International Journal of Mathematics Research.
ISSN 0976-5840 Volume 12, Number 1 (2020), pp. 65-67

https://dx.doi.org/10.37624/IJMR/12.1.2020.65-67

A n Dimensional Convex Hull in a $n+1$ Dimesion

Israel Ramirez Nuñez

High School Student

"Universidad del Valle de México", campus Chihuahua, Chihuahua, México.

Abstract

Suppose $\mathbb{e}^n$ defines the surface of $(n-1)-$sphere then we prove for a n dimensional convex hull C such that C is a subset of an open ball, so C is the projection of a set of points B in a $n+1$ dimension such that B is a subset of $\mathbb{e}^{n+1}$. From this we present a proof where for each set B there exist a polytope P such that B is a subset of P.The results are generalize for polytopes.

Keywords. Polytopes, Convex hull, Arrangements of points, flats.

1. INTRODUCTION

Suppose we have a polar coordinate (α, r) and for some positive integer i

$$\left(\frac{a_i}{z}\right) \coloneqq \alpha$$

Where if we have a number t such that t divides α and 360 then

$$z = \frac{360}{t}$$

For all i

$$[a_i \in \mathbb{N}^+ | 0 \leq a_i \leq z]$$

To generalize this for a n dimensional space with coordinates $(\alpha, \beta, \dots, r)$ then t must divide 360 and all the angles.

We call this kind of coordinates as the "n −polytope coordinate system". Now We present a $e_n - polytope$ as a n dimensional polytope P where all its points $P(v)$ are defined by the n −polytope coordinates. From this we define $\mathbb{P}_{rn}$ as a n −polytope coordinate system for some radio r.

2. CONVEX HULLS

Definition 2.1 $\mathbb{e}^n$ is the surface of $(n-1)$ −sphere

Definition 2.2 A n dimensional open ball of radio r is defined by $B_r(x)_n$

Suppose we have a n dimensional convex hull $\boldsymbol{C}$ where $\boldsymbol{C} \subseteq B_r(x)_n$ therefore we obtain the next theorem.

Theorem 2.1 $\boldsymbol{C} = [\boldsymbol{R}(B)|B \subseteq P]$ where P is a $e_{n+1} - polytope$, B is a set of points in a $n+1$ dimension and the function $\boldsymbol{R}(B)$ is the projection of the set B in a n dimensional space.

Proof.

Suppose the set C belongs to a 2-dimensional space and using the Euclidian coordinates we define the elements of C as

$$(x_k, y_k)$$

Therefore, for each point k of C there exist a number z_k such that

$$(x_k, y_k, z_k) \in \mathbb{e}^3$$

This means that the coordinates (x_k, y_k, z_k) can be expressed in the 3 −polytope coordinate system, Therefore we can construct a $e_3 - polytope$ which goes through all points (x_k, y_k, z_k).

To generalize this in any dimension we can say that for any point $(x_1, x_2 \dots x_n)$ there exist a number x_{n+1} such that $(x_1, x_2 \dots x_{n+1}) \in \mathbb{e}^{n+1}$

Corollary 2.1 For a n dimensional polytope C such that $\subseteq B_r(x)_n$, then there exist B such that $B \subseteq P$ and $\boldsymbol{R}(B) = C$

In general, if we have a set of points C in a n dimensional space such that $C \nsubseteq \mathbb{P}_{rn}$ and $C \subseteq B_r(x)_n$ therefore there exist a set B such that $B \subseteq P \subseteq \mathbb{P}_{r(n+1)}$ and $R(B) = C$

CONCLUSION.

Here we propose a future use of this paper. From the recent article of Stephen Wolfram, he proposes that universe is a hyperplane, but we think it could be a $e_4 - polytope$.

REFERENCES

[1] Martini, H. Convex polytopes whose projection bodies and difference sets are polars. Discrete Comput Geom 6, 83–91 (1991). https://doi.org/10.1007/BF02574676

[2] Shephard, G. (1972). Sections and projections of convex Shephard, G. (1972). Sections and projections of convex polytopes. *Mathematika, 19*(2), 144-162. doi:10.1112/S0025579300005593

[3] Clinch, K., Nixon, A., Schulze, B. *et al.* Pairing Symmetries for Euclidean and Spherical Frameworks. *Discrete Comput Geom* (2020). https://doi.org/10.1007/s00454-020-00198-9

[4] Kohl, F., Olsen, M. & Sanyal, R. Unconditional Reflexive Polytopes. *Discrete Comput Geom* (2020). https://doi.org/10.1007/s00454-020-00199-8

[5] Nguyen, D., Pak, I. On the Number of Integer Points in Translated and Expanded Polyhedra. *Discrete Comput Geom* (2020). https://doi.org/10.1007/s00454-020-00178-z