International Journal of Mathematics Research.
ISSN 0976-5840 Volume 9, Number 2 (2017), pp. 125-134

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New Results for Fuzzy Generalized Continuous Functions

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Abstract

In this paper, a new class of functions called fuzzy continuous function, fuzzy generalized continuous function and fuzzy δg-irresolute mappings has been defined and its properties are investigated. Examples and counter examples are given.

Keywords and phrases: Fuzzy δ-continuity, fuzzy δ-generalized continuity, fuzzy δg-irresolute mappings, fuzzy topological space, fuzzy generalized closed set, fuzzy δ-generalized closed set, fuzzy continuous function, fuzzy generalized continuous function, fuzzy δg-irresolute mappings.

Mathematics subject classification: 54A40

1. INTRODUCTION

Zadeh [6] introduced the fundamental concept of fuzzy sets. The concept of fuzzy topological spaces was introduced in [5]. The concept of extension of fuzzy topological spaces introduced by [2]. And the generalized fuzzy continuous functions was introduced by [3].

Let (X, τ) be a fuzzy topological space and $\tau \subset \tau^*$ then τ^* will be called a simple extension of τ[2] if there exists of $\delta \notin \tau$ such that

$$\tau^* = \{\lambda \vee (\mu \wedge \delta)/\lambda, \mu \in \tau\}$$

In this case, we write $\tau^* = \tau(\delta)$. In this paper, we introduce a new definitions of fuzzy δ- continuous function, fuzzy δ- closed set, fuzzy δg- continuous functions. Throughout this paper X and Y represents the fuzzy topological spaces (X, τ) and (Y, σ).

2. PRELIMINARIES

A fuzzy topology τ [5] on X is a collection of subsets of τ such that

(i) $0, 1 \in \tau$ (or $o_x, 1_x \in \tau$)

(ii) If $\lambda, \mu \in \tau$ then $\lambda \vee \mu \in \tau$

(iii) $\lambda_i \in \tau$ for each $i \in \tau$ then $\vee \lambda_i \in \tau$

The ordered pair (X, τ) is called a fuzzy topological space (in short fts) and members of τ are called τ-fuzzy open sets or simply fuzzy open sets.

A fuzzy set λ in a fuzzy topological space is called a fuzzy closed set if its complement $(1-\lambda)$ is fuzzy open set. Cl (λ) denotes the closure of λ and is given by

$$\text{Cl}(\lambda) = \wedge\{\mu/\mu \text{ is fuzzy closed and } \mu \geq \lambda\}.$$

Int (λ) denotes the interior of λ and is given by

$$\text{Int}(\lambda) = \vee\{\mu/\mu \text{ is fuzzy open and } \mu \leq \lambda\}$$

Definition: 2.1

A fuzzy set λ in a fts (X, τ) is called

(i) fuzzy semi open [1] if $\lambda \leq \text{Cl}(\text{Int}(\lambda))$.

(ii) fuzzy pre open [4] if $\lambda \leq \text{Int}(\text{Cl}(\lambda))$.

(iii) fuzzy α open [4] if $\lambda \leq \text{Int}(\text{Cl}(\text{Int}(\lambda)))$.

(iv) fuzzy generalized closed (Briefly,fg-closed) [1] if $\text{Cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open in (X, τ).

Definition: 2.2

A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be

(i) fuzzy continuous [5] if $f^{-1}(\lambda)$ is fuzzy open set in (X, τ), for each fuzzy open set λ in (Y, σ).

(ii) fuzzy pre Continuous [4] if $f^{-1}(\lambda)$ is fuzzy pre open set in (X, τ), for each fuzzy open set λ in (Y, σ).

(iii) fuzzy α-continuous [4] if $f^{-1}(\lambda)$ is fuzzy α-open set in (X, τ), for each fuzzy open set λ in (Y, σ) .

(iv) fuzzy g-continuous [3] if $f^{-1}(\lambda)$ is fg-closed set in (X, τ), for each fuzzy closed set λ in (Y, σ) .

3. NEW FORMS OF FUZZY CONTINUITY BY SUITABLE CHOICE OF δ

Definition: 3.1

Let $f : (X, \tau) \to (Y,\sigma)$ and δ be a non fuzzy open set of (X, τ) then f is called a fuzzy δ- continuous function, if $f^{-1}(\lambda)$ is an fuzzy open set in (X, τ^*) for every fuzzy open set λ in (Y,σ).

Preposition: 3.1

Let $f:(X, \tau) \to (Y, \sigma)$ be a fuzzy continuous function and let δ be a non- fuzzy open subset in (Y, σ) if $f^{-1}(\delta)$ is a fuzzy open set of (X, τ) then $f : (X, \tau) \to (Y, \sigma^*)$ is a fuzzy continuous function.

Proof:

Let $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy continuous function and $\delta \notin \sigma$ be a non fuzzy open subset in (Y, σ) and let $\sigma \subset \sigma^*$ .Let β be a fuzzy open set in (Y, σ^*) Then $\beta = \lambda\vee(\mu \wedge \delta)$ where λ and μ are fuzzy open sets in (Y, σ).

$$f^{-1}(\beta) = f^{-1}(\lambda \vee (\mu \wedge \delta))$$

$$= f^{-1}(\lambda) \vee (f^{-1}(\mu) \wedge f^{-1}(\delta))$$

Since f is a fuzzy continuous function and assumption that $f^{-1}(\delta)$ is a fuzzy open set of (X, τ) then $f^{-1}(\beta)$ is fuzzy open subset of (X, τ) which implies that

$f: (X, \tau) \to (Y, \sigma^*)$ is a fuzzy continuous function.

Preposition: 3.2

Let $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy continuous function and $\delta \notin \tau$ be a non fuzzy set in (X, τ) then $f : (X, \tau^*) \to (Y, \sigma)$ is a fuzzy continuous function.

Proof:

Let $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy continuous function and $\delta \notin \tau$ be a non fuzzy open subset in (X, τ). Let $\tau \subset \tau^*$ and Consider $f : (X, \tau^*) \to (Y, \sigma)$. Then this function f is fuzzy continuous function, since every fuzzy open set in (X, τ) is an fuzzy open set in (X, τ^*) and since $f : (X, \tau) \to (Y,\sigma)$ is a fuzzy continuous functions.

Preposition: 3.3

Let $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy continuous function and δ_1 be a non fuzzy open subset in (X, τ) and δ_2 be a non fuzzy open subset in (Y, σ), if $f^{-1}(\delta_2)$ is a fuzzy open set in $(X, \tau(\delta_1))$, then $f: (X, \tau(\delta_1)) \to (Y,\sigma(\delta_2))$ is a fuzzy continuous function.

Proof

Let $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy continuous function and $\delta \notin \tau$ be a non fuzzy open set in (X, τ) and let $\tau \subset \tau^*$. Consider $f : (X, \tau(\delta_1)) \to (Y,\sigma)$ since every fuzzy open set in (X, τ) is an fuzzy open set in $(X, \tau(\delta_1))$ and since $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy continuous function. Then $f : (X, \tau(\delta_1)) \to (Y,\sigma)$ is a fuzzy continuous function.

Let $\delta_2 \notin \sigma$ be a non fuzzy open subset in (Y, σ) Let $\sigma \subset \sigma^*$, where σ^* be a simple extension of σ if and only if there exist $\delta_2 \notin \sigma$ such that $\sigma^* = \{\lambda\vee(\mu\wedge\delta)/\lambda, \mu\in\sigma\}$. In this case, we write $\sigma^* = \sigma(\delta_2)$. Let β be an fuzzy open set in $(Y, \sigma(\delta_2))$, then $\beta= \lambda\vee(\mu \wedge \delta_2)$ where λ and μ are fuzzy open set in (Y, σ)

$$f^{-1}(\beta) = f^{-1}(\lambda\vee(\mu\wedge\delta_2))$$

$$= f^{-1}(\lambda)\vee(f^{-1}(\mu)\wedge f^{-1}(\delta_2))$$

Since f is a fuzzy continuous function and the assumption that $f^{-1}(\delta_2)$is an fuzzy open set in $(X, \tau(\delta_1))$, $f^{-1}(\beta)$ is a in fuzzy open set in $(X, \tau(\delta_1))$ then $f : (X, \tau(\delta_1)) \to (Y,\sigma((\delta_2))$ is a fuzzy continuous function.

Preposition : 3.4

Every fuzzy continuous function is a fuzzy δ- continuous function.

Proof:

Let $f : (X, \tau) \to (Y,\sigma)$ is a fuzzy continuous function then $f^{-1}(\lambda)$ is a fuzzy open set in (X, τ) for every fuzzy open set λ in (Y, σ). Let us consider $\delta \notin \tau$ be a non fuzzy open subset in (X, τ), then consider $\tau \subset \tau^*$. Consider $f : (X, \tau^*) \to (Y,\sigma)$. Since $f : (X, \tau) \to (Y, \sigma)$ is a fuzzy continuous function and every fuzzy open set in (X, τ) is a fuzzy open set in (X, τ^*) , then $f^{-1}(\lambda)$ is an fuzzy open set in (X, τ^*).

Hence $f : (X, \tau^*) \to (Y,\sigma)$ is a fuzzy δ- continuous function.

Remark :3.1

The converse of preposition 3.2 is not always true as shown by the following example.

Example :3.1

Let $X = \{a, b, c\}$, $\tau = \{1, 0, \lambda\}$ where $\lambda: X \to [0, 1]$ is defined by $\lambda(a) = 0$, $\lambda(b) = 0$, $\lambda(c) =1$. Then (X, τ) is a fuzzy topological space. Let δ be a non fuzzy open set in (X, τ) where $\delta : X \to [0,1]$ is defined by $\delta(a) = 0$, $\delta(b) = 1$, $\delta(c) = 0$ then $\tau(\delta) = \{0, 1, \lambda, \delta, \lambda\vee\delta\}$ be the fuzzy topology on X and let $Y= \{a,b,c\}$, $\sigma = \{1, 0, \lambda\}$ is the fuzzy topology on Y, where $\lambda(a) = 0$, $\lambda(b) = 0$, $\lambda(c) = 1$.

If $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = a$, $f(b) = c$, $f(c) = b$ then $f^{-1}(\lambda) = \delta$ since δ is a fuzzy open set in (X, τ^*). Then f is a fuzzy δ-continuous function , but not a fuzzy continuous function.

4. NEW FORMS OF FUZZY GENERALIZED CONTINUITY BY SUITABLE CHOICE OF δ

Definition: 4.1

A fuzzy subset λ of a space (X, τ) is said to be fuzzy δ- generalized closed set (Briefly, fuzzy δg-closed) if $\delta Cl(\lambda) \leq \beta$ whenever $\lambda \leq \beta$ and β is fuzzy open in (X, τ) where $\delta Cl(\lambda)$ is given by $\delta Cl(\lambda) =\wedge\{\gamma \leq 1:\lambda\leq \gamma$, whenever γ is a fuzzy closed set $\tau^*\}$.

A fuzzy subset of X belonging to τ^* is denoted by fuzzy δ- open set, the complement of fuzzy δ-open set is denoted by fuzzy δ- closed set .

The family of all fuzzy δ- open set is denoted by FδO(X) and the family of all δ-fuzzy closed sets is denoted by FδC(X).

Definition: 4.2

A function f : (X, τ) $\rightarrow$ (Y, σ) is called a fuzzy δ-continuous function if $f^{-1}(\lambda)$ is a δ-fuzzy closed set in (X, τ), for every closed set λ in (Y,σ).

Definition: 4.3

A function f : (X, τ) $\rightarrow$ (Y, σ) is called δg-fuzzy δg- continuous function if $f^{-1}(\lambda)$ is a fuzzy δg- closed set in (X, τ), for every closed set λ in (Y, σ).

Preposition : 4.4

For a fuzzy subset of a space (X, τ) and a function f : (X, τ) $\rightarrow$ (Y, σ) from the definition stated above, we have the following diagram of implications.

$$\begin{array}{ccc} \text{fuzzy continuous function} & \xrightarrow{(3)} & \text{g-fuzzy continuous function} \\ (1)\downarrow & & \downarrow(4) \\ \text{fuzzy } \delta\text{-continuous function} & \xrightarrow{(2)} & \text{fuzzy } \delta\text{g- continuous function} \end{array}$$

Proof

1) Since f is a fuzzy continuous function then $f^{-1}(\lambda)$ is a fuzzy open set in (X, τ) for every fuzzy open set λ in (Y, σ) but every fuzzy open set in (X, τ) is a fuzzy open set in (X, τ*) => $f^{-1}(\lambda)$ is a fuzzy δ- open set in (X, τ*) for every fuzzy open set λ in (Y, σ) => f is a fuzzy δ-continuous function.

2) Since f is a δ-fuzzy continuous function then $f^{-1}(\lambda)$ is a fuzzy δ-closed set in (X, τ) for every fuzzy closed set λ in (Y, σ). But every fuzzy δ- closed set in (X, τ) is a fuzzy δg-closed set in (X,τ)=> $f^{-1}(\lambda)$ is a fuzzy δg- closed set in (X, τ) for every fuzzy closed set λ in (Y, σ) => f is a fuzzy δg-continuous function.

3) Since f is a fuzzy continuous function then $f^{-1}(\lambda)$ is a fuzzy closed set in (X,τ) for every fuzzy closed set λ in (Y,σ) But every fuzzy closed set in (X,τ) is a g-fuzzy closed set in (X,τ) then $f^{-1}(\lambda)$ is a g-fuzzy closed set in (X, τ) for every fuzzy closed set λ in (Y, σ) => f is a g-fuzzy continuous function.

4) Since f is g-fuzzy continuous function then $f^{-1}(\lambda)$ is a g-fuzzy closed set in

(X,τ) is a fuzzy δ-closed set in (X,τ*) => every fuzzy δ-closed set is a δg-fuzzy closed set => $f^{-1}(\lambda)$ is a δg-fuzzy closed set in (X, τ*) for every fuzzy closed set λ in (Y, σ) => f is a fuzzy δg- continuous function.

Example : 4.6

Let X = Y = {a, b, c} Define fuzzy sets λ, δ, β: X = Y$\rightarrow$ [0, 1] by the equation λ(a) = 0.5, λ(b) = 0, λ(c) = 0

δ(a) = 0, δ(b) = 0.6, δ(c) = 0, and

β(a) = 0.6, β(b) = 0.6, β(c) = 1 then

τ = {1, 0, λ , β} is a fuzzy topology on X and σ = {1, 0, δ} is a fuzzy topology on Y. Let δ be the non fuzzy open set in (X, τ) then τ (δ) ={1, 0, λ, δ, $\lambda\vee\delta$}. Let λ_1(a) = 0.4, λ_1(b) = 0, λ_1(c) = 0 be the fuzzy subset in (X, τ). If f : (X, τ) $\rightarrow$ (Y, σ) defined by,

a) f (a) = a, f (b)= b and f (c) = c then f is a fuzzy δ-continuous function but not a fuzzy continuous function.
b) f (a) = f (c) = b, and f (b) = a then f is a fuzzy g-continuous function but not a fuzzy continuous function.

5. ON FUZZY δg-CONTINUOUS FUNCTION

Definition: 5.1

A mapping f:(X,τ)$\rightarrow$(Y,σ) is said to be fuzzy δ-irresolute (briefly, fδ-irresolute) if $f^{-1}(\lambda)$ fuzzy δ-closed set in X, for every closed set λ in Y.

Definition : 5.2

A mapping f:(X,τ)$\rightarrow$(Y,σ) is said to be fuzzy δ-generalized irresolute (briefly, fδg-irresolute) if $f^{-1}(\lambda)$ is fδg-closed set in X, for every δg-closed set λ in Y.

Theorem : 5.1

Every fδg-irresolute mapping is fδg-continuous.

Proof:

Let f : (X, τ) $\rightarrow$ (Y, σ) is fδg-irresolute. Let λ be a fuzzy closed set in Y. Then λ is fδg-closed fuzzy set in Y. Since f is fδg-irresolute. $f^{-1}(\lambda)$ is fδg-closed set in X. Hence f is fδg-continuous.

Remark : 5.1

However the converse of the above theorem need not be true as seen from the following example.

Example : 5.1

Let X = Y = {a, b, c}. Define fuzzy sets λ, δ_1, δ_2 : X $\rightarrow$ [0, 1] by the equation , λ (a) = 0.5, λ (b) = 0, λ (c) = 1,

$\delta_1(a) = 0$, $\delta_1(b) = 0.6$, $\delta_1(c) = 0$,

$\delta_2(a) = 0$, $\delta_2(b) = 0$, $\delta_2(c) = 0.7$

and γ : Y $\rightarrow$ [0, 1] defined by $\gamma(a) =1$, $\gamma(b) =0.5$, $\gamma(c) =0$.

$\tau = \{1, 0, \lambda\}$ is a fuzzy topology on (X, τ). $\sigma = \{1, 0, \gamma\}$ is a fuzzy topology on (Y, σ). Let δ_1 be the non fuzzy open set in (X, τ), then $\tau(\delta_1) = \{1, 0, \lambda, \delta_1, \lambda\vee\delta_1\}$ and δ_2 be the non fuzzy open set in (Y,σ), then $\sigma(\delta_2)=\{1, 0, \gamma, \delta_2, \gamma\vee\delta_2\}$. Let f : (X, τ) $\rightarrow$ (Y, σ) defined by f (a) = b, f (b) = a, f (c) = c, then f is fuzzy δg-continuous function. But f is not fδg-irresolute.

Definition:5.3

A mapping f : (X, τ) $\rightarrow$ (Y, σ) is said to be fuzzy δ-closed mappings if f(λ) is fuzzy δ-closed in (Y, σ), for every fuzzy closed set λ in X.

Definition:5.4

A mappings f : (X, τ) $\rightarrow$ (Y, σ) is said to be fuzzy δg-closed mappings if f(λ) is fuzzy δg-closed in (Y, σ), for every fuzzy closed set λ in X.

Definition:5.5

If every fuzzy δg-closed in X is fδ-closed in X,then the space can be denoted as $f\delta T_{1/2}$-space.

Theorem:5.2

A fuzzy topological space (X, τ) is $f\delta T_{1/2}$-space if and only if $F\delta O(X, \tau) = FG\delta O(X, \tau)$.

Proof:(Necessity)

Let (X, τ) be $f\delta T_{1/2}$-space.Let $\lambda \in FG\delta O(X, \tau)$, then $1-\lambda$ is a $f\delta g$-closed set.Thus $\lambda \in F\delta O(X, \tau)$. Hence $F\delta O(X, \tau) = FG\delta O(X, \tau)$.

(sufficiency)

Let $F\delta O(X, \tau) = FG\delta O(X, \tau)$. Let λ is a $f\delta g$-closed.Then $1-\lambda$ is a $f\delta g$-open.Hence $1-\lambda \in F\delta O(X, \tau)$.Thus λ is a $f\delta$-closed set.Therefore (X, τ) is a $f\delta T_{1/2}$-space.

Theorem:5.3

Let $f : (X, \tau) \to (Y, \sigma)$ be $f\delta g$-continuous.Then f is fuzzy δ-continuous if (X, τ) is $f\delta T_{1/2}$-space.

Proof:

Let λ be a fuzzy closed of (Y, σ). Since f is $f\delta g$-continuous, $f^{-1}(\lambda)$ is $f\delta g$-closed set of (X, τ). Again (X, τ) is a $f\delta T_{1/2}$-space and hence $f^{-1}(\lambda)$ is a $f\delta$-closed set of (X, τ).This implies that f is fuzzy δ-continuous.

Example:5.2

Let $X = Y = \{a, b, c\}$. Define fuzzy sets $\lambda, \delta : X \to [0, 1]$ by the equation ,

$\lambda(a) = 0.5, \quad \lambda(b) = 0, \quad \lambda(c) = 1,$

$\delta(a) = 0, \quad \delta(b) = 0.6, \quad \delta(c) = 0,$

and $\gamma : Y \to [0, 1]$ defined by $\gamma(a) = 1$, $\gamma(b) = 0.5$, $\gamma(c) = 1$.

$\tau = \{1, 0, \lambda\}$ is a fuzzy topology on (X, τ). $\sigma = \{1, 0, \gamma\}$ is a fuzzy topology on (Y, σ). Let δ be the non fuzzy open set in (X, τ), then $\tau(\delta) = \{1, 0, \lambda, \delta, \lambda \vee \delta\}$. Let $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = b$, $f(b) = a$, $f(c) = c$, then f is fuzzy δg-continuous function. But λ is not $f\delta T_{1/2}$-space as λ is not $f\delta$-closed set of (X, τ).Then f is not fuzzy δ-continuous.

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