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Sum of Polynomial Function

Monishwaran Maheswaran

STUDENT-FIITJEE PU COLLEGE

Bangalore

Abstract

We obtain an integral value for all the sums of the form $\sum_{x=1}^{\infty}\frac{p(x)}{2^x}$, where x ranges from 1 to ∞. $p(x)$ is a polynomial of degree n for $n \in$ W and the coefficients of The polynomial ∈ I.

Keywords: Polynomial, integer, Geometric progression

1 INTRODUCTION

The summation of a polynomial

$$\sum_{x=1}^{\infty}\frac{p(x)}{2^x}$$

Where x ranges from one to infinity. We observe that this sum always outputs an integral value. This summation of

$$p(x) = ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + ex^{n-4} \ldots ....$$

For the coefficients a, b, c, d, e belonging to integers over 2^x always converges by the $ratio\ test$.

$$\sum_{x=1}^{\infty}\frac{ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + ex^{n-4} \ldots \ldots .}{2^x}$$

2 REQUIRED IDENTITIES

Telescoping the summation identities:

→1.1 For constant *c* and sequences *a1, a2, ..., an* and *b1, b2, ..., bn*

$$\sum_{k=1}^{n}(ca_k + b_k) = c\sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

→1.2 Infinite sum can be expressed as

$$\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty}\sum_{k=1}^{n} a_k$$

→1.3 Geometric or exponential series for real x $\neq$1

$$\sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n \qquad \sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$$

→1.4 Infinite decreasing geometric series , |x|<1 has values of:

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

3 MAIN RESULTS:

Theorem :

For any function of the form $\sum_{x=1}^{\infty}\frac{p(x)}{2^x}$ where the polynomial $p(x)$ has integral coefficients and has degree $n \in W$. Such a function results in an integer.

$$\sum_{x=1}^{\infty}\frac{p(x)}{2^x} = I$$

Proof:

$$p(x) = ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + ex^{n-4} \dots\dots$$

By substituting $p(x)$

$$\sum_{x=1}^{\infty}\frac{p(x)}{2^x} = \sum_{x=1}^{\infty}\frac{ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + ex^{n-4}\dots\dots}{2^x}$$

and making use of identity 1.1

$$\sum_{x=1}^{\infty}\frac{ax^n}{2^x}+\sum_{x=1}^{\infty}\frac{bx^{n-1}}{2^x}+\sum_{x=1}^{\infty}\frac{cx^{n-2}}{2^x}+\sum_{x=1}^{\infty}\frac{dx^{n-3}}{2^x}+\sum_{x=1}^{\infty}\frac{ex^{n-4}}{2^x}\dots\dots\dots$$

Further simplifying

$$a\sum_{x=1}^{\infty}\frac{x^n}{2^x}+b\sum_{x=1}^{\infty}\frac{x^{n-1}}{2^x}+c\sum_{x=1}^{\infty}\frac{x^{n-2}}{2^x}+d\sum_{x=1}^{\infty}\frac{x^{n-3}}{2^x}+e\sum_{x=1}^{\infty}\frac{x^{n-4}}{2^x}\dots\dots\dots$$

For integer coefficients for the polynomial that is $a, b, c, d, e, a_n, a_0 .. \in I$

$$\sum_{x=1}^{\infty}(a_0 2^{-x}+ax2^{-x}+bx^2 2^{-x}+cx^3 2^{-x}+dx^4 2^{-x}\dots\dots\dots.a_n x^n)$$

$\sum_{x=1}^{\infty} a_0 2^{-x}+\sum_{x=1}^{\infty} ax2^{-x}+\sum_{x=1}^{\infty} bx^2 2^{-x}+\sum_{x=1}^{\infty} cx^3 2^{-x}+\sum_{x=1}^{\infty} dx^4 2^{-x}\dots.$

The first term converges to give an integer.

$\sum_{x=1}^{\infty} a_0 2^{-x}$ since a_0 is an integer constant and 2^{-x} converges to 1.

$$\sum_{x=1}^{\infty} a_0 2^{-x}$$

Using identity 1.1 and simplifying, we obtain

$$a_0\sum_{x=1}^{\infty} 2^{-x}$$

$$\sum_{x=1}^{\infty} a_0 2^{-x}=a_0$$

The second term converges to 2 by the following method

$$S=\sum_{x=1}^{\infty}\frac{x}{2^x}=\sum_{x=0}^{\infty}\frac{x}{2^x}$$

$$S\quad=\sum_{x=1}^{\infty}\frac{(x-1)}{2^{(x-1)}}=2\sum_{x=1}^{\infty}\frac{(x-1)}{2^x}$$

$$S-\frac{S}{2}=\frac{1}{2}S=\sum_{x=1}^{\infty}\frac{x}{2^x}-\sum_{x=1}^{\infty}\frac{(x-1)}{2^x}$$

We observe that $\frac{1}{2^x}$ is a geometric progression that converges to one by making use of identitiy 1.4 $\frac{2^{-1}}{1-2^{-1}}=1$

the function becomes

$$S = 2\sum_{x=1}^{\infty}\frac{(1)}{2^x} = 2$$

The third term converges in a similar manner.

$$S = \sum\nolimits_{x=1}^{\infty}\frac{x^2}{2^x} = \sum_{x=0}^{\infty}\frac{x^2}{2^x}$$

$$S \quad = \sum_{x=1}^{\infty}\frac{(x-1)^2}{2^{(x-1)}} = 2\sum_{x=1}^{\infty}\frac{(x-1)^2}{2^x}$$

$$S - \frac{S}{2} = \frac{1}{2}S = \sum_{x=1}^{\infty}\frac{x^2}{2^x} - \sum_{x=1}^{\infty}\frac{(x-1)^2}{2^x}$$

$$S = 2\sum_{x=1}^{\infty}\frac{(2x-1)}{2^x}$$

Extract I term

$$S = 2[\frac{1}{2} + \sum_{x=2}^{\infty}\frac{(2x-1)}{2^x}]$$

Shift n

$$S = 1 + 2\sum_{x=1}^{\infty}\frac{(2x+1)}{2^{x+1}}$$

$$S = 1 + \sum_{x=1}^{\infty}\frac{(2x+1)}{2^x}$$

$$S = 2S - S$$

$$= 2[1 + \sum_{x=1}^{\infty}\frac{(2x+1)}{2^x}] - [2\sum_{x=1}^{\infty}\frac{(2x-1)}{2^x}]$$

$$= 2 + 2\sum_{x=1}^{\infty}\frac{(2x+1)}{2^x}] - [2\sum_{x=1}^{\infty}\frac{(2x-1)}{2^x}]$$

$$= 2 + 2\sum_{x=1}^{\infty}\frac{2}{2^x}$$

We observe that $\frac{1}{2^x}$ is a geometric progression that converges to one by making use of identitiy 1.4 $\frac{2^{-1}}{1-2^{-1}} = 1$

the function becomes

$$= 2 + 4\sum_{x=1}^{\infty} \frac{1}{2^x}$$

$$= 2 + 4 = 6$$

We obtain similar results for the remaining number of terms using the above method.

Hence, we can say that the summation of polynomial is always an integer.

$$\sum_{x=1}^{\infty} \frac{p(x)}{2^x} \in I$$

REFERENCES

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