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Flow Shop Scheduling Problem with Loading and Unloading Time with Four Machines

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Abstract:

Maximum number of scheduling problems based on transportation time like loading time, unloading time etc. or it may be considered in processing time. Scheduling problems with setup time, transportation time, loading time, unloading time minimizes the total cost of production, total workload and release process. In this paper we are considering four machines with five jobs arranged in series. Also loading time and unloading time is considered separately. Johnson's rule is applied for organizing the sequence of jobs. Pre-emption of jobs is not allowed. Effect of breakdown interval on jobs is calculated. The expansion of algorithm is useful for solving the numerical problem. Objective of the study is to minimize the total working duration of the product as well as to minimize the cost of production.

Keywords: Flow shop scheduling, loading time, unloading time, Johnson's rule, set up time, transportation time.

1.1 Introduction:

Operation Research is nothing but the research of operations. There are different ways of defining Operation Research. According to Churchman, Ackoff and Arnoff Operation Research in the most general sense can be characterized as the application of scientific methods, tools and techniques problem involving the operations of systems so as to provide those in control of the operations with optimum solutions to the problems. Scheduling is the branch of operation research. Effective scheduling leads success in each area of research.

In today's economic system of India, industries play an important role in increasing rate of overall growth that is GDP as well as GNP of India and also increases the employment. Production can be increased if there is a proper utilization of available resources in an optimized way. Optimization is possible if there is effective scheduling which is an important feature of manufacturing. . In Johnson's research work from1954 to 1980 there is not concept of job importance or weightage of jobs [1]. When inventory cost is involved in scheduling, job weightage becomes one of the major factors in arranging the jobs in order. Duan Gang et al.[2] have discussed inventory problem of manufacturing by taking into consideration the demand at different levels of manufacturing. .

Johnson gives an algorithm for two machine problem. Ali Allahverdi [3] described the problem of three machine flow shop scheduling with set up time and weightage of jobs. T. C. Edwin Cheng et.al [4] studied two types of scheduling problems one is static flow shop scheduling problem and another one is deterministic flow shop scheduling problem with set up time of machines Deepak Gupta et.al.[5] also proved an algorithm for 3machines n jobs flow shop scheduling problem with transporting and processing time. Sultana Parveen et al.[6] has described multi criteria scheduling problem which is the most important issue in current manufacturing. Vladimir Modrak et al.[7] proposed the algorithm which converts the problem of m machines into the problem two machines. Peterkofsky et.al [8] focused on the time consuming problem of cargo ships. 60% of total time used up in port by cargo ship because of the method using for loading and unloading of goods. Kathryn E. Stecke et.al [9] used loading time for solving the problems related to system of interconnection of nine machines, checking station and waiting line by automatic mechanism. J. Riezebos et.al [10] represents flow shop scheduling problems with numerous operations, time intervals and waiting time. Y.YIH [18] worked on scheduling problems in the field of manufacturing where buffer is not available in the middle of two workstations. For ex.- flexible PCB electroplating line. With the objective of total working time that is makespan, other criteria's like tardiness, setup time, flow time and idle time etc. are also considered by different researchers[13,14,1]

In this paper we have introduced the concept of loading and unloading time, transporting time for all the jobs as loading and unloading time is one of the important aspect when amount of the articles is huge. Also breakdown interval and importance of jobs that is weights of jobs considered. Johnson's algorithm is used for finding the optimum solution of the problem of four machines arranged in series. By using Johnson's rule Sequence of machines has been decided.[11,12,1]. Every job is performed on each machine in given order. Pre-emption of job is not allowed[12].

With the help of result given in [1] we can prove the theorem given in 1.2

If $\min\left\{P_i + LT_{PQi} + a_i + UL_{PQI}\right\} \geq max\{Q_i + LT_{PQi} + a_i + UL_{PQI}\}$, then

$CP_m + LT_{PQm} + a_m + UL_{PQm} \geq CQ_{m-1}$,where P_i and Q_i are machines, a_i is transportation time from machine P to Q, LT_{PQi} is loading time at machine P_i and UL_{PQi} is unloading time at machine Q_i .

We can prove this result by using the method of mathematical induction.

1.2 Aim and method of the proof for optimizing solution:

When the articles i-1, i, i+1 are arranged in sequence, the optimal solution can be obtained so that

$$\text{Min}\ (\ P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i}\ ,\ LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1}\ )$$

$$< Min(P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}}\ ,\ LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i\ )$$

Let X and X' denotes sequences of items.

X = $\{T_1, T_2, - - - - - T_{i-1}, T_i, T_{i+1}, T_{i+2}, - - - T_n\}$

X' = $\{T'_1, T'_2, - - - - - T'_{i-1}, T'_i, T'_{i+1}, T'_{i+2}, - - - T'_n\}$

Let processing time of any item j on machine Y (ie. P, Q , R, S) for sequences X , X' be given by $(Y_{j,}\ Y'_j)$

Similarly completion time of any item p on machine Y (ie. P, Q, R, S) for sequences X, X' be given by $(CY_{j,}\ CY'_j)$

Let $(a_{j,}a'_{j,})$, $(b_{j,}b'_{j})$, $(c_{j,}c'_{j})$ are transportation time of item j from machine P to Q, Q to R and R to S resp.

Let $p_{j,}p'_{j}$,$q_{j,}q'_{j}$, $r_{j,}r'_{j}$, $s_{j,}s'_{j}$ set up time of item on machines P, Q ,R, S resp. for sequences X and X'.

Let LT_{PQi} denotes loading time of item i at the machine P_i , LT_{QRi} at Q_i ,LT_{RSi} at R_i .

UL_{PQi} denotes unloading time of item i at the machine Q_i ,UL_{QRi} at R_i ,UL_{RSi} at S_i .

The time required on machines Q , R & S for completion of j^{th} item is given by

$CQ_j = \max\left(cP_j + LT_{PQ_j} + a_j + UL_{PQ_j}, cQ_{j-1} \right) + Q_j$

$\quad = cP_j + LT_{PQ_j} + a_j + UL_{PQ_j} + Q_j$

$CR_j = \max\left(cQ_j + LT_{QR_j} + b_j + UL_{QR_j}, cR_{j-1}\right) + R_j$

$\quad = cQ_j + LT_{QR_j} + b_j + UL_{QR_j} + R_j$

$CS_j = \max\left(cR_j + LT_{RS_j} + c_j + UL_{RS_j}, cS_{j-1}\right) + S_j$

$\quad = \max\left(cP_j + LT_{PQ_j} + a_j + UL_{PQ_j} + Q_j + LT_{QR_j} + b_j + UL_{QR_j} + R_j + LT_{RS_j} + c_j + UL_{RS_j}, cS_{j-1}\right) + S_j$ -------------- (1)

The sequence X will be selected in such a way that if $cS_m < c'S_m$ --------- (2)

Max $\left(cP_m + LT_{PQ_m} + a_m + UL_{PQ_m} + Q_m + LT_{QR_m} + b_m + UL_{QR_m} + R_m + LT_{RS_m} + c_m + UL_{RS_m}, cS_{m-1}\right) + S_m$

$< \max\left(c'P_m + LT'_{PQ_m} + a'_m + UL'_{PQ_m} + Q'_m + LT'_{QR_m} + b'_m + UL'_{QR_m} + R'_m + LT'_{RS_m} + c'_m + UL'_{RS_m}, c'S_{m-1}\right) + S'_m$ but

$cP_m + LT_{PQ_m} + a_m + UL_{PQ_m} + Q_m + LT_{QR_m} + b_m + UL_{QR_m} + R_m + LT_{RS_m} + c_m + UL_{RS_m} = c'P_m + LT'_{PQ_m} + a'_m + UL'_{PQ_m} + Q'_m + LT'_{QR_m} + b'_m + UL'_{QR_m} + R'_m + LT'_{RS_m} + c'_m + UL'_{RS_m}$

And $S_m = S'_m$ Equation 2 will be satisfied only if $cS_{m-1} < c'S_{m-1}$ -------- (3)

Using all above results, we can say that equation 2 is true if

$c\,S_j < c'S_j$ (j = i+1,i+2-----,m & i= 1,2,-----m-1) -------------(4)

To find the values of cS_{i+1} & $c'S_{i+1}$

$S_{i+1} = \max\left\{ cR_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}}, cS_i\right\} + S_{i+1}$

$\quad = \max\left\{cP_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}}, cS_i\right\} + S_{i+1}$

$= \max \{cP_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}}, \max (cR_i + LT_{RS_i} + c_i + UL_{RS_i}, cS_{i-1}) + S_i\} + S_{i+1}$

$= \max \{cP_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}}, cR_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i, cS_{i-1} + S_i\} + S_{i+1}$

$= \max \{cP_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}}, \max(cP_i + LT_{PQ_i} + a_i + UL_{PQ_i} \quad cQ_{i-1}) + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i, LT_{RS_i} + c_i + UL_{RS_i} + S_i, cS_{i-1} + S_i\} + S_{i+1}$

$cS_{i+1} = \max \{cP_{i-1} + P_i + P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1}, cP_{i-1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + S_{i+1}, cS_{i-1} + S_i + S_{i+1}\}$ -------(5)

Similarly $c'S_{i+1} = \max \{c'P_{i-1} + P'_i + P'_{i+1} + LT'_{PQ_{i+1}} + a'_{i+1} + UL'_{PQ_{i+1}} + Q'_{i+1} + LT'_{QR_{i+1}} + b'_{i+1} + UL'_{QR_{i+1}} + R'_{i+1} + LT'_{RS_{i+1}} + c'_{i+1} + UL'_{RS_{i+1}} + S'_{i+1}, c'P_{i-1} + P'_i + LT'_{PQ_i} + a'_i + UL'_{PQ_i} + Q'_i + LT'_{QR_i} + b'_i + UL'_{QR_i} + R'_i + LT'RSi + c'_i + UL'_{RS_i} + S'_i + S'_{i+1}, cS_{i-1} + S_i + S_{i+1}\}$ --(6)

Comparing sequences S & S',

we get c P_{i-1} = c′ P_{i-1} & c S_{i-1}= c′ S_{i-1}

$Y_i = Y'_{i+1}$, $Y_{i+1} = Y'_i$ where Y = P, Q, R, or S ------------(7)

Also $a_i = a'_{i+1}$, $a_{i+1} = a'_i$, $b_i = b'_{i+1}$,$b_{i+1} = b'_i$, $c_i = c'_{i+1}$,$c_{i+1} = c'_i$

$LT_{PQ_i} = LT'_{PQ_{i+1}}$, $LT_{QR_i} = LT'_{QR_{i+1}}$, $LT_{RS_i} = LT'_{RS_{i+1}}$

$UL_{PQ_i} = UL'_{PQ_{i+1}}$, $UL_{QR_i} = UL'_{QR_{i+1}}$, $UL_{RS_i} = UL'_{RS_{i+1}}$

$LT_{PQ_{i+1}} = LT'_{PQ_i}$, $LT_{QR_{i+1}} = LT'_{QR_i}$, $LT_{RS_{i+1}} = LT'_{RS_i}$, ,

$UL_{PQ_{i+1}} = UL'_{PQ_i}$, $UL_{QR_{i+1}} = UL'_{QR_i}$, $UL_{RS_{i+1}} = UL'_{RS_i}$

Writing eq. 4 for j = i+1 & using eq. 7, we get

$max\{cP_{i-1} + P_i + P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1}, cP_{i-1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + S_{i+1}, cS_{i-1} + S_i + S_{i+1}\} < max\{cP_{i-1} + P_{i+1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i, cP_{i-1} + P_{i+1} +$

$LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + S_i,\ cS_{i-1} + S_{i+1} + S_i \}$ -------- (8)

Subtracting last term from both sides, we get

Max $\{cP_{i-1} + P_i + P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1},\ cP_{i-1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i + S_{i+1}\} <$

$max\{cP_{i-1} + P_{i+1} + P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + Q_i + LT_{QR_i} + b_i + UL_{QR_i} + R_i + LT_{RS_i} + c_i + UL_{RS_i} + S_i,\ cP_{i-1} + P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + Q_{i+1} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + R_{i+1} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + S_{i+1} + S_i\}$

Now Subtracting

$max\{cP_{i-1} + P_i + P_{i+1} + LT_{PQ_i} + a_i + UL_{PQ_i} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + LT_{QR_i} + b_i + UL_{QR_i} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + LT_{RS_i} + c_i + UL_{RS_i} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + Q_i + Q_{i+1} + R_i + R_{i+1} + S_i + S_{i+1}\}$

Therefore

Min $\{ P_i + LT_{PQ_i} + a_i + UL_{PQ_i} + LT_{QR_i} + b_i + UL_{QR_i} + LT_{RS_i} + c_i + UL_{RS_i} + Q_i + R_i,\ LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + Q_{i+1} + R_{i+1} + S_{i+1} \} <$

Min $\{P_{i+1} + LT_{PQ_{i+1}} + a_{i+1} + UL_{PQ_{i+1}} + LT_{QR_{i+1}} + b_{i+1} + UL_{QR_{i+1}} + LT_{RS_{i+1}} + c_{i+1} + UL_{RS_{i+1}} + Q_{i+1} + R_{i+1},\ LT_{PQ_i} + a_i + UL_{PQ_i} + b_i + UL_{QR_i} + LT_{QR_i} + LT_{RS_i} + c_i + UL_{RS_i} + Q_i + R_i + S_i \}$

1.3 Designing of Problem

To prove the utility of above theorem numerical problem is solved.

In the given problem four machines namely P, Q, R, S are arranged in series loading time, unloading time and transport time of articles A_1, A_2, A_3, ------ A_n are considered. The transporting agent transport the items from machine P to machine Q, machine Q to machine R, machine R to machine S in such a way that after delivering the articles to machine S without delay come back to machine P for transferring the next item. All notations are same as mentioned in the theorem.

All machines required time for loading items transported by transport agent so that machines will start processing, this time will be considered as loading time. After finishing the working of machine the articles will be transferred to the next machine, the time required for unloading of these articles on the particular machine is considered as unloading time.

Step I:

Supposed three machines E, F and G are assumed with the service time E_i, F_i and G_i resp.

Where, $E_i = P_i + LT_{PQi} + a_i + UL_{PQi} + Q_i + LT_{QRi} + b_i + UL_{QRi}$

$F_i = LT_{PQi} + a_i + UL_{PQi} + Q_i + LT_{QRi} + b_i + UL_{QRi} + R_i$

$G_i = LT_{QRi} + b_i + UL_{QRi} + R_i + LT_{RSi} + c_i + UL_{RSi} + S_i$

1) Min $(Pi + LT_{PQi} + a_i + UL_{PQi}) \geq$ Max $(LT_{PQi} + a_i + UL_{PQi} + Q_i)$

2) Min $(LT_{QRi} + b_i + UL_{QRi} + R_i) \geq$ Max $(Q_i + LT_{QRi} + b_i + UL_{QRi})$

3) Min $(LT_{Rsi} + c_i + UL_{RSi} + S_i) \geq$ Max $(R_i + LT_{Rsi} + c_i + UL_{RSi})$

Step II:

Now this problem will be reduced into two machine problem by considering two fictitious machines H and K where

$Hi = E_i + F_i$ and $Ki = F_i + G_i$. And

1) If min $(H,K) = H_i$ then $H'_i = H_i - wt_i$ and $K'_i = K_i$

2) If min $(H,K) = K_i$ then $H'_i = H_i$ and $K'_I = K_i + wt_i$

Article	E_i	F_i	G_i	wt_1
A_1	E_1	F_1	G_1	wt_1
A_2	E_2	F_2	G_2	wt_2
A_3	E_3	F_3	G_3	wt_3
A_4	E_4	F_4	G_4	wt_4
A_5	E_5	F_5	G_5	wt_5

Step III:

For getting the proper sequence the new problem is defined as follows ;

$$H'i = H'i/ wt_i \quad \text{and} \quad K'i = K'i/ wt_i$$

Article	H'_i / wt_i	K_i / wt_i	wt_i
A_1	H'_1 / wt_1	K'_1 / wt_1	wt_1
A_2	H'_2/ wt_2	K'_2 / wt_2	wt_2
A_3	H'_3 / wt_3	K'_3 / wt_3	wt_3
A_4	H'_4 / wt_4	K'_4 / wt_4	wt_4
A_5	H'_5 / wt_5	K'_5 / wt_5	Wt_5

Step IV:

Considering the breakdown interval and the effect of this time interval should be observed on all jobs. If the jobs are affected by this time interval then the difference of the interval will be added. With the effect of this breakdown interval, the problem will be redefined as follows:

1) If the job is affected by the breakdown interval then

$$P'_i = P_i + (u-v)\ , \quad Q'_i = Q_i + (u-v)\ , \quad R'_i = R_i + (u-v)\ , \quad S'_i = S_i + (u-v)$$

2) If the job is not affected by the breakdown interval then

$$P'_i = P_i,\ Q'_i = Q_i,\ R'_i = R_i\ ,\ S'_i = S_i$$

Step V: Applying steps I, II, III, IV the problem has been solved for getting the optimal sequence. The scheduling of all the course of action is in such a way that the minimum time should be required for getting the optimum solution or whole production .For finding the algorithm the above information can be symbolized and represented in table for finding the sequence by using Johnson's rule of sequencing.

1.4 Numerical Example:

Ai	P_i	LT_{PQi}	a_i	UL_{PQi}	Q_i	LT_{QRi}	b_i	UL_{QRi}	R_i	LT_{RSi}	c_i	UL_{RSi}	S_i	wt_i
A_1	6	2	4	3	5	2	3	4	6	2	2	5	6	2
A_2	5	3	6	3	2	2	5	2	5	1	3	6	5	4
A_3	4	3	5	2	4	2	2	4	6	2	3	4	6	3
A_4	5	4	3	2	4	3	4	2	7	2	2	3	8	5
A_5	8	2	3	1	3	2	6	1	8	1	2	4	8	2

$Pi+ LT_{PQi} + a_i+ UL_{PQi} \geq \max(LT_{PQi} + a_i+ UL_{PQi} + Q_i)$

$LT_{QRi}+ b_i + UL_{QRi} + R_i \geq max\,(Q_i +LT_{QRi} + b_i+ UL_{QRi})$

$LT_{Rsi}+ c_i + UL_{RSi} + S_i \geq \max\,(R_i +LT_{Rsi}+ c_i + UL_{RSi})$

Step I: Supposed three machines E, F and G are assumed with the service time E_i, F_i and G_i resp.

Article	E_i	F_i	G_i	wt_i
A_1	29	29	30	2
A_2	28	28	29	4
A_3	26	28	29	3
A_4	27	29	31	5
A_5	26	26	32	2

Step II: Now this problem will be reduced into two machine problem by considering two fictitious machines H and K, where $Hi = E_i + F_i$ and $Ki = F_i + G_i$

Article	H	K	wt_i
A_1	58	59	2
A_2	56	57	4
A_3	54	57	3
A_4	56	60	5
A_5	52	58	2

1) If $\min (H,K) = H_i$ then $H'_i = H_i - wt_i$ and $K'_i = K_i$

2) If $\min (H,K) = K_i$ then $H'_i = H_i$ and $K'_I = K_i + wt_i$

Article	H'_i	K'_i	wt_i
A_1	56	59	2
A_2	52	57	4
A_3	51	57	3
A_4	51	60	5
A_5	50	58	2

Step III;

For getting the proper sequence the new problem is defined as follows ;

Article	H'_i / wt_i	K'_i / wt_i	wt_i
A_1	28	29.5	2
A_2	13	14.25	4
A_3	17	19	3
A_4	10.2	12	5
A_5	25	29	2

By Johnson's rule the optimal sequence obtained for above reduced problem is 4, 2, 3, 5, 1. Then the time required for total processing of articles by using above scheduling sequence i.e minimum time for entire production can be calculated by considering the time required by the transporting agent when it returns back to machine $M_1$to load the next article and also the time when it reaches to machine M_2 for unloading of an article.

Ai	P_i		LT_{PQi}	a_i	UL_{PQi}	Q_i		LT_{QRi}	b_i	UL_{QRi}	R_i		LT_{RSi}	c_i	UL_{RSi}	S_i		wt_i
	I	O				I	O				I	O						
A_4	0	5	4	3	2	14	18	3	4	2	27	34	2	2	3	41	49	5
A_2	5	10	3	6	3	22	24	2	5	2	36	41	1	3	6	55	60	4
A_3	10	14	3	5	2	26	30	2	2	4	45	51	2	3	4	64	70	3
A_5	14	22	2	3	1	31	34	2	6	1	52	60	1	2	4	74	82	2
A_1	**22**	**28**	2	4	3	37	42	2	3	4	64	70	2	2	5	87	93	2

Effect of breakdown interval:[35,41]

The effect of breakdown interval is on jobs Q_i1 and R_i2, hence the original problem is converted into new problem

Ai	P_i	LT_{PQi}	a_i	UL_{PQi}	Q_i	LT_{QRi}	b_i	UL_{QRi}	R_i	LT_{RSi}	c_i	UL_{RSi}	S_i	wt_i
A_1	6	2	4	3	**11**	2	3	4	6	2	2	5	6	2
A_2	5	3	6	3	2	2	5	2	11	1	3	6	5	4
A_3	4	3	5	2	4	2	2	4	6	2	3	4	6	3
A_4	5	4	3	2	4	3	4	2	7	2	2	3	8	5
A_5	8	2	3	1	**3**	2	6	1	8	1	2	4	8	2

Repeating the same procedure of step 1,2,3 for finding the sequence and getting the optimal solution.

Article	E_i	F_i	G_i	wt_i
A_1	35	35	30	2
A_2	28	34	35	4
A_3	26	28	29	3
A_4	27	29	31	5
A_5	26	26	32	2

Step II:

Now this problem will be reduced into two machine problem by considering two fictitious machines H and K where

Article	H	K	wt_i
A_1	70	65	2
A_2	64	69	4
A_3	54	57	3
A_4	56	60	5
A_5	52	58	2

1) If min $(H,K) = H_i$ then $H'_i = H_i - wt_i$ and $K'_i = K_i$

2) If min $(H,K) = K_i$ then $H'_i = H_i$ and $K'_I = K_i + wt_i$

Article	H'_i	K'_i	wt_i
A_1	70	67	2
A_2	60	69	4
A_3	51	57	3
A_4	51	60	5
A_5	50	68	2

Step III;

For getting the proper sequence the new problem is defined as follows ;

Article	H'_i / wt_i	K'_i / wt_i	wt_i
A_1	35	33.5	2
A_2	15	17.25	4
A_3	17	19	3
A_4	10.2	12	5
A_5	25	34	2

By Johnson's rule the optimal sequence obtained for above reduced problem is 4, 2, 3, 5, 1. Then the time required for total processing of articles by using above scheduling sequence i.e minimum time for entire production can be calculated by considering the time required by the transporting agent when it returns back to machine $M_1$to load the next article and also the time when it reaches to machine M_2 for unloading of an article.

Ai	P_i		LT_{PQi}	a_i	UL_{PQi}	Q_i		LT_{QRi}	b_i	UL_{QRi}	R_i		LT_{RSi}	c_i	UL_{RSi}	S_i		wt_i
	I	O				I	O				I	O				I	O	
A_4	0	5	4	3	2	14	18	3	4	2	27	34	2	2	3	41	49	5
A_2	5	10	3	6	3	22	24	2	5	2	36	47	1	3	6	57	62	4
A_3	10	14	3	5	2	26	30	2	2	4	51	57	2	3	4	66	72	3
A_5	14	22	2	3	1	31	34	2	6	1	58	66	1	2	4	76	84	2
A_1	**22**	**28**	2	4	3	37	48	2	3	4	70	76	2	2	5	89	95	2

Minimum weighted flow time (MWFT)

= (49*5) + (62- 5) *4 + (72-10)*3 +(84-14)*2 + (95- 22)*2

= 245 + 228+ 186 + 140 + 146 / 5+4+3+2+2

= 945 / 16

= 59 hours

1.5 Conclusion:

The total elapsed time for the complete process is 95 hrs and minimum weighted flow time is 59 hrs. From above table it is shown that the time gets reduced for total production by using the sequence obtained with the help of Johnson’s rule. The proposed method of optimization is useful for finding the optimum solution. The algorithm is useful by reducing the number of machines into two machines.

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