

Calculus Method of Solving Quadratic Equations

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Abstract

This paper introduces a new application of differential calculus and integral calculus to solve various forms of quadratic equations.

Keywords: quadratic equations, differential calculus, integral calculus, infinitesimal integral calculus, constant of infinity.

I. INTRODUCTION

This paper is based on the usage of the principles of differential calculus and integral calculus to solve quadratic equations. It is hypothesized that if we use calculus to solve quadratic equations, we will arrive at the same solution that we may get from the quadratic formula method. It is also hypothesized that the quadratic formula can be derived from the quadratic formula.

The paper involves the use of differential and integral calculus which takes solving of quadratic equations to the next level. The formulas which we will come across the paper, or use them, are shown in Table 1 under Heading IV.

The paper shall be dealt under 2 headings. Heading II will deal with the use of differential calculus in solving quadratic equations while Heading III will deal with the use of integral calculus in quadratic equations, as far as solving of quadratic equations is concerned. Heading V is hugely concerned about the advantages of using calculus to solve quadratic equation.

II. USE OF DIFFERENTIAL CALCULUS TO SOLVE QUADRATIC EQUATIONS

To solve quadratic equations, we shall require the use of first order differential calculus only.

Let us start with an example.

Let $f(x) = x^2 - 5x + 6$. Then if we solve it either by the quadratic formula method or factorisation method, we will arrive at the answer that $x = 2$ or $x = 3$.

We know that the above equation is a quadratic of the general form $ax^2 + bx + c = 0$. Now if we differentiate the above equations with the help of first order differentiation, the derivation will come as follows:

$$ax^2 + bx + c = 0$$

$$\text{Let } f(x) \text{ be } ax^2 + bx + c = 0$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta f(x)}{x} \equiv \frac{d}{dx}(ax^2 + bx + c) = \frac{d}{dx}(0)$$

$$\frac{d}{dx}(ax^2 + bx + c) = \frac{d}{dx}(0) \equiv \frac{d}{dx}ax^2 + \frac{d}{dx}bx + \frac{d}{dx}c = \frac{d}{dx}0$$

$$\frac{d}{dx}ax^2 + \frac{d}{dx}bx + \frac{d}{dx}c = \frac{d}{dx}0 \equiv 2ax + b = 0$$

$$2ax + b = 0 \equiv 2ax = -b \text{ or } x = \frac{-b}{2a}$$

But the value of x is same as the variable $\frac{-b}{2a}$ in the quadratic formula. By hit and trial method, we found out that the other variable required for the solution of x had the value same as of the expression $\frac{\sqrt{D}}{2a}$ in the quadratic formula. Thus, we can use differential calculus to solve quadratic equations by treating the quadratic function as a first order derivative.

Now, the question arises that whether it will be useful to solve quadratic equations of other variations. The answer to this question is 'YES'. Check out this example.

Let the quadratic equation be $\frac{2^{x^2}}{2^x} = 1$

$$\text{Let } f(x) = \frac{2^{x^2}}{2^x} = 1$$

$$\frac{2^{x^2}}{2^x} = 1 \equiv 2^{x^2-x} = 2^0$$

$$2^{x^2-x} = 2^0 \equiv x^2 - x = 0$$

$$\frac{d}{dx}(x^2 - x) = \frac{d}{dx}(0) \equiv 2x - 1 = 0$$

$$2x - 1 = 0 \text{ or } x = \frac{1}{2}$$

$$\text{Discriminant } D = b^2 - 4ac \equiv 1^2 - 4 \times 1 \times 0 \equiv 1$$

Note in the above example that the value of the discriminant is zero and the initial value of $x=1$. Since $D=1$ and initial value of $x=1/2$ (or $-1/2$ in other cases), we can stop here and the solution to our quadratic equation is 1. This can be treated as a special case while solving quadratic equations with the help of calculus.

But let us remind you that in the above example, if we differentiate the equation in the beginning, we will arrive at a totally different answer which is wrong way of solving the given example by calculus method.

We can use this method in solving various quadratic equations. Also, it is helping us in doubly prove that the quadratic formula is universal in nature.

Thus, involving differential calculus in solving quadratic equations may sound a bit weird, but it is helping all of us to solve with much more ease without the need of a calculator by your side rather than using the quadratic formula which is pretty much cumbersome.

Thus, the following steps should be done to solve a quadratic equation with the help of differential calculus:

1. Differentiate both L.H.S as well as R.H.S of the quadratic equation.
2. Reduce it to linear equation.
3. Find initial value of the variable x .
4. Find out the value of the Discriminant of the quadratic equation and then calculate the value of the expression $\frac{\sqrt{D}}{2a}$
5. Find the actual value of variable x viz. the solution of quadratic equation by conducting the \pm function with the initial value of variable x and expression $\frac{\sqrt{D}}{2a}$.

III. USE OF INFINITESIMAL INTEGRAL CALCULUS TO SOLVE QUADRATIC EQUATIONS

Students and teachers have a general tendency to think that integral calculus is pretty much easier than differential calculus. But if we involve the use of infinitesimal integral calculus in solving quadratic equations, it will change their mind as it requires insanely

large amount of patience.

Let us take the same example that the quadratic equation is $x^2 - 5x + 6 = 0$. Then to solve the above quadratic equation with the help of infinitesimal integral calculus, the following steps should be done.

1. Integrate the L.H.S as well as R.H.S of the equation

Here, we will integrate the above equation without any limits, which means that the variable $\Delta x \rightarrow \infty$.

$$\begin{aligned} \lim_{\Delta x \rightarrow \infty} \Sigma f(x) \cdot \Delta x &\equiv \int f(x) \cdot dx \\ \int f(x) \cdot dx &\equiv \int (x^2 - 5x + 6) \cdot dx \\ \int (x^2 - 5x + 6) \cdot dx & \\ &\equiv \int x^2 \cdot dx - \int 5x \cdot dx + \int 6 \cdot dx \\ &= \int 0 \cdot dx - \text{Exp. 1} \end{aligned}$$

$$\begin{aligned} \text{From Exp. 1, } \frac{x^3}{3} - 5 \cdot \frac{x^2}{2} + 6x + c &= 0 \\ \frac{x^3}{3} - 5 \cdot \frac{x^2}{2} + 6x + c = 0 &\equiv \frac{2x^3 - 15x^2 + 36x + 6c}{6} = 0 \\ \frac{2x^3 - 15x^2 + 36x + 6c}{6} = 0 &\equiv 2x^3 - 15x^2 + 36x + 6c = 0 \end{aligned}$$

At this juncture, we will get confused that how could this possibly the way to solve a quadratic equation. But fear not, the above expression does have a solution so that you can arrive to the solution of a quadratic equation.

2. Substitution and division of the expression obtained from integration of the quadratic equation.

This is the most important of all the steps while solving quadratic equations with the help of infinitesimal integral calculus.

Let us assume that the value of the constant of infinity be 1. Substituting the value of the constant of infinity in the above expression, we get-

$$2x^3 - 15x^2 + 36x + 6 = 0$$

Now let us divide the above expression with the original quadratic equation.

It will look something of this sort-

$$2x^3 - 15x^2 + 36x + 6 \div x^2 - 5x + 6 = 0 \div x^2 - 5x + 6$$

$$2x^3 - 15x^2 + 36x + 6 \div x^2 - 5x + 6 = 0$$

Here although we may obtain a remainder, we shall neglect it and take into consideration of the value of the quotient only. Thus, from the above expression, we get

$$\text{Since } 2x^3 - 15x^2 + 36x + 6 \div x^2 - 5x + 6 = 2x - 5(\text{quotient})$$

$$\therefore 2x^3 - 15x^2 + 36x + 6 \div x^2 - 5x + 6 = 0 \equiv 2x - 5 = 0$$

3. Find out the value of the discriminant of the quadratic equation, check whether the initial value of $x=1$ or -1 and the value of discriminant = 1.

4. Add the discriminant to the initial value of x and divide it by the expression $2a$ to calculate the solution of the quadratic equation.

Interestingly, we will observe that the value of the quotient obtained in the above process is same as the one that we obtained in the differential calculus method.

Table I: Formulas to Be Incorporated To Solve Quadratic Equations

Serial Number	Formula
1.	$\frac{d}{dx}x^n = nx^{n-1}$
2.	$\int x^n \cdot dx = \frac{x^{n+1}}{n+1}$
3.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

These formulas should be kept in mind while solving quadratic equations with the help of calculus.

