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Convergence of Fuzzy f – Matrix

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Abstract

In this paper a table method of determining the f-matrix of the fuzzy matrix is introduced. This paper introduces a processing method of determining the f-matrix using the table method by which the algorithm realized. This paper shows by the example that the convergence of fuzzy f-matrix from the convergence of fuzzy matrix also this paper discusses about the properties of fuzzy f-matrix

Keywords: Fuzzy matrix, Fuzzy f-matrix, Convergence of fuzzy f-matrix, Power of fuzzy f-matrix.

1. INTRODUCTION

K.H. Kim and F.W. Roush[4] have put forward the concept of the generalized inverse of the fuzzy matrix in the extract. Luo Ching - Zhong [5] has given the definition method and the decision condition of finding f – matrix of the fuzzy matrix. The definition method is very difficult in particular as the order of the matrix is very large. This paper is aimed at this weak point of the definition method and gives a table method of solving the f-matrix of all the g-inverse of the fuzzy matrix.

A nxn matrix A = [a_{ij}] with all the a_{ij} in [0,1] is called a fuzzy matrix. We compute powers of A using the max-min composition of fuzzy matrices. Use min for multiplication and max for addition. Define A^2 =AA, A^3 = A^2A, etc.

It is well known that [3] the sequence {A^n}, n = 1, 2, 3,……… either converges or oscillates. By convergence we mean that there is a positive integer c so that A^n =A^c for

$n \geq c$. Convergence of powers of a fuzzy matrix has been investigated by many researchers. In preceding investigation, some conditions for convergence of the powers of a fuzzy matrix are shown [3]. When a fuzzy matrix represents a fuzzy transitive relation, its powers always converge. In this case, precise properties about convergence are obtained [1].

2. ALGORITHM OF f-MATRIX

2.1 Regular [4]

A matrix A is regular if and only if there exist a matrix X such that AXA = A such a matrix is called a generalized inverse or g-inverse of A.

2.2 Definition [4]

For any fuzzy matrix $A = [a_{ij}]_{nxm}$.

$$X_{jk} = \min \{a_{st} / a_{st} < (a_{sj} \wedge a_{kt})\},$$

$$j = 1,2, \ldots m$$

$$k = 1,2, \ldots n.$$

and specify the minimum of null set is equal to 1, then all the X_{jk} compose of a fuzzy matrix $X = [x_{jk}]_{mxn}$ too, then the matrix X is called f-matrix of the matrix A.

2.3 Definition

Let a fuzzy matrix A which has the minor of $a_{11} \in A$ is unit matrix, we can determine the f-matrix X of the matrix A from the above definition, then the f-matrix X is the generalized inverse of the matrix A.

(i.e) AXA = A, simply X is g-inverse of A.

2.4 Algorithm

Suppose A is a fuzzy matrix with minor of a_{11} is unit matrix, on the basis of the definition,

$$X_{jk} = \min \{a_{st} / a_{st} < (a_{sj} \wedge a_{kt})\},$$

$$j = 1,2, \ldots m$$

$$k = 1,2, \ldots n.$$

We deploy its into all the terms and have formula,

$$
\begin{aligned}
X_{jk} = \min \{a_{st} / \; & a_{11} < (a_{1j} \wedge a_{k1}),\ a_{12} < (a_{1j} \wedge a_{k2}),\ \ldots\ldots\ldots,\ a_{1m} < (a_{1j} \wedge a_{km}), \\
& a_{21} < (a_{2j} \wedge a_{k1}),\ a_{22} < (a_{2j} \wedge a_{k2}),\ \ldots\ldots\ldots,\ a_{2m} < (a_{2j} \wedge a_{km}), \\
& \ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots \\
& \ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots \\
& \ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots \\
& a_{n1} < (a_{nj} \wedge a_{k1}),\ a_{n2} < (a_{nj} \wedge a_{k2}),\ \ldots\ldots\ldots,\ a_{nm} < (a_{nj} \wedge a_{km})\}
\end{aligned}
$$

From this we may construct a table as shown by the table consisting from the matrix A and j^{th} column and the k^{th} row of the matrix A. We treat the table by the different way. Thus we have

$$
\begin{pmatrix}
a_{11} & a_{12} & \ldots\ldots\ldots\ldots\ldots\ldots & a_{1m} \\
a_{21} & a_{22} & \ldots\ldots\ldots\ldots\ldots\ldots & a_{2m} \\
.. & .. & \ldots\ldots\ldots\ldots\ldots\ldots & .. \\
a_{n1} & a_{n2} & \ldots\ldots\ldots\ldots\ldots\ldots & a_{nm}
\end{pmatrix}
$$

STEP (1): Reconstruct Set B:

We reconstruct the set B by the content of the table. The elements of the set are taken out in the way. We draw respectively a horizontal line and a vertical line from every element a_{il}, i=1,2,……n and l=1,2,…….m of the matrix A and we compare a_{il} with the corresponding element a_{il} in the 0^{th} column, and a_{kl} in the 0^{th} row respectively. We put a_{il} into the set B if a_{ij} and a_{kl} both are greater than a_{il} or else put the null value Φ into the set B.

STEP (2): Solve for minimum:

We solve for the minimum of the set B reconstructed from the relation x_{jk} (if the minimum is equal to 1 if elements of the set B is all null value Φ).

STEP (3): To construct the f-matrix:

In the way after treating all the table consisted from all the j^{th} columns (j=1,2,…m) and the k rows (k=1,2,…n) throughout the matrix A, we constructed row by row the matrix X with all the x_{jk} obtained above. The matrix X_{mxn} is namely f-matrix of the matrix $A_{nxm.}$

2.5 Theorem

The matrix $X=[x_{jk}]$ is composed by the relation $X_{jk} = \min \{a_{st} / a_{st} < (a_{sj} \wedge a_{kt})\}$ in the fuzzy matrix A which minor of $a_{11} \in A$ is unit matrix. Then the matrix x is the generalized inverse (g-inverse) of A.

Proof

Let A be an nxm matrix.

The relation $X_{jk} = \min \{a_{st} / a_{st} < (a_{sj} \wedge a_{kt})\}$,

$j = 1,2, \ldots m$

$k = 1,2, \ldots n$.

can be written as

$$
\begin{aligned}
X_{jk} = \min \{a_{st} / \; & a_{11} < (a_{1j} \wedge a_{k1}),\ a_{12} < (a_{1j} \wedge a_{k2}),\ \ldots\ldots,\ a_{1m} < (a_{1j} \wedge a_{km}),\\
& a_{21} < (a_{2j} \wedge a_{k1}),\ a_{22} < (a_{2j} \wedge a_{k2}),\ \ldots\ldots,\ a_{2m} < (a_{2j} \wedge a_{km}),\\
& \ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots\\
& \ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots\\
& \ldots\ldots \qquad \ldots\ldots \qquad \ldots\ldots\\
& a_{n1} < (a_{nj} \wedge a_{k1}),\ a_{n2} < (a_{nj} \wedge a_{k2}),\ \ldots\ldots,\ a_{nm} < (a_{nj} \wedge a_{km})\}
\end{aligned}
$$

From the above process we can get f-matrix X of the fuzzy matrix A.

Now we have to show the f-matrix X is g-inverse of A. (i.e) AXA=A.

Since A is a fuzzy matrix of order nxm, then the f-matrix X is of order mxn.

Now to check the relation AXA=A.

$AX = \Sigma a_{ij} . x_{ji}$ i=1,2,…,n and j=1,2,…,m

Assume that the product of the fuzzy matrix AX=B, B is the matrix of order nxn, elements in the matrix B is b_{ii}.

If a_{ij} is less than or equal to x_{ji} for every j, then $b_{ii} = \max(a_{ij})$.

If x_{ji} is less than or equal to a_{ij} for every j, then $b_{ij} = \max(x_{ji})$.

Therefore, the matrix B=AX is the element of X or the element of A.

Also $BA = AXA = \Sigma b_{ii} .a_{ij} = D$, where D contains the elements d_{ij}, i=1,2,…n, j=1,2,….m

$$d_{ij} = \begin{cases} b_{ii}, & \text{if } b_{ii} \le a_{ij} \\ a_{ij}, & \text{if } a_{ij} \le b_{ii} \end{cases}$$

Here the product of the fuzzy matrix BA is of order nxm from the above multiplication.

The above relation we easily identify that the product of the fuzzy matrix BA

[i.e, AXA] contains only in the elements of the fuzzy matrix A.

(i.e) AXA =A

Therefore f-matrix X is generalized inverse of A.

Hence the proof.

2.6 Corollary

(i) Let X= [x_{jk}] j=1,2,…..m is the f-matrix of the matrix A then the eqn. XAX=X is satisfied trivially from the above theorem.

(ii) Let X=[x_{jk}] is the f-matrix of the matrix A, then the f-matrix X is the reflexive generalized inverse of the matrix A. i.e) X satisfies both eqn.

i) AXA=A and ii) XAX = X.

2.7 Example

Let A be a square matrix of order nxn.

$$A = \begin{pmatrix} 1 & 0.3 & 0.5 \\ 0.2 & 1 & 1 \\ 0.4 & 1 & 1 \end{pmatrix}$$

From the definition we have

$$X_{jk} = \min \{a_{st} / a_{st} < (a_{sj} \wedge a_{kt})\},$$

j = 1,2, ….. m

k = 1,2, ….. n.

Therefore we found out the f-matrix X of the matrix A.

$$\text{i.e)} \quad X = \begin{pmatrix} 1 & 0.3 & 0.3 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.3 & 0.2 \end{pmatrix}$$

3. WEAK-GENERALIZED INVERSE

3.1 Definition

For any fuzzy matrix A, we can compose the f-matrix X of the matrix A using the relation

$$X_{jk} = \min \{a_{st} / a_{st} < (a_{sj} \wedge a_{kt})\},$$

$$j = 1,2, \ldots\ldots m$$

$$k = 1,2, \ldots\ldots n.$$

then the f-matrix X is weak-generalized inverse of A if X satisfies the relation

$XAX = X$

3.2 Different Classes of generalized Inverse [2]

Every finite matrix A of real or complex elements, there is a unique matrix X satisfying the four equations.

(i). $AXA = A$ (ii). $XAX = X$ (iii). $(AX)^* = AX$ and (iv). $(XA)^* = XA$

where A^* denotes the conjugate transpose of A.

3.3 Proposition

Every f-matrix X of the matrix A is a reflexive g-inverse of A.

Proof

We already prove the f-matrix X is g-inverse of the matrix A.

It is enough to show $XAX = X$.

Consider the f-matrix X of the matrix A. then $XA = \Sigma\, x_{ij}.a_{ji}$ $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$

where $x_{ij} \,.\, a_{ji} = \min\{x_{ij}, a_{ji}\}$

Assume the above product XA as Y. Therefore, XA is the element of y_{ii}.

$$YX = XAX = \Sigma\, y_{ii} \,.\, x_{ij.} \qquad i = 1,2,\ldots\ldots..n$$

$= X$ {because X is the smallest element obtain from the matrix element A.}

3.4 Proposition

Every f-matrix X of the matrix A is need not Moore – Penrose inverse of the matrix A.

Remark

If all the elements in the fuzzy matrix A_{nxn} have the same elements except in the minor of $a_{11,}$ then the f-matrix X is same as the fuzzy matrix A. i.e) A has the g-inverse itself.

4. CONVERGENCE OF POWER OF A FUZZY MATRIX

A nxn matrix A = [a_{ij}] with all the a_{ij} in [0,1] is called a fuzzy matrix. We compute powers of A using the max-min composition of fuzzy matrices. Use min for multiplication and max for addition. Define A^2 =AA, $A^3 = A^2A$, etc.

4.1 Definition:[1]

If there exist an integer c such that $A^n = A^c$, for all n is greater than or equal to c, then the sequence of fuzzy matrix {A^n}.

4.2 Definition:[1]

If there is a finite set of fuzzy matrices {A_1, A_2, ………,A_L} so that there is a positive integer c such that $A^c = A_1$, $A^{c+1} = A_2$, ………………, $A^{c+L} = A_1$………, then the sequence of fuzzy matrix {A^n} diverges (or) oscillates.

4.3 Example:

Suppose $A = \begin{pmatrix} 1 & 0.4 & 0.6 \\ 0.5 & 0.2 & 0.3 \\ 0.4 & 0.1 & 1 \end{pmatrix}$

To find the sequence $\{A^n\}$ of fuzzy matrix.

$$A^2 = A.A = \begin{pmatrix} 1 & 0.4 & 0.6 \\ 0.5 & 0.2 & 0.3 \\ 0.4 & 0.1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.4 & 0.6 \\ 0.5 & 0.2 & 0.3 \\ 0.4 & 0.1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0.4 & 0.6 \\ 0.5 & 0.4 & 0.5 \\ 0.4 & 0.4 & 1 \end{pmatrix}$$

$$A^3 = A^2$$

Similarly we can find the sequence $\{A^n\}$ of fuzzy matrix that converges to A^2.

(i.e) $A^n = A^2$ for all $n \geq 2$, ($A \leq A^2 = A^3 = \ldots\ldots\ldots\ldots$)

Therefore the sequence $\{A^n\}$ converges to A^2.

5. CONVERGENCE OF F – MATRIX:

5.1 Proposition:

Any f-matrix X of the matrix A_{nxn} is converges to itself. (i.e) $X = X^2 = X^3 = \ldots\ldots$ We have the f- matrix X of A

(i.e) a fuzzy matrix $A = \begin{pmatrix} 1 & 0.3 & 0.5 \\ 0.2 & 1 & 1 \\ 0.4 & 1 & 1 \end{pmatrix}$ has the f-matrix $X = \begin{pmatrix} 1 & 0.3 & 0.3 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.3 & 0.2 \end{pmatrix}$

Now to find the f-matrix X is converges to itself. i.e) $X^n = X$ for all $n \geq 1$.

$$X^2 = X.X = \begin{pmatrix} 1 & 0.3 & 0.3 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.3 & 0.2 \end{pmatrix} \begin{pmatrix} 1 & 0.3 & 0.3 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.3 & 0.2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0.3 & 0.3 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.3 & 0.2 \end{pmatrix} = X$$

Similarly $X^3 = X^4 = X^5 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = X$.

Therefore f-matrix X is converges to itself.

5.2. PERIOD OF POWER OF FUZZY MATRIX

Let A = [a_{ij}] be an nxn fuzzy matrix whose components $a_{ij} \in [0,1]$ and the power of a is defined as A^k = A.A…………A (k-times) and the elements of A^k is denoted by $a_{ij}^{(k)}$.

5.3 Definition [8]

If there exist an integer k such that $A^{k+1} = A^k$ holds, then the powers of fuzzy matrix are said to converse. Otherwise they are said to diverge or oscillate.

5.4 Proposition [8]

The powers of a fuzzy matrix either converges to A^c for a finite c, or oscillate with a finite period. That is a fuzzy matrix is simply said to converge (diverge) when its powers converge (diverge) and the smallest period of powers of a fuzzy matrix is simply said to be its period.

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