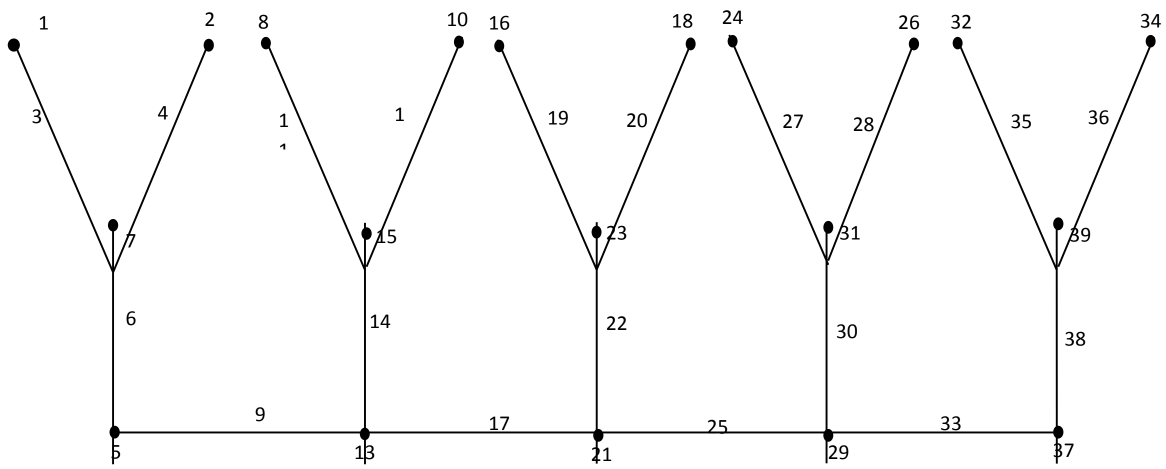

Figure :5

Theorem: 2.11

A graph obtained by attaching a triangle at each pendant vertex of a Comb is a Super Heronian mean graph.

Proof:

Let G_1 be a comb and G be the graph obtained by attaching a triangle at each pendant vertex of G_1.Let its vertices be u_i, v_i, w_i, x_i, $1\leq i\leq n$.

Define a function f: $V(G) \to \{1,2, \ldots, p+q\}$ by,

$f(u_i)=9i-1$; $1 \le i \le n$

$f(v_i)=9i-3$; $1 \le i \le n$

$f(w_1)=1$

$f(w_i)=9i-9$; $2 \le i \le n$

$f(x_i)=9i-5$; $1 \le i \le n$

Edges are labeled with,

$f(u_iu_{i+1})=9i+3$; $1 \le i \le n-1$

$f(u_iv_i)=9i-2$; $1 \le i \le n$

$f(v_1w_1)=3$, $f(v_iw_i)=9i-7$; $2 \le i \le n$

$f(v_ix_i)=9i-4$; $1 \le i \le n$

$f(w_1x_1)=2$, $f(w_ix_i)=9i-8$; $2 \le i \le n$

Thus f provides a Super Heronian mean graph.

Example:2.1 A Super Heronian mean labeling of $G=(P_4 \odot K_1) \odot K_{1,2}$ is given below,

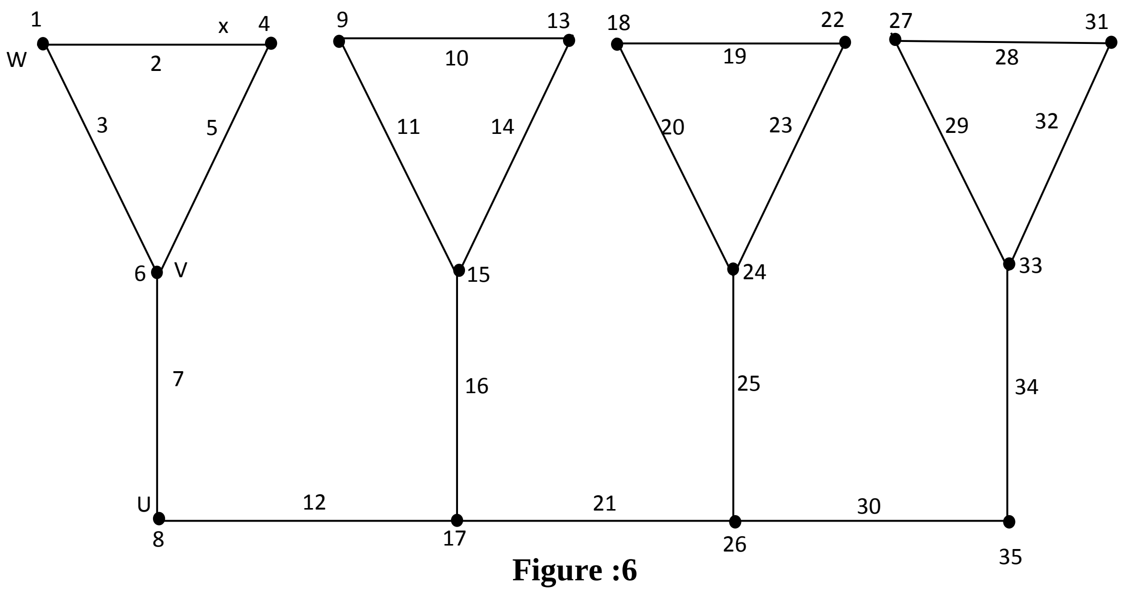

Figure :6

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