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Prime Labeling for Some Planter Related Graphs

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Abstract

Here we investigate prime labeling for some planter related graphs. We also discuss prime labeling in the context of some graph operations namely duplication, fusion, and vertex switching in planter R_n.

Keywords: Prime Labeling, Prime Graph, Duplication, Fusion, Switching.

1. INTRODUCTION

In this paper, we consider only simple, finite, undirected and non – trivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. For notations and terminology we refer to Bondy and Murthy[1]. The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout. A(1982 P 365 – 368)[8]. Many researchers have studied prime graph for example in Fu. H(1994 P 181 – 186)[4] have proved that the path P_n on *n* vertices is a prime graph. In Deretsky. T(1991 P 359 – 369)[3] have proved that the Cycle C_n on *n* vertices is a prime graph. Lee. S(1998 P 59 – 67)[6] have proved that Wheel W_n is a prime graph iff. *n* is even. In [7] S. Meena and K. Vaithilingam have proved the prime labeling for some Fan related graphs. For latest survey on graph labeling we refer to [5] (Gallian. J. A 2009). Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past four decades.

2. PRELIMINARY DEFINITIONS

Definition [7]

Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f : V(G) \to \{1, 2, \dots, p\}$ is called a *prime labeling* if for each edge $e = uv, \gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a *prime graph*.

Definition [7]

Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k' with $N(v_k') = N(v_k)$. In other words a vertex v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k' also.

Definition [7]

Let *u* and *v* be two distinct vertices of a graph G. A new graph G_1 is constructed by *identifying(fusing)* two vertices *u* and *v* by a single vertex *x* is such that every edge which was incident with either *u* or *v* in G is now incident with *x* in G_1.

Definition [7]

A *vertex switching* G_v of a graph G is obtained by taking a vertex *v* of G, removing all the entire edges incident with *v* and adding edges joining *v* to every vertex which are not adjacent to *v* in G.

Definition [2]

A k – coloring of a graph $G = (V, E)$ is a function $c : V \to C$, where $|c| = k$. (Most often we use $c = [k]$). Vertices of the same color form a color class. A coloring is *proper* if adjacent vertices have different colors. A graph is *k – colorable* if there is a proper *k* – coloring. The chromatic number $\chi(G)$ of a graph G is the minimum *k* such that G is *k* – colorable.

3. PRIME LABELING FOR SOME PLANTER RELATED GRAPHS

3.1. Planter Graph

The *Planter graph* $R_n, (n \geq 3)$ can be constructed by joining a fan graph $F_n, (n \geq 2)$ and cycle graph $C_n, (n \geq 3)$ with sharing a common vertex, where *n* is any positive integer. i.e., $R_n = F_n + C_n$.

Example 3.2.

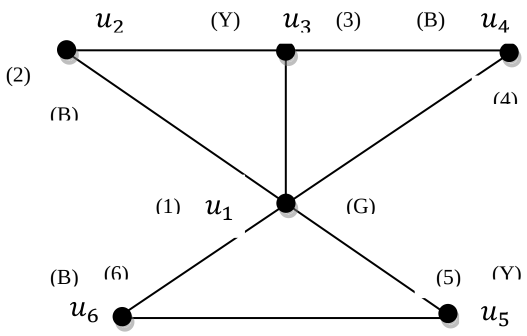

Figure 3.1. The Planter graph R_3.

Theorem 3.3. A planter graph $R_n, (n \geq 3)$ admits prime graph, where *n* is any positive integer.

Proof. Let G be the graph of planter graph R_n. Let $\{u_1, u_2, \ldots, u_{2n}\}$ be the vertices of R_n. Let $E(R_n)$ be the edges of the planter graph where $E(R_n) = \{u_1 u_i / 2 \leq i \leq 2n\} \cup \{u_i u_{i+1} / 2 \leq i \leq n+1\} \cup \{u_i u_{i+1} / n+2 \leq i \leq 2n-1\}$. Here $|V(R_n)| = 2n$.

Define a labeling $f : V(R_n) \to \{1, 2, \ldots, 2n\}$ as follows.

$$f(u_i) = i \quad \text{for} \quad 1 \leq i \leq 2n.$$

Clearly vertex labels are distinct. Then for any edge $e = u_1 u_i \in R_n$, $\gcd\big(f(u_1), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1$ and for any edge $e = u_i u_{i+1} \in R_n$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$. Since it is consecutive positive integers. Then f admits prime labeling. Thus R_n is a prime graph.

Example 3.4.

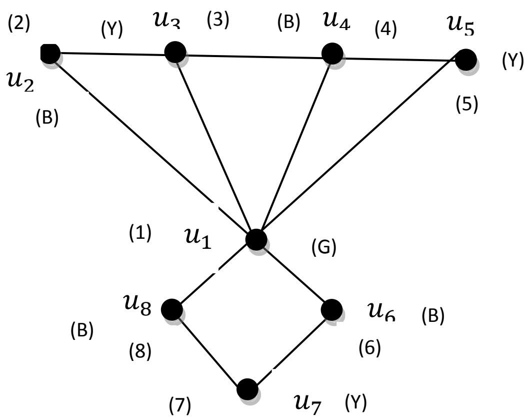

Figure 3.2. Prime labeling for R_4 .

Theorem 3.5. The graph obtained by duplication of any vertex u_k to $u_k{}'$ of the planter graph R_n, $(n \geq 3)$ is a prime graph, where *n* is any positive integer.

Proof. Let G be the graph of planter graph R_n. Let $V(R_n) = \{u_1, u_2, \ldots, u_{2n}\}$ and $E(R_n) = \{u_1u_i / 2 \leq i \leq 2n\} \cup \{u_iu_{i+1} / 2 \leq i \leq n+1\} \cup \{u_iu_{i+1} / n+2 \leq i \leq 2n-1\}$. Let u_k be any vertex of the planter graph R_n, $u_k{}'$ be its duplicated vertex and G_k be the graph resulted due to duplication of the vertex u_k in R_n, where *n* is any positive integer. Let $u_k{}'$ be the duplication of u_k in G_k. Then $|V(G_k)| = 2n + 1$.

We define a labeling $f : V(G_k) \to \{1, 2, \ldots, 2n+1\}$ as follows.

$$f(u_i) = i \qquad \text{for } 1 \leq i \leq 2n$$

$$f(u_k{}') = 2n + 1$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.6.

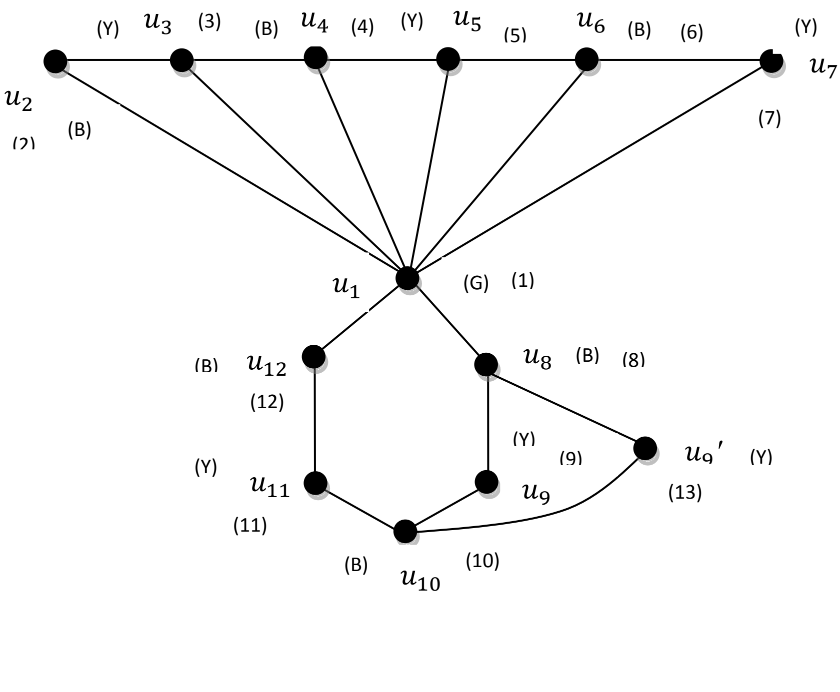

Figure 3.3. Duplication of u_9 in R_6.

Theorem 3.7. The graph obtained by duplicating of an apex vertex u_1 to $u_1{}'$ in the planter graph R_n is a prime graph, where *n* is any positive integer.

Proof. Let G be the graph of planter graph R_n. Let u_1 be an apex vertex of the planter graph R_n, $u_1{}'$ be its duplicated of an apex vertex and G_k be the graph resulted due to duplication of an apex vertex u_1 in O_n, where *n* is any positive integer. Let $u_1{}'$ be the duplication of an apex vertex u_1 in G_k. Then $|V(G_k)| = 2n + 1$.

We define a labeling $f : V(G_k) \to \{1, 2, \dots, 2n + 1\}$ as follows.

$$f(u_i) = i \qquad \text{for } 1 \leq i \leq 2n$$

$$f(u_1{}') = 2n + 1$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.8.

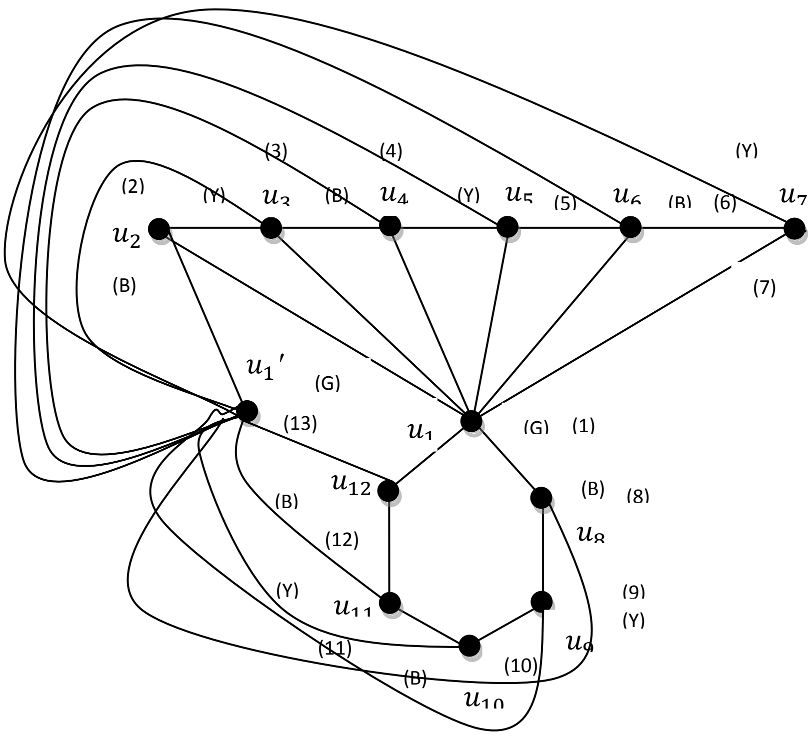

Figure 3.4. Duplication of an apex vertex u_1 in R_6.

Theorem 3.9. The graph obtained by fusing any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) in the planter graph R_n is a prime graph, where *n* is any positive integer.

Proof. Let R_n, $(n \geq 3)$ be the planter graph with vertices $u_1, u_2, \dots, u_{2n}$ and the vertex u_i be fused with u_k. Denote the resultant graph as G_k. Here we note that $|V(G_k)| = 2n - 1$.

We define a labeling $f: V(G_k) \to \{1,2, \dots, 2n-1\}$ as follows

$$f(u_1) = 1$$
$$f(u_2) = 2 = f(u_3)$$
$$f(u_i) = i - 1 \qquad \text{for } 4 \leq i \leq 2n$$

According to this pattern the vertices are labeled such that for any edge $e = u_i u_k \in G_k$, $\gcd\big(f(u_i), f(u_k)\big) = 1$. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of the planter graph R_n, $(n \geq 3)$ is a prime graph.

Example 3.10.

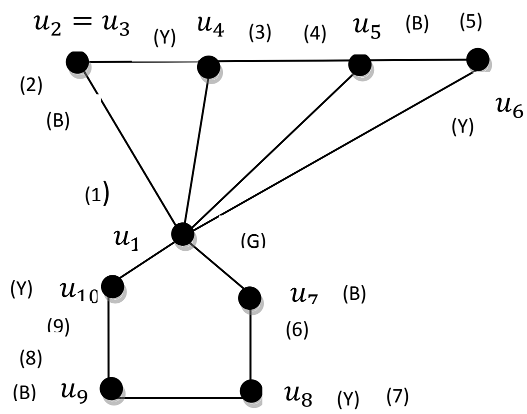

Figure 3.5. Fusion of u_2 and u_3 in R_5.

Theorem 3.11. The graph obtained by identifying an apex vertex u_1 with any vertex u_k (where $d(u_1, u_k) \geq 3$) in the planter graph R_n is a prime graph, where *n* is any positive integer.

Proof. Let R_n, $(n \geq 3)$ be the planter graph with vertices $u_1, u_2, \dots, u_{2n}$ and an apex vertex u_1 be fused with u_k. Denote the resultant graph as G_k. Here we note that $|V(G_k)| = 2n - 1$.

We define a labeling $f: V(G_k) \to \{1,2, \dots, 2n-1\}$ as follows

$$f(u_1) = 1 = f(u_7)$$
$$f(u_i) = i \qquad \text{for } 2 \leq i \leq n+1$$
$$f(u_i) = i - 1 \qquad \text{for } n+3 \leq i \leq 2n$$

According to this pattern the vertices are labeled such that for any edge $e = u_1 u_k \in G_k$, $\gcd\big(f(u_1), f(u_k)\big) = 1$. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) an apex vertex u_1 with any vertex u_k (where $d(u_1, u_k) \geq 3$) of the planter graph R_n, $(n \geq 3)$ is a prime graph.

Example 3.12.

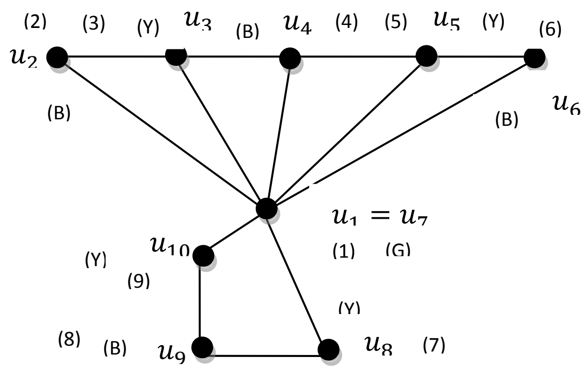

Figure 3.6. Fusion of u_1 and u_7 in R_5.

Theorem 3.13. The switching of any vertex u_k in the planter graph $R_n,(n \geq 3)$ produces a Prime graph, where n is any positive integer.

Proof. Let $G = R_n$, $(n \geq 3)$ be the planter graph and $u_1, u_2, \ldots, u_{2n}$ be the successive vertices of planter graph R_n, $(n \geq 3)$ and G_u denotes the graph obtained by vertex switching of G with respect to the vertex u. It is obvious that $|V(G_u)| = 2n$.

Define a labeling $f: V(G_u) \to \{1,2, \ldots, 2n\}$ as follows

$$f(u_i) = i \quad \text{for } 1 \leq i \leq n+1$$
$$f(u_i) = i - 1 \quad \text{for } n+3 \leq i \leq n+4$$
$$f(u_9) = 11$$
$$f(u_i) = i \quad \text{for } n+5 \leq i \leq 2n$$

Then for any edge $e = u_i u_{i+1} \in G_u$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$. Thus f is a prime labeling and consequently G_u is a prime graph. That is, the switching of any vertex in the planter graph R_n, $(n \geq 3)$ produces a prime graph.

Example 3.14.

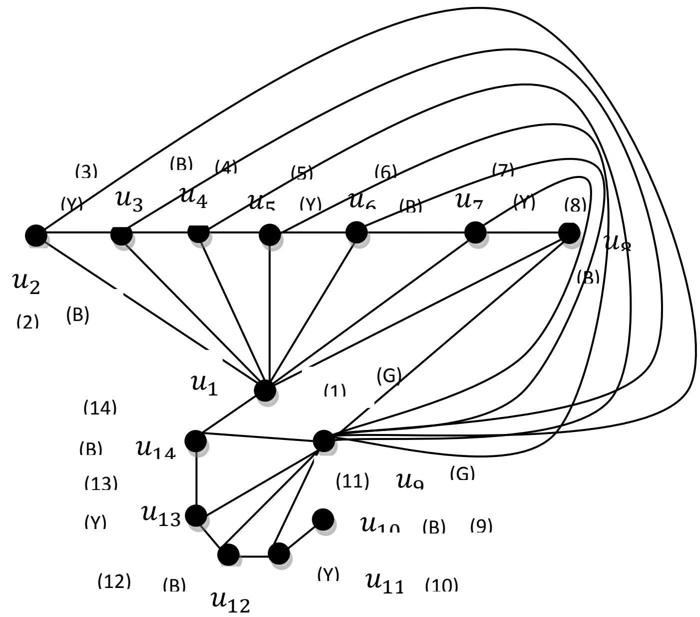

Figure 3.7. Switching the vertex u_9 in R_7.

Theorem 3.15. The switching of an apex vertex u_1 in the planter graph $R_n, (n \geq 3)$ produces a Prime graph, where n is any positive integer.

Proof. Let $G = R_n$, $(n \geq 3)$ be the planter graph and $u_1, u_2, \ldots, u_{2n}$ be the successive vertices of planter graph R_n, $(n \geq 3)$ and G_u denotes the graph obtained by an apex vertex switching of G with respect to the vertex u_1. It is obvious that $|V(G_u)| = 2n$. Without loss of generality, we initiate the labeling from u_1 and proceed in the clock – wise direction.

Define a labeling $f: V(G_u) \rightarrow \{1,2, \ldots, 2n\}$ as follows

$$f(u_i) = i \qquad \text{for } 1 \leq i \leq 2n$$

Then for any edge $e = u_1 u_i \in G_u$, $\gcd\big(f(u_1), f(u_i)\big) = 1$. Thus f is a prime labeling and consequently G_u is a prime graph. That is, the switching of an apex vertex u_1 in the planter graph R_n, $(n \geq 3)$ produces a prime graph and it is a disconnected graph.

Example 3.16.

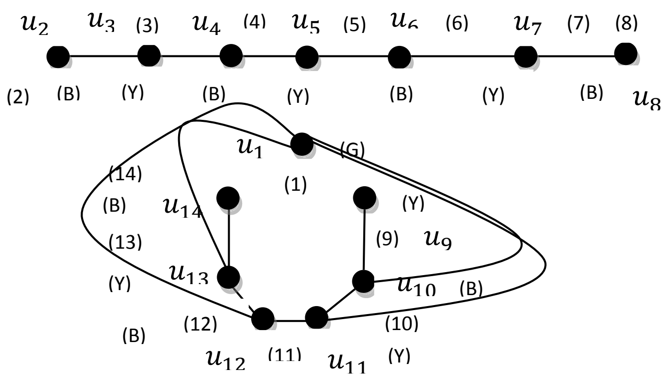

Figure 3.8. Switching an apex vertex u_1 in R_7.

Theorem 3.17. The graph obtained by joining two copies of the planter graph R_n, $(n \geq 3)$ by a path P_k is a prime graph.

Proof. Let G be the graph obtained by joining two copies of the planter graph R_n, $(n \geq 3)$ by a path P_k. Let $u_1, u_2, \ldots, u_{2n}$ be the vertices of first copy of planter graph R_n, $(n \geq 3)$ and let $v_1, v_2, \ldots, v_{2n}$ be the vertices of second copy of planter graph R_n,

$(n \geq 3)$. Let $w_1, w_2, \ldots, w_{2n}$ be the vertices of path P_k with $u_1 = w_1$ and $v_1 = w_k$. We note that $|V(G)| = 4n + k - 2$ and $|E(G)| = 5n + k$.

To define a labeling $f : V(G) \to \{1, 2, \ldots, 4n, 4n+1, \ldots, 4n+k-2\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \leq i \leq 2n.$$

$$f(v_i) = 2n + i \quad \text{for } 1 \leq i \leq 2n.$$

$$f(w_j) = 4n + k - j \quad \text{for } 1 < j < k.$$

Clearly vertex labels are distinct. Thus function defined above provides prime labeling for a graph G. That is, the graph obtained by joining two copies of the planter graph R_n, $(n \geq 3)$ by a path P_k is a prime graph.

Example 3.18.

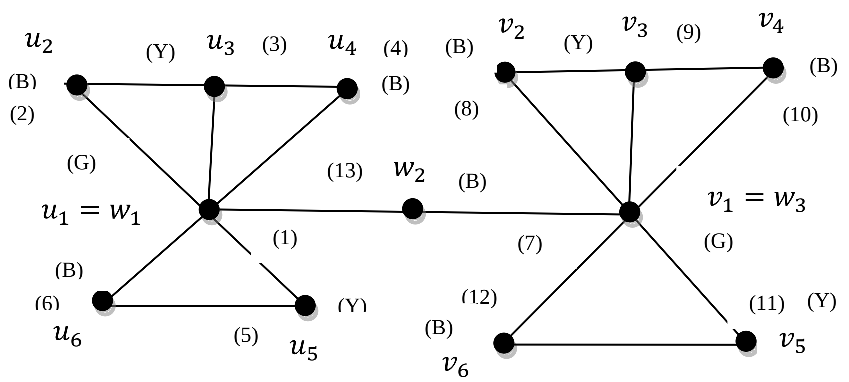

Figure 3.9. Joining two copies of Planter graph R_3 by a path P_3.

CONCLUSION

In this paper we proved that the planter graph R_n , duplication of the planter graph R_n, fusing of the planter graph R_n, switching of the planter graph R_n and also joining two copies of planter graph R_n by a path P_k are prime graphs. There may be many interesting prime graphs can be constructed in future.

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