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# Strong λ- Bi Near Subtraction Semigroups

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**Abstract**

*In this paper we introduce the notion of* Strong λ- *bi-near subtraction semigroup. Also we give characterizations of* Strong λ- *bi-near subtraction semigroup.*

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**Key words**: Strong λ bi-near subtraction semigroup, $S_1$- bi-near subtraction semigroup, $S_2$- bi-near subtraction semigroup, Nil near subtraction semigroup.

## 1. INTRODUCTION

In 2007, Dheena[1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz[4]. Zekiye Ciloglu, Yilmaz Ceven [5] gave the notation of Fuzzy Near Subtraction semigroups. Seydali Fathima et.al[2,3] introduced the notation of $S_1$-near subtraction semigroup and $S_2$-near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of Strong $\lambda$- Bi near subtraction semigroup .

## 2. PRELIMINARIES

A non-empty subset X together with two binary operations "–" and "." is said to be subtraction semigroup If (i) (X,–) is a subtraction algebra (ii) (X, .) is a semi group (iii) $x(y-z)=xy-xz$ and $(x-y)z= xz-yz$ for every x, y, $z\in X$. A non-empty subset X together with two binary operations "–" and "." is said to be near subtraction semigroup if (i) (X,–) is a subtraction algebra (ii) (X,.) is a semi group and (iii) $(x-y)z= xz-yz$ for every x, y, $z\in X$. A non-empty subset X is said to be **$S_1$-near subtraction semigroup** if for every $a\in X$ there exists $x\in X^*$ such that axa=xa. A non-empty subset X is said to be **$S_2$-near subtraction semigroup** if for every $a\in X$ there exists $x\in X^*$ such that axa=ax. A non-empty subset X is said to be **nil-near subtraction semigroup** if there exists a positive integer k>1 such that $a^k=0$ Which implies that xa=0 where $x=a^{k-1}$.

## 3. STRONG $\lambda$-BI NEAR SUBTRACTION SEMIGROUP

**Definition 3.1**

A non-empty subset X together with two binary operations"-" and "." Is said to be **Strong $\lambda$- bi near subtraction semigroup**. Then X is the both strong $S_1$ and strong $S_2$-near subtraction semigroup

**Example 3.2**

Let X={0,a,b,1} in which "-" and "." be defined by

| - | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | a | 0 | c | b |
| b | b | 0 | 0 | 0 |
| c | c | 0 | c | 0 |

| . | 0 | a | b | c |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | a | a | a |
| b | 0 | 0 | b | b |
| c | 0 | a | b | c |

Then X is a Strong $\lambda$- bi near-subtraction semi group

**Result 3.3**

Every Strong λ-bi near Subtraction Semigroup is a λ- bi near Subtraction Semigroup

Proof:

Let X be a Strong λ -bi near Subtraction Semigroup where X is the both Strong $S_1$ and Strong $S_2$ near subtraction semigroup ⇒ For a,b∈X, aba=ba and aba=ab ⇒ For a∈X, aba=ba and aba=ab for some b∈X. Therefore X is both $S_1$ and $S_2$ near subtractionsemigroup. Thus X is a λ- bi near Subtraction Semigroup.

**Remark 3.4**

The converse of the above result need not be true shown by a following example.

Let X={0,a,b,c} in which "–" and "." be defined by

| - | 0 | a | b | 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | a | 0 | a | a |
| b | b | b | 0 | b |
| 1 | 1 | 1 | 1 | 0 |

| . | 0 | a | b | 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| a | a | a | b | c |
| b | 0 | 0 | 0 | 0 |
| 1 | 0 | a | b | 1 |

Thus X is an λ- bi near-subtraction semi group but not Strong λ - bi-near subtraction semi group

Hence, every λ- bi-near subtraction semi group need not be a Strong λ- bi-near subtraction semi group.

**Theorem 3.5**

Let X be a Boolean near subtraction semigroup. Each of the following statement implies that X is a Strong λ-bi near Subtraction Semigroup

1. X is commutative.
2. X is of Type I and Type II.
3. aXa=Xa and aX=aXa for all a∈X (That is, X is $P_1'$ and $P_1$ near subtraction semigroup).
4. X is sub commutative.

Proof :

Let X be a Boolean near subtraction semigroup

Let X be a commutative near subtraction semigroup.and let a,b ∈ X. Now, aba = a(ba) = a(ab) (Since X be a commutative) =$a^2$b =ab(Since X be a Boolean)=ba

(Since X be a commutative) and let a,b $\in$ X. Now, aba = (ab)a = (ba)a (Since X be a commutative) =ba$^2$ =ba(Since X be a Boolean)=ab (Since X be a commutative). Thus X is a Strong $\lambda$-bi near subtraction semigroup.

Let X be of Type I and Type II near subtraction semigroup and let a,b$\in$X. Then aba=baa=ba$^2$=ba [Since X is Boolean] and aba=aab=a$^2$b=ab [Since X is Boolean]. That is, aba=ba and aba=ab. Thus X is a F$^*$-bi near subtraction semigroup.

Let a$\in$X. Since Xa=aXa and aX=aXa, for every b$\in$X there exists y$\in$X such that ba=aya and ab=aya. Now aba=a(ba)=a(aya)=a$^2$ya=aya [Since X is Boolean]=ba and aba=(ab)a=(aya)a=aya$^2$=aya[Since X is Boolean]=ab. Thus X is a Strong $\lambda$-bi near subtraction semigroup.

Let X be a Sub-commutative Let a $\in$ X, aX=Xa. Therefore for every b$\in$ $X_1$ there exists c$\in$ $X_1$ such that ba=ac and ab=ca. Now,aba a(ba) = a(ac) =ac$^2$=ac(Since X be a Boolean)= ba and aba=(ab)a = (ca)a=ca$^2$=ca(Since X be a Boolean )=ab. Thus X is a Strong $\lambda$-bi near Subtraction

**Theorem 3.6**

Any homomorphic image of a Strong $\lambda$-bi near Subtraction Semigroup is a Strong $\lambda$-bi near Subtraction Semigroup.

**Proof :**

Let f: X$\rightarrow$ Y be a homomorphism. Since X be a Strong $\lambda$--bi near subtraction semigroup $\Rightarrow$ aba =ba and aba = ab for all a,b$\in$ X. Let $y_1, y_2 \in$ Y then there exists $x_1,x_2 \in$ Y such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Clearly then $x_1y_1x_1 = y_1x_1$ and $x_1y_1x_1 = x_1y_1$. And the desired result now follows.

## 4. RESULTS ON STRONG $\lambda$-BI NEAR SUBTRACTION SEMIGROUP.

**Proposition 4.1**

If X is a Strong $\lambda$-bi near subtraction semigroup then E$\subset$ C(X).

Proof

For e$\in$E, eX = Xe [Since X is Sub-commutative] =Xe$^2$[X is left bipotent]. for e$\in$E, eX = Xe [Since X is Sub-commutative]$\Rightarrow$ eeX = eXe $\Rightarrow$ e$^2$X = eXe $\Rightarrow$ eX = eXe [since e$^2$=e].

Again Xe=eX$\Rightarrow$ Xee=eXe $\Rightarrow$ Xe$^2$=eXe $\Rightarrow$ Xe=eXe[since e$^2$=e].Clearly, Then for every x$\in$X, there exists u and v in y such that ex=eue and xe=eveexe=euee=eue$^2$=eue=ex and exe=eeve= e$^2$ve=eve=xe.. Thus ex=exe=xe for all x$\in$X. Consequently E$\subset$ C(X).

**Theorem 4.2**

Let X be a left self distributive S-near subtraction semigroup Then X is a Strong λ-bi near subtraction semigroup if and only if X is a GNF.

**Proof**

Assume that X is a GNF. Now for a$\in$X, Since X is a S-bi near subtraction semigroup.Let a$\in$ax =axa. Thus X is a regular... Let a,b$\in$ X. Then a = aba [Since X is regular] =abaa[Since X is self distributive] = aa [Since X is regular] =a$^2$. Therefore, X is Boolean and regular implies that X is Strong λ-bi near subtraction semigroup

Conversely, assume that X is a Strong λ-bi near subtraction semigroup. Since X is a S-bi near subtraction semigroup.Let a$\in$ax =axa. Thus X is a regular. Again by Proposition 4.1, E$\subseteq$ C(x). Therefore X is GNF.

**Theorem 4.3**

Let X be a S-near subtraction semigroup Then X is a Strong λ-bi near subtraction semigroup if and only if for every x$\in$X there exists a unique central idempotent e such that Xx = Xe and X is regular.

**Proof**

For the only if part let x$\in$X. Since X is a Strong λ-bi near subtraction semigroup and from by previous Theorem, X is a GNF. Therefore X is regular and E$\subset$C(x). Let a,x$\in$X. Since X is regular, x = xax, Now Xx = Xax = Xe, Where e = ax$\in$E. Since E$\in$ C(x), the idempotent e is central and Xx = Xe, for all x$\in$X. Let $e_1\in$E and X$e_1$ = Xx, for some central idempotent $e_1$. Now $e_1$ = $e_1^2\in$X$e_1$ = Xx = Xe and so $e_1$= ne, for some n$\in$X. Consequently, $e_1$ = ne = ne$^2$ = (ne)e = $e_1$e and e = e$^2\in$Xe = X $e_1$ and so e = u$e_1$, for some u$\in$X. This implies that e = u$e_1$ = u$e_1^2$ = (u$e_1$)$e_1$ = e$e_1$. Since e is a central idempotent, e$e_1$ = $e_1$e and so e = $e_1$. Thus e is the unique central idempotent.

Conversely, since X is regular and idempotent central, X is a GNF. Therefore by Previous Theorem, X is a Strong λ-bi near subtraction semigroup.

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