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Approximation of the Series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{an^2+bn+c}$ where a,b,c ∈ R with a≠0

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Abstract

Here we give approximation of an alternating series using remainder term of the series. Here we introduce a new term called correction term. The correction term plays a vital role in series approximation.

Keywords: Correction function, error function, remainder term, alternating series, rational approximation, Dirichlet's series.

INTRODUCTION

The illusturious mathematician Madhava of 14th century introduces correction function for the series for pi. The Madhava series is

$C = \frac{4d}{1} - \frac{4d}{3} + \frac{4d}{5} - \cdots + (-1)^{n-1}\frac{4d}{2n-1} + (-1)^n \frac{4d(2n)/2}{(2n)^2+1}$, where C is the circumference of a circle of diameter d.

Here the remainder term is $(-1)^n 4d\, G_n$ where $G_n = \frac{(2n)/2}{(2n)^2+1}$ is the correction term. The introduction of the correction term improves the value of C and gives a better approximation for it.

RATIONAL APPROXIMATION OF ALTERNATING SERIES $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{an^2+bn+c}$

where a,b,c ∈ R with a≠0 and $\sqrt{b^2-4ac} \neq 2a$.

The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{an^2+bn+c}$ satisfies the conditions of alternating series test and so it is convergent.

Theorem

The correction function for the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{an^2+bn+c}$ where a,b,c ∈ R with a≠ 0 is $G_n = \frac{1}{\{2an^2+(2b+2a)n+(2c+b+2a)\}}$

Proof

If G_n is the correction function after n terms of the series ,then

we have $G_n + G_{n+1} = \frac{1}{an^2+(2a+b)n+a+b+c}$

The error function is $E_n = G_n + G_{n+1} - \frac{1}{an^2+(2a+b)n+a+b+c}$

Let $G_n(r_1, r_2) = \frac{1}{\{2an^2+(4a+2b)n+(2a+2b+2c)\}-(r_1 n+r_2)}$ where $r_1, r_2 \in R$ and n is fixed.

Then error function $|E_n(r_1, r_2)|$ is minimum for $r_1 = 2a$, $r_2 = b$

Hence for $r_1 = 2a$, $r_2 = b$, both G_n and E_n are functions of a single variable n.

Thus the correction function for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{an^2+bn+c}$ is

$$G_n = \frac{1}{\{2an^2+(2b+2a)n+(2c+b+2a)\}}$$

The corresponding error function is

$$|E_n| = \frac{|(b^2-4ac)-4a^2|}{\{2an^2+(2b+2a)n+(2c+b+2a)\}\{(2an^2+(2b+6a)n+(6a+3b+2c)\}\{(an^2+(2a+b)n+(a+b+c)\}}$$

Hence the proof.

REMARK

Clearly G_n is less than the absolute value of the $(n+1)^{th}$ term.

APPLICATION

1. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \eta(2)$

We have $\eta(2) = 0.8224670334$, using a calculator.

The correction function for the series is $G_n = \frac{1}{2n^2+2n+2}$

For n= 10 , the series approximation after applying correction function is given below

Number of terms	S_n	$S_n + (-1)^n G_n$
10	**0.8**179621756	**0.82246**666801

2. THE ALTERNATING SERIES $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)}$

The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n+1)}$ is convergent and converges to 2log2-1.

We have 2log2-1 = 0.3862943611, using a calculator.

The correction function for the series is $G_n = \frac{1}{2(n+1)^2+1^2}$

For n= 10 , the series approximation after applying correction function is given below

Number of terms	S_n	$S_n + (-1)^n G_n$
10	0.**38**21789321	**0.3863**3283098

CONCLUSION

The introduction of correction function improves the sum of the series and gives a better approximation.

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