International Journal of Mathematics Research.
ISSN 0976-5840 Volume 9, Number 2 (2017), pp. 109-119

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The Non - Split Distance - 2 Domination in Graphs

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Abstract

A distance -2 dominating set D$\subseteq$ V of a graph G is a non-split distance -2 dominating set if the induced sub graph <V-D> is connected. The non-split distance -2 domination number $\gamma_{ns\leq 2}(G)$ is the minimum cardinality of a non-split distance -2 dominating set. In this paper, we define the notion of non-split distance -2 domination in a graph. We get many bounds on non- split distance -2 domination number. Exact values of this new parameter are obtained for some standard graphs. Nordhaus - Gaddum type results are also obtained for this new parameter.

Keywords: Dominating set, non-split dominating set, distance -2 dominating set, non-split distance -2 dominating set, non- split distance -2 domination number.

1. INTRODUCTION

All graphs considered here are simple, finite and undirected. Let n and m denote the order and size of a graph G. We use the terminology of [12].Let $\Delta(G)$ ($\delta(G)$) denote the maximum (minimum) degree. The independence number β_0(G) is the maximum cardinality among the independent set of vertices of G. The lower independence number i(G) is the minimum cardinality among the maximum independent set of vertices of G. The vertex covering number α_0(G) is the minimum cardinality of vertex covering of G. The girth g(G) of a graph G is the length of a shortest cycle in G. The circumference c(G) is the length of a longest cycle. The radius of G is rad(G) = min{ecc(v): v$\in V$} and diam(G)= max{ecc(v):v$\in V$},where ecc(v) is eccentricity of a vertex which is defined as max{dis (u,v): v$\in V$ }in [11].

A non empty set D$\subseteq$V (G) is said to be a dominating set of G if every vertex not in D is adjacent to at least one vertex in D. A dominating set D$\subseteq$ V of a graph G is a non-split (split) dominating set if the induced sub graph <V-D> is connected (disconnected). The non-split (split) domination number $\gamma_{ns}(G)$ ($\gamma_s(G)$) is the minimum cardinality of a non-split (split) dominating set. A set D of vertices in a graph G is a distance -2 dominating set if every vertex in V-D is within distance 2 of atleast one vertex in D. The distance -2 domination number $\gamma_{\leq 2}(G)$ is the minimum cardinality of a distance -2 dominating set in G. A distance -2 dominating set D$\subseteq$ V of a graph G is a split distance -2 dominating set if the induced sub graph <V-D> is disconnected. The split distance -2 domination number $\gamma_{s\leq 2}(G)$ is the minimum cardinality of a split distance -2 dominating set.

Kulli V.R. and Janakiram B. introduced the concept of non-split domination in graph in [13]. The purpose of this paper is to introduce the concept of non-split distance -2 domination in graphs.

Definition 1.1

A distance -2 dominating set D$\subseteq$ V of a graph G is a non-split distance -2 dominating set if the induced sub graph <V-D> is connected. The non-split distance -2 domination number $\gamma_{ns\leq 2}(G)$ is the minimum cardinality of a non-split distance -2 dominating set.

The minimal non-split distance -2 dominating set in a graph G is a non-split distance -2 dominating set that contains no non-split distance -2 dominating set as a proper subset.

The distance -2 open neighborhood of a vertex $v \in V$ is the set, $N_{\leq 2}(v)$ of vertices within a distance of two of (v).

Example: 1.2

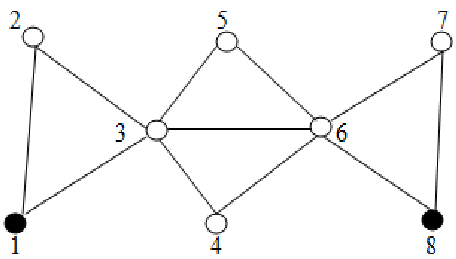

Figure.1

Here $D=\{1,8\}$, $\gamma_{ns\le 2}(G)=2$

2. EXACT VALUES OF $\gamma_{ns\le 2}(G)$ FOR SOME STANDARD GRAPHS.

2.1: **Observation:**

1. For any path P_n, for n $\geq$ 7

$$\gamma_{ns\le 2}(P_n)=n-4$$

2. For any cycle C_n, for n $\geq$ 5

$$\gamma_{ns\le 2}(C_n)=n-4$$

3. For any wheel graph W_n, for n $\geq$ 3

$$\gamma_{ns\le 2}(W_n)=1$$

4. For any friendship graph F_n, for n $\geq$ 2

$$\gamma_{ns\le 2}(F_n)=1$$

5. For any complete graph K_n, for n $\geq$ 3

$$\gamma_{ns\le 2}(K_n)=1$$

6. For any star graph $K_{1,m}$, for m $\geq$1

$$\gamma_{ns\le 2}(K_{1,m})=1$$

7. For any complete bipartite graph $K_{n,m}$, for m $\geq$ n,

$$\gamma_{ns\le 2}(K_{n,m})=1$$

8. For any Book graph B_n, for n $\geq$3

$$\gamma_{ns\le 2}(B_n)=1$$

9. For any helm graph H_n, for n $\geq$3

$$\gamma_{ns\le 2}(H_n)=1$$

3. BOUNDS ON THE NON-SPLIT DISTANCE -2 DOMINATION NUMBER $\gamma_{ns\leq 2}(G)$

Theorem 3.1

For any graph G, $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$

Proof

Every non-split distance -2 dominating set of G is a distance -2 dominating set of G,

We have $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$

Note 3.2

For any graph G, $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$

Theorem 3.3

For any graph G, $\gamma_{ns\leq 2}(G) \leq \gamma_{ns}(G)$

Proof

Every non-split dominating set of G is a non-split distance -2 dominating set of G,

We have $\gamma_{ns\leq 2}(G) \leq \gamma_{ns}(G)$

Theorem 3.4

For any graph G, $\gamma_{\leq 2}(G) \leq \min(\gamma_{ns\leq 2}(G), \gamma_{s\leq 2}(G))$

Proof

Every non-split distance -2 dominating set and every split distance -2 dominating set of G is a distance -2 dominating set of G,

We have $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$, $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$

Hence $\gamma_{\leq 2}(G) \leq \min(\gamma_{ns\leq 2}(G), \gamma_{s\leq 2}(G))$

Proposition 3.5

For any graph G, $\gamma(G) = \gamma_{ns}(G) = \gamma_{\leq 2}(G) = \gamma_{ns\leq 2}(G)$ if and only if G is a wheel graph W_n.

Proposition 3.6

For any graph G, $\gamma_{ns\leq 2}(G) = \gamma_{s\leq 2}(G) = \gamma_s(G) = \gamma(G) = \gamma_{\leq 2}(G)$ if and only if G is a star graph $K_{1,m}$, for m > 1.

Proposition 3.7

For any graph G, $\gamma_{ns\le 2}(G)=\gamma_{s\le 2}(G)=\gamma_{s}(G)=\gamma(G)=\gamma_{\le 2}(G)$ if and only if G is a friendship graph Fn.

Proposition 3.8

For any graph G, $\gamma_{\le 2}(G)=\gamma_{ns\le 2}(G)$ if and only if G is a bipartite graph $K_{n, m}$, for n< m.

Proposition 3.9

For any graph G, $\gamma_{\le 2}(G)=\gamma_{ns\le 2}(G)=\gamma_{ns}(G)=\gamma(G)$ if and only if G is a complete graph $K_{n,}$ for n > 2.

Theorem 3.10

A non-split distance -2 dominating set D of G is minimal if and only if for each vertex $v\in D$, one of the following conditions is satisfied.

(i) There exists a vertex $u\in V-D$ such that

Case (a): $N_{\le 2}(u)\cap D=D$ when D is connected.

Case (b): $N_{\le 2}(u)\cap D=\{v\}$ when D is disconnected.

(ii) v is an isolated vertex in <D>

(iii) $N_{\le 2}(u)\cap(V-D)\ne\emptyset$

Proof

Suppose D is a minimal non-split distance dominating set of G.

Suppose the contrary.

That is, if there exists a vertex $v\in D,$ such that v does not satisfy any of the given conditions, then by theorem given by Kulli V.R and Janakiram B.(1997), there exists a distance -2 dominating set D_1=D- $\{v\}$ such that the induced sub graph <V-D_1> is connected. This implies that D_1 is a non-split distance -2 dominating set of G contradicting the minimality of D. Therefore, the condition is necessary.

Sufficiency follows from the given conditions.

Theorem 3.11

If H is a connected spanning sub graph of G, then $\gamma_{ns\le 2}(G)\le\gamma_{ns\le 2}(H)$

Theorem 3.12

For any graph G, $\gamma_{ns\le 2}(G)=p-\Delta(G)$ if and only if G is a star graph $K_{1, m}$, for m>

1,where p is number of vertices.

Theorem 3.13

For any graph G, $\gamma_{ns\leq 2}(G) = \delta(G)$ if and only if G is a helm graph Hn.

Theorem 3.14

For any graph G, which is not a tree then $\gamma_{ns\leq 2}(G) \leq c(G)$ where $c(G)$ is circumference of a graph G.

Theorem 3.15

For any graph G, which is not a tree then $\gamma_{ns\leq 2}(G) \leq g(G)$ where $g(G)$ is girth of a graph G.

Theorem 3.16

Let G be any connected graph of order greater than or equal to 3, then $\gamma_{ns\leq 2}(G) \leq n - 3$, where n is the number of vertices.

Proof

Since G is connected, there is a spanning tree T of G with (n-1) vertices. If v is a pendant vertex of T then (n-3) vertices of T other than v form a minimal non-split distance -2 dominating set of G, hence $\gamma_{ns\leq 2}(G) \leq n - 3$.

Nordhas - Gaddum Type results

Theorem 3.17

Let G be a graph such that both G and $\bar{G}$ have no isolates. Then,

(i) $\gamma_{ns\leq 2}(G) + \gamma_{ns\leq 2}(\bar{G}) \leq 2(n-3)$

(ii) $\gamma_{ns\leq 2}(G).\gamma_{ns\leq 2}(\bar{G}) \leq (n-3)^2$

Proof

The results follow from Theorem 3.16

Theorem 3.18

For any graph G, $\gamma_{ns\leq 2}(G) \leq n - \Delta(G)$

Theorem 3.19

For any tree T_n, $\gamma_{ns\leq 2}(G) \leq n - p$ where n is number of vertices and p is number of end vertices.

Note 3.20

For any tree T_n, which is a star graph $\gamma_{ns\leq 2}(G) = n - p$ where n is number of vertices and p is number of end vertices.

Theorem 3.21 (Kulli and Janakiram, 1997)

For any graph G, $\gamma_s(G) \leq \alpha_0(G)$

Theorem 3.22

For any graph G, $\gamma_{ns\leq 2}(G) \leq \alpha_0(G)$

Proof

Since $\gamma_{ns\leq 2}(G) \leq \gamma_{ns}(G)$ and $\gamma_{ns}(G) \leq \alpha_0(G)$ [By Theorem 3.3]

We have $\gamma_{ns\leq 2}(G) \leq \alpha_0(G)$

Theorem 3.23

For any graph G, $\gamma_{\leq 2}(G) + \gamma_{ns\leq 2}(G) \leq n$

Proof

Since $\gamma(G) \leq \beta_0(G)$, $\gamma_{\leq 2}(G) \leq \gamma(G)$ and $\gamma_{ns\leq 2}(G) \leq \alpha_0(G)$ [By Theorem 3.22]

Thus $\gamma_{\leq 2}(G) + \gamma_{ns\leq 2}(G) \leq \alpha_0(G) + \beta_0(G)$

We have $\gamma_{\leq 2}(G) + \gamma_{ns\leq 2}(G) \leq n$

Theorem 3.24

For any graph G, $i(G) + \gamma_{ns\leq 2}(G) \leq n$

Proof

Since $i(G) \leq \beta_0(G)$, and $\gamma_{ns}(G) \leq \alpha_0(G)$ [By Theorem 3.22]

Thus $i(G) + \gamma_{ns\leq 2}(G) \leq \alpha_0(G) + \beta_0(G)$

We have $i(G) + \gamma_{ns\leq 2}(G) \leq n$

Lemma 3.24

For $k \geq 1$, every connected graph G has a spanning tree T such that $\gamma_k(G) = \gamma_k(T)$

in [24]

Lemma 3.25

For $k \geq 1$, every connected graph G has a spanning tree T such that $\gamma_{ns\leq 2}(G) = \gamma_{ns\leq 2}(T)$

Proof

Since $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$ and $\gamma_{\leq 2}(T) = \gamma_{\leq 2}(G)$ [By Theorem 3.1 and Lemma 3.25]

We have $\gamma_{ns\leq 2}(G) = \gamma_{ns\leq 2}(T)$

Theorem 3.26

For $k \geq 1$, if G is a connected graph with diameter d, then $\gamma_k(G) \geq \frac{d+1}{2k+1}$ in [24]

Theorem 3.27

For any graph G is a connected graph with diameter d, then $\gamma_{ns\leq 2}(G) \geq \frac{d+1}{2k+1}$,where k=2.

Proof

Since $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$ and $\gamma_{\leq 2}(G) \geq \dfrac{d+1}{2k+1}$ [By Theorem 3.26]

We have $\gamma_{ns\leq 2}(G) \geq \frac{d+1}{5}$

Theorem 3.28

If G=P_n where $n \equiv 0\ mod(2k+1)$, then $\gamma_k(G) = \frac{diam(G)+1}{2k+1}$ in [24]

Theorem 3.29

If G=P_n where $n \equiv 0\ mod(2k+1)$, then $\gamma_{ns\leq 2}(G) = \frac{diam(G)+1}{2k+1}$ where k =2.

Proof

Since $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$ and $\gamma_{\leq 2}(G) = \dfrac{diam(G)+1}{2k+1}$ [By Theorem3.28]

We have $\gamma_{ns\leq 2}(G) \geq \frac{diam(G)+1}{5}$

Theorem 3.30

For any graph G is a connected graph with radius r, then $\gamma_k(G) \geq \frac{2r}{2k+1}$ in [24]

Theorem 3.31

For any graph G is a connected graph with radius r, then $\gamma_{ns\leq 2}(G) \geq \frac{2r}{2k+1}$, where k = 2.

Proof

Since $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$ and $\gamma_{\leq 2}(G) \geq \dfrac{2r}{2k+1}$ [By Theorem 3.30]

We have $\gamma_{ns\leq 2}(G) \geq \frac{2r}{5}$

Theorem 3.32

For any graph G is a connected graph with girth g, then$\gamma_k(G) \geq \frac{g}{2k+1}$ in [24]

Theorem 3.33

For any graph G is a connected graph with girth g, then $\gamma_{ns\leq 2}(G) \geq \frac{g}{2k+1}$, where k =2.

Proof

Since $\gamma_{\leq 2}(G) \leq \gamma_{ns\leq 2}(G)$ and $\gamma_{\leq 2}(G) \geq \dfrac{g}{2k+1}$ [By Theorem 3.33]

We have $\gamma_{ns\leq 2}(G) \geq \frac{g}{5}$

CONCLUSION

In the communication network, n cities are linked via communication channels. A transmitting group is a subset of those cities that are able to transmit messages to every city in the network. Such a transmitting group is nothing else than a non-split dominating set in the network graph, and non-split distance –2 domination number of this graph is the minimum number of disjoints transmitting groups in the network.

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