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ON FUZZY WEAKLY SIMPLY LINDELÖF SPACES

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Abstract:
In this paper, the concept of fuzzy weakly simply Lindelof spaces is introduced and several characterizations of fuzzy weakly simply Lindelof spaces are given.

Keywords : Fuzzy dense set, Fuzzy nowhere dense set, Fuzzy simply open set, Fuzzy residual set,Fuzzy second category space, fuzzy submaximal space, Fuzzy strongly irresolvable space, Fuzzy resolvable space.

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1. INTRODUCTION

The concept for the Fuzzy Sets was introduced by L.A. Zadeh[14]in 1965, as a new approach to modeling unknown. This definition has since penetrated almost every branch of mathematics. In 1968 C.L.Chang[4] presented and developed the theory of fuzzy topological spaces. Chang's paper paved the way for the enormous subsequent development of various fuzzy topological concepts. Among the numerous covering properties of fuzzy topological spaces, there has been a great deal of attention given to those coverings that include fuzzy open and fuzzy regular open sets.

A.S.Bin Shahna[3] introduced the notion of fuzzy spaces at Lindelof and explored some of their properties. G.Thangaraj and G.Balasubramanian[6] introduced and studied fuzzy nearly Lindelof spaces, fuzzy almost Lindelof spaces,fuzzy weakly Lindelof spaces.G.Thangaraj and K.Dinakaran[9] developed and researched the definition of ' fuzzy simply * open sets'. The purpose of this paper is to introduce and study fuzzy simply* Lindelof spaces by means of fuzzy simply* open sets. Several characterizations of fuzzy simply* Lindelof spaces are established. Examples are given to illustrate the concepts introduced in this paper.

2. PRELIMINARIES

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I which takes 1 only.

Definition 2.1[6]: Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior $int(\lambda)$ and the closure $cl(\lambda)$ are defined respectively as follows:
(i). $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$.
(ii). $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Lemma 2.1[1]: For a fuzzy set λ of a fuzzy topological space X,
(i). $1 - int(\lambda) = cl(1 - \lambda)$,
(ii). $1 - cl(\lambda) = int(1 - \lambda)$.

Lemma 2.2[1]: For a family $A = \{\lambda_\alpha\}$ of a fuzzy sets of fuzzy space X, $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case A is a finite set $\vee cl(\lambda_\alpha) = cl(\vee\lambda_\alpha)$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$.

Definition 2.2: A fuzzy set λ in a fuzzy topological space (X, T) is called

(i) fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$.[12]

(ii) fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is., $int[cl(\lambda)] = 0$, in (X, T).[12]

(iii) fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.[12]

(iv) fuzzy simply open set if $Bd(\lambda)$ is a fuzzy nowhere dense set in (X, T). That is, λ is a fuzzy simply open set in (X, T) if $intcl[(cl(\lambda) \wedge cl(1 - \lambda)] = 0$, in (X, T).[8]

(v) fuzzy first category set in a fuzzy topological space (X, T). Then $1 - \lambda$ is called a fuzzy residual set in (X, T).[10]

(vi) fuzzy regular open set in (X, T), if $intcl(\lambda) = \lambda$.[1]

Definition 2.3: A fuzzy topological space (X, T) is called a

(i) fuzzy Baire space if $int[\vee_{i=1}^{\infty}(\lambda_i)] = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T).[10]

(ii) fuzzy sub maximal space if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, $\lambda \in T$ in (X, T).[3]

(iii) fuzzy hyper connected space if every non-null fuzzy open subset of (X, T) is fuzzy dense set in (X, T). That is., a fuzzy topological space (X, T) is hyper-connected if $cl(\mu_i) = 1$, for all $\mu_i \in T$.[9]

(iv) fuzzy first category space if the fuzzy set 1_X is a fuzzy first category set in (X, T). That is, $1_X = \vee_{i=1}^{\infty}(\lambda_i)$ where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T). Otherwise (X, T) will be called a fuzzy second category space.[12]

(v) fuzzy resolvable space if there exists a fuzzy dense set λ in (X, T) such that $1 - \lambda$ is also a fuzzy dense set in (X, T) Otherwise (X, T) is called a fuzzy irresolvable space.[13]

(v) fuzzy strongly irresolvable space if for every fuzzy dense set λ in (X, T), $clint(\lambda) = 1$ in (X, T). That is, $cl(\lambda) = 1$ implies that $clint(\lambda) = 1$, in (X, T).[15]

(vi) fuzzy simply Lindelof if each cover of X by fuzzy simply open sets has a countable subcover.That is, (X, T) is a fuzzy simply Lindelof space if $\bigvee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, where $intcl[bd(\lambda_\alpha)] = 0$ in (X, T), then $\vee_{n \in N}\{\lambda_{\alpha_n}\} = 1$ in (X, T).[14]

3. FUZZY WEAKLY SIMPLY LINDELÖF SPACES

Definition 3.1: A fuzzy topological space (X, T) is said to be fuzzy weakly simply Lindelof space if $\vee_{\alpha \in \Delta}\{\lambda_\alpha\} = 1$, where $intcl[bd(\lambda_\alpha)] = 0$ in (X, T), then $cl[\vee_{n \in N}\{\lambda_{\alpha_n}\}] = 1$ in (X, T).

Example 3.1: Let $X = \{a, b, c\}$. The fuzzy sets λ,μ,δ α, β, and γ are defined on X, as follows: $\lambda : X \to [0, 1]$ defined as $\lambda(a) = 0.2$; $\lambda(b) = 0.3$; $\lambda(c) = 1$;

$\mu : X \to [0, 1]$ defined as $\mu(a) = 0.5$; $\mu(b) = 1$; $\mu(c) = 0.7$;

$\delta : X \to [0, 1]$ defined as $\delta(a) = 1$; $\delta(b) = 0.5$; $\delta(c) = 0.6$;

$\alpha : X \to [0, 1]$ defined as $\alpha(a) = 0.2$; $\alpha(b) = 0.4$; $\alpha(c) = 1$;

$\beta : X \to [0,1]$ defined as $\beta(a) = 0.5;$ $\beta(b) = 1,$ $\beta(c) = 0.6;$
$\gamma : X \to [0,1]$ defined as $\gamma(a) = 1;$ $\gamma(b) = 0.4;$ $\gamma(c) = 0.4;$
Then $T = \{0, \lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \delta \vee (\lambda \wedge \mu), \mu \wedge (\lambda \vee \delta), 1\}$ is a fuzzy topology on X. On computation,we see that the fuzzy sets in (X,T), are $\lambda, \mu, \delta, \lambda \vee \mu, \lambda \vee \delta, \mu \vee \delta, \lambda \wedge \mu, \lambda \wedge \delta, \mu \wedge \delta, \lambda \vee (\mu \wedge \delta), \delta \vee (\lambda \wedge \mu), \mu \wedge (\lambda \vee \delta)$ are fuzzy open and fuzzy dense sets in (X,T). Hence (X,T) is a fuzzy weakly simply Lindelof space.

Proposition 3.1: A fuzzy topological space (X,T) is said to be fuzzy weakly simply Lindelof if for any cover of X by fuzzy closed sets $\{\lambda_\alpha\}_{\alpha\in\Delta}$ such that $\wedge_{\alpha\in\Delta}\{\lambda_\alpha\} = 0$, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ suchthat $int[\wedge_{n\in N}\{\lambda_{\alpha_n}\}] = 0$ in (X,T).

Proof: Let (X,T) be a fuzzy weakly simply Lindelof space. Let $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a cover of fuzzy closed sets such that $\wedge_{\alpha\in\Delta}\{\lambda_\alpha\} = 0$. Then $1 - \wedge_{\alpha\in\Delta}\{\lambda_\alpha\} = 1 - 0 = 1$ implies that $\vee_{\alpha\in\Delta}\{1-\lambda_\alpha\} = 1$. Now (λ_α) is fuzzy closed implies $(1-\lambda_\alpha)$ is fuzzy open. Since (X,T) is fuzzy weakly simply lindelof space and $\vee_{\alpha\in\Delta}\{1-\lambda_\alpha\} = 1$, there exists a countable subcover such that $cl[\vee_{n\in N}\{1-\lambda_{\alpha_n}\}] = 1$. Therefore $1 - cl[\vee_{n\in N}\{1-\lambda_{\alpha_n}\}] = 0$ implies that $int[1 - \vee_{n\in N}\{1-\lambda_{\alpha_n}\}] = 0$. That is $int[\wedge_{n\in N}\{\lambda_{\alpha_n}\}] = 0$

Proposition 3.2: A fuzzy topological space (X,T) is said to be fuzzy weakly simply Lindelof if for any cover $\{\lambda_\alpha\}_{\alpha\in\Delta}$ of X by fuzzy regular open sets of X admits a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ suchthat $cl[\vee_{n\in N}\{\lambda_{\alpha_n}\}] = 1$ in (X,T).

Proof: Let $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a cover of X by fuzzy regular open sets of (X,T). Then $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a fuzzy open cover of (X,T). Since (X,T) is fuzzy weakly simply Lindelof, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ suchthat $cl[\vee_{n\in N}\{\lambda_{\alpha_n}\}] = 1$ in (X,T).

Theorem 3.1: If λ is a fuzzy open and fuzzy dense set in a fuzzy topological space (X,T), then λ is a fuzzy simply open set in (X,T).

Proposition 3.3: If $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where (λ_α)'s are fuzzy open sets in a fuzzy weakly simply Lindelof and fuzzy hyperconnected space (X,T) then there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ for X.

Proof: Let (X,T) be a fuzzy weakly simply lindelof and hyperconnected space such that $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where $(\lambda_\alpha) \in T$. Since (X,T) is a fuzzy hyperconnected space, the fuzzy open sets (λ_α)'s are fuzzy dense sets in (X,T). Then (λ_α)'s are fuzzy open sets and fuzzy dense sets in (X,T). Then by theorem 3.1, (λ_α)'s are fuzzy simply open sets in (X,T). Since (X,T) is a fuzzy weakly simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ for X.

Proposition 3.4: If (X,T) is a fuzzy weakly Lindelof space and if $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where $(\lambda_\alpha) \in T$. and $cl(\lambda_\alpha) = 1$ in (X,T), then $\wedge_{n\in N}\{\mu_{\alpha_n}\}] = 0$, where μ_{α_n}'s are fuzzy nowhere dense sets in (X,T).

Proof: Let (X,T) be a fuzzy simply Lindelof space. By hypothesis, the fuzzy sets (λ_α)'s are fuzzy open and fuzzy dense sets in (X,T). Then by theorem 3.1, (λ_α)'s are fuzzy simply open sets in (X,T). Now $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where (λ_α)'s are fuzzy simply open sets in (X,T), implies that $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a fuzzy simply open cover of X. Since (X,T) is a fuzzy weakly simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ fuzzy simply open sets for X and then $\vee_{n\in N}\{\lambda_{\alpha_n}\} = 1$ in (X,T).Then $1 - \vee_{n\in N}\{\lambda_{\alpha_n}\} = 0$ and hence $\wedge_{n\in N}\{1-\lambda_{\alpha_n}\} = 0$. Let $\mu_{\alpha_n} = 1 - \lambda_{\alpha_n}$. Now $intcl(1-\lambda_{\alpha_n}) = 1 - clint(\lambda_{\alpha_n}) = 1 - cl(\lambda_{\alpha_n}) = 1 - 1 = 0$, and thus $(1-\lambda_{\alpha_n})$'s are fuzzy nowhere dense set in (X,T). Hence $\wedge_{n\in N}\{\mu_{\alpha_n}\} = 0$, where (μ_{α_n})'s are fuzzy nowhere dense sets in (X,T).

Proposition 3.5: If (X,T) is a fuzzy weakly simply Lindelof space and fuzzy submaximal space and if $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where (λ_α)'s are fuzzy dense sets in (X,T), then $\wedge_{n\in N}\{\mu_{\alpha_n}\}] = 0$, where μ_{α_n}'sare fuzzy nowhere dense sets in (X,T).

Proof: Let (X,T) be a fuzzy weakly simply Lindelof space and fuzzy submaximal space. By hypothesis $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where $cl(\lambda_\alpha) = 1$ in (X,T). Since (X,T) is a fuzzy submaximal space, the fuzzy dense sets (λ_α)'s are fuzzy open sets in (X,T). Hence (λ_α)'s are fuzzy dense sets and fuzzy dense sets in (X, T). Then, by proposition 3.4, $\wedge_{n\in N}\{\mu_{\alpha_n}\} = 0$, where (μ_{α_n})'sare fuzzy nowhere dense sets in (X, T).

Theorem 3.2: If λ is a fuzzy simply open set in a fuzzy topological space (X, T), then $\lambda \wedge (1 - \lambda)$ is a fuzzy nowhere dense set in (X, T).

Proposition 3.6: If (X, T) is a fuzzy weakly simply Lindelof space, Then (X, T) is a fuzzy second category space.

Proof: Let (X, T) be a fuzzy weakly simply Lindelof space and $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a cover of X by fuzzy simply open sets in (X, T). By theorem 3.2, $(\{\lambda_{\alpha_n}\} \wedge \{1 - \lambda_{\alpha_n}\})$'s are fuzzy nowhere dense sets in (X, T). Since (X, T) is a fuzzy weakly simply Lindelof space,there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ for X. Then $\vee_{n\in N}\{\lambda_{\alpha_n}\} = 1$ in (X, T). Now $(\{\lambda_{\alpha_n}\} \wedge \{1 - \lambda_{\alpha_n}\}) \leq \{\lambda_{\alpha_n}\}$ in (X, T) implies that $\vee_{n\in N}[\{\lambda_{\alpha_n}\} \wedge \{1 - \lambda_{\alpha_n}\}] \leq \vee_{n\in N}\{\lambda_{\alpha_n}\}$ and then $\vee_{n\in N}[\{\lambda_{\alpha_n}\} \wedge \{1 - \lambda_{\alpha_n}\}] \leq 1$ implies $cl[\vee_{n\in N}[\{\lambda_{\alpha_n}\} \wedge \{1 - \lambda_{\alpha_n}\}]] \leq cl(1)$ [Since $cl(\lambda) = 1$.] Then $cl[\vee_{n\in N}[\{\lambda_{\alpha_n}\} \wedge \{1 - \lambda_{\alpha_n}\}]] \neq 1$, where $(\{\lambda_{\alpha_n}\} \wedge \{1 - \lambda_{\alpha_n}\})$'s are fuzzy nowhere dense sets, implies that (X, T) is not a fuzzy first category space and hence (X, T) is a fuzzy second category space.

Theorem 3.3: If the fuzzy topological space (X, T) is a fuzzy Baire space, then (X, T) is a fuzzy second category space.

Proposition 3.7: If (X, T) is a fuzzy weakly simply Lindelof space, Then (x, T) is a fuzzy Baire space.

Proof: The proof follows from proposition 3.6 and the theorem 3.3

Theorem 3.4: If λ is a fuzzy nowhere dense set in a fuzzy topological space (X, T), then λ is a fuzzy simply open set in (X, T).

Proposition 3.8: If $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a cover of X by fuzzy nowhere dense sets in a fuzzy weakly simply Lindelof space (X, T), then there is a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ for X.

Proof: Let $\{\lambda_\alpha\}_{\alpha\in\Delta}$ be a cover of X by fuzzy nowhere dense sets in (X, T), then $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where $intcl(\lambda_\alpha) = 0$ in (X, T). by theorem 3.4, the fuzzy nowhere dense sets (λ_α)'s are fuzzy simply open sets in (X, T) and thus $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a cover of X by fuzzy simply open sets in (X, T). Since (X, T) is a fuzzy weakly simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ for X. That is, $\vee_{n\in N}\{\lambda_{\alpha_n}\} = 1$, implies $cl[\vee_{n\in N}\{\lambda_{\alpha_n}\}] = 1$, where $intcl[bd(\lambda_{\alpha_n})] = 0$ in (X, T).

Theorem 3.5: If λ is a fuzzy simply open set in a fuzzy topological space (X, T), then $cl(\lambda)$ is a fuzzy simply open set in (X, T).

Proposition 3.9: If $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where$intcl(\lambda_\alpha) = 0$ in a fuzzy weakly simply Lindelof space (X, T), then there exists a fuzzy first category set λ in (X, T) such that $cl(\lambda) = 1$ in (X, T).

Proof: Let (X, T) be a fuzzy weakly simply Lindelof space such that $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$ and $intcl(\lambda_\alpha) = 0$. Now $intcl(\lambda_\alpha) = 0$ in (X, T) implies that $(\lambda_\alpha$'s are fuzzy nowhere dense set in (X, T). Then by theorem 3.5, $\{cl(\lambda_\alpha)\}$'sare fuzzy simply open sets in (X, T). Now $\lambda_\alpha \leq cl(\lambda_\alpha)$ implies that $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} \leq \vee_{\alpha\in\Delta}\{cl(\lambda_\alpha)\}$ and then $1 \leq \vee_{\alpha\in\Delta}\{cl(\lambda_\alpha)\}$. That is$\vee_{\alpha\in\Delta}\{cl(\lambda_\alpha)\} = 1$, where $\{cl(\lambda_\alpha)\}$'s are fuzzy simply open sets in the fuzzy weakly simply Lindelof space (X, T). Then there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ for X by fuzzy simply open sets in (X, T). That is, $\vee_{n\in N}\{cl(\lambda_{\alpha_n})\} = 1$. Now by lemma 2.2, $\vee_{n\in N}cl(\lambda_{\alpha_n}) \leq cl[\vee_{n\in N}(\lambda_{\alpha_n})]$ implies that $1 \leq cl[\vee_{n\in N}(\lambda_{\alpha_n})]$. That is, $cl[\vee_{n\in N}(\lambda_{\alpha_n})] = 1$. Then λ is a fuzzy first category set in (X, T) such that $cl(\lambda) = 1$, in (X, T).

Proposition 3.10: If $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where (λ_α)'s are fuzzy nowhere dense sets in a fuzzy weakly simply Lindelof and fuzzy Baire space (X, T) then (X, T) is a fuzzy resolvable space.

Proof: Let (X, T) be a fuzzy weakly simply Lindelof space such that $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where $intcl(\lambda_\alpha) = 0$ in (X, T). Then by proposition 3.9, there exists a fuzzy first category set λ in (X, T) such that $cl(\lambda) = 1$. Since (X, T) is a fuzzy Baire space, $int(\lambda) = 0$ in (X, T). Now $cl(1 - \lambda) = 1 - int(\lambda) = 1 - 0 = 1$. Thus $cl(\lambda) = 1$ and $cl(1 - \lambda) = 1$ in (X, T). Hence (X, T) is a fuzzy resolvable space.

Proposition 3.11: If $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where (λ_α)'s are fuzzy nowhere dense sets in a fuzzy weakly simply Lindelof and fuzzy second category space (X, T) then (X, T) is a fuzzy resolvable space.

Proof: The proof follows from proposition 3.10 and the theorem 3.3.

Theorem 3.6: If λ in a fuzzy closed set with $int(\lambda) = 0$, in a fuzzy topological space (X, T), then λ is a fuzzy simply open set in (X, T).

Proposition 3.12: If (X, T) is a fuzzy weakly simply Lindelof space and if $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where (λ_α)'s are fuzzy closed sets with $int(\lambda_\alpha) = 0$ in (X, T), then $cl[\vee_{n\in N}(\lambda_{\alpha_n})] = 1$, in (X, T)

Proof: Let (X, T) be a fuzzy weakly simply Lindelof space. By hypothesis,the fuzzy sets (λ_α)'s are fuzzy closed sets with $int(\lambda_\alpha) = 0$ in (X, T). Then by theorem 3.6,(λ_α)'s are fuzzy simply open sets in (X, T). Now $\vee_{\alpha\in\Delta}\{\lambda_\alpha\} = 1$, where (λ_α)'s are fuzzy simply open sets in (X, T), implies that $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a fuzzy simply open cover of X. Since (X, T) is fuzzy weakly simply Lindelof space, there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$of fuzzy simply open sets, for X. Hence $cl[\vee_{n\in N}(\lambda_{\alpha_n})] = 1$, where $(1 - \lambda_{\alpha_n}) \in T$ and $int(\lambda_{\alpha_n}) = 0$, in (X, T).

Theorem 3.7: If $int(\lambda) = 0$, for a fuzzy set λ in a fuzzy topological space (X, T), then λ is a fuzzy simply open set in (X, T).

Proposition 3.13: If $\{\lambda_\alpha\}_{\alpha\in\Delta}$ is a cover of X by fuzzy sets $int(\lambda_\alpha) = 0$ in a fuzzy strongly irresolvable and fuzzy weakly simply Lindelof space in (X, T), then $cl[\vee_{n\in N}(\lambda_{\alpha_n})] = 1$, in (X, T)

Proof: Suppose that $\vee_{\alpha\in\Delta}(\lambda_\alpha) = 1$, where $int(\lambda_\alpha) = 0$ in (X, T). Since (X, T) is a fuzzy strongly irresolvable space, by theorem 3.7, $\{\lambda_{\alpha_n}\}'s$ are fuzzy simply open sets in (X, T). Since (X, t) is a fuzzy weakly simply Lindelof space, $\vee_{n\in N}(\lambda_\alpha)] = 1$, where $\{\lambda_\alpha\}'s$ are fuzzy simply open sets implies that there exists a countable subcover $\{\lambda_{\alpha_n}\}_{n\in N}$ for X by simply open sets. That is, $cl[\vee_{n\in N}(\lambda_{\alpha_n})] = 1$, where $int(\lambda_{\alpha_n}) = 0$ in (X, T).

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