

Exact Value of pi (π) : ($17 - 8\sqrt{3}$)

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Abstract

Finding the accurate value of pi (π) is a 4000 year problem. During the decades with the help of computers, scientists are able to find value up to 31 trillion digits (10^{13}) which is also not an accurate value. During time, scientists mentioned that it is impossible to find accurate value.

In my innovation paper, I have solved above problem by using easy ways of Geometry and challenging Algebra mathematical calculations, equations and formulas. Pi (π) is a universal constant in mathematics. An exact value of pi is derived as $17 - 8\sqrt{3}$ & approximate value is 3.14359353944... The derivation of this value is supported by number of geometrical constructions, arithmetic calculations and use of some simple algebraic formulae. As pi is the ratio of circumference and diameter of a circle, this attempt is made to find a solution for a global puzzling problem of equating the area of a square with that of a circle.

INTRODUCTION

The value of pi is important not only in mathematics, science, engineering but also in research oriented disciplines and many other applications. However, the problem of the exact value of pi is well known worldwide and has been discussed for more than thousand years ago. Though, the exact value of pi can be computed more than 31 trillion digits with the help of computer to achieve more accuracy, it is still difficult/impossible to reach the exact value of pi. If the value of pi is exact then it will equalize area of circle to the area of square.

Fibonacci series is very well known to everyone which is 1 1 2 3 5 8 13 21 (Computed as $1+1=2$, $2+1=3$, $3+2=5$...). Similarly, a new series is proposed as below

1	2
3	4
6	9
13	19
28	41
60	88
129	189
277	406
595	872
.....

From this series, it is observed that the division of numbers at specific positions evaluates to 3.14789. For example, $(189/60=3.14...)$, $(872/277=3.14...)$, $(595/189=3.14...)$, $(129/41=3.14...)$ and so on. This calculation gives a clue that there exists a relationship between this series and the value of pi which motivated to research on exact value of pi.

Many mathematicians have tried to divide a circle into n-sides polygon to a great extent in order to get more accurate value of pi. However, computations performed using this value of pi will never give exact results but approaching towards exactness. Due to this globally it is assumed that pi is a transcendental number.

If pi is transcendental i.e. $\pi = 3.1415926535897...$ (Current value)

Then $(\pi - 3) = 0.1415926535897...$ and $(4 - \pi) = 0.8584073464102...$

$(\pi - 3)$ represents inscribed dodecagon and $(4 - \pi)$ represents circumscribed square.

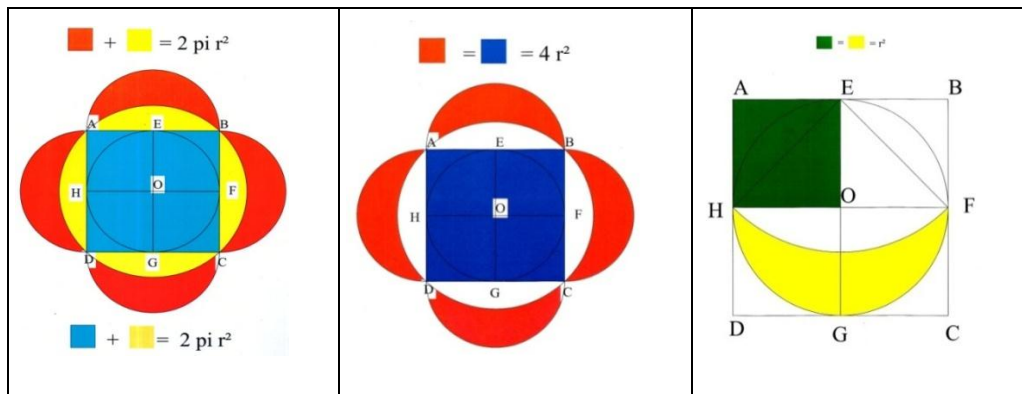
$$(\pi - 3) + (4 - \pi) = 1$$

$(0.1415926535897... + 0.8584073464102...) = 0.999999999999... \approx 1$ but not exactly equal to 1

Here use of $(\pi - 3)$ and $(4 - \pi)$ is explained in detail in section algebra and geometry proofs. Similarly, even if the value of pi is calculated up to trillion digits or infinite digits by an arithmetic method then also it will be difficult to get an exact value. In this paper the exact value of pi (π) is proposed as $(17 - 8\sqrt{3})$ based on algebraic method which is demonstrated with number of proofs. Computations carried out using this suggested value of pi produce exact results.

The unsolvable world problem: To equate the area of a square with that of a circle.

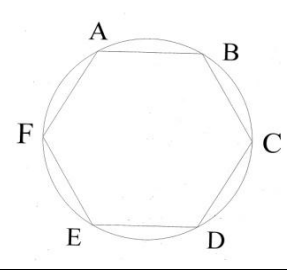
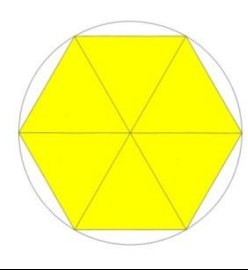
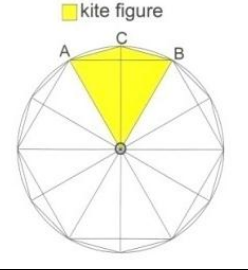
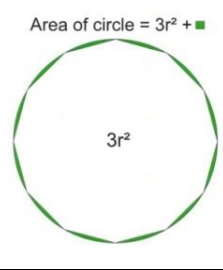
As per mathematicians' opinion, it is not possible to equate the area of a square to that of a circle. However, in this paper it is proved to get same areas of both geometries is possible. This mentioned claim is supported using following figures.



In the first section of above figure, area of red and yellow part is equal to $2\pi r^2$ and it is also same for blue and yellow part. In the second section, area of red and blue part is $4r^2$. From the first and second sections, it can be derived that area of green and yellow part is r^2 , which is shown in section three.

Basic figures

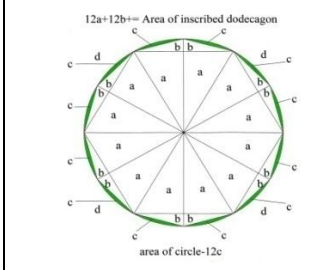
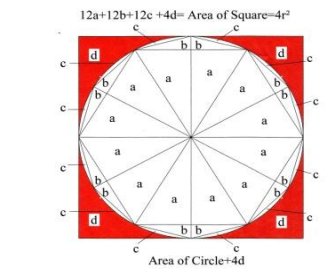
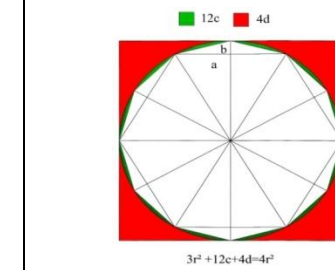
The following basic figure represents an example by which area of all generated kite figures (one shown with yellow color) is calculated as $3r^2$. The area of circle is $3r^2 + \text{green part}$. So here the focus on finding the area of green part is explained in the remaining section.

			
Inscribed hexagon Side of r	Six equilateral triangle Side of r	Area of kite figure = $r^2/2$ Area of ins. Dodd. = $3r^2$	Area of circle = $3r^2 + \blacksquare$ Area of circle = $3r^2$ = $(3r^2 + \text{remaining part.})$

How to find area of remaining part?

Algebra and geometry proofs

Basic information: Note: let a, b, c and d each part shows area in following figures

		
Area of inscribed dodecagon = $(12a + 12b) = 3r^2$ = $(\pi r^2 - 12c)$	Area of circumscribed square = $(12a + 12b + 12c + 4d) = 4r^2$ = $(\pi r^2 + 4d)$	$(4 - \pi) r^2 = 4d$ $(\pi - 3) r^2 = 12c$ $(4d + 12c) = [(4 - \pi) + (\pi - 3)] r^2 = 1$

$$\text{Area of inscribed hexagon} = 12a = (1.5\sqrt{3}) r^2 \quad a = (0.125\sqrt{3}) r^2$$

$$\text{Area of inscribed dodecagon} = (12a + 12b) = 3r^2 = (\pi r^2 - 12c)$$

$$(12a + 12b) - 12a = 12b = (3 - 1.5\sqrt{3}) r^2 \quad b = (0.25 - 0.125\sqrt{3}) r^2$$

$$\text{Area of circumscribed square} = (12a + 12b + 12c + 4d)$$

$$= 4r^2$$

$$= (\pi r^2 + 4d)$$

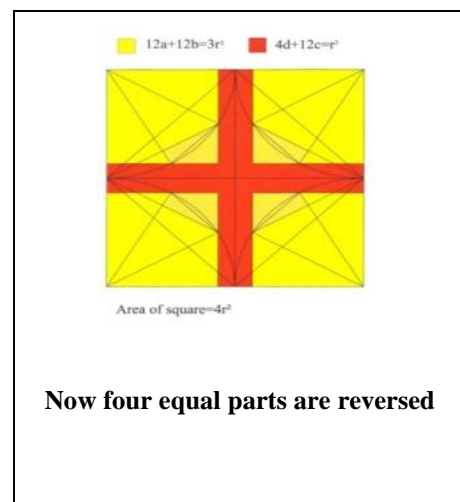
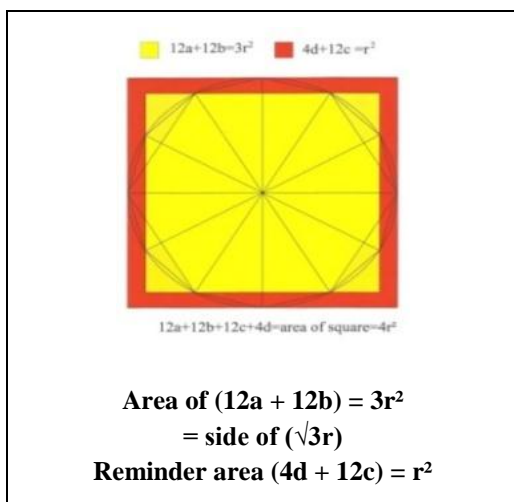
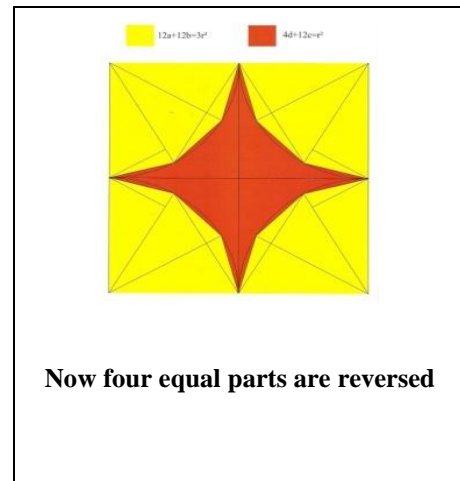
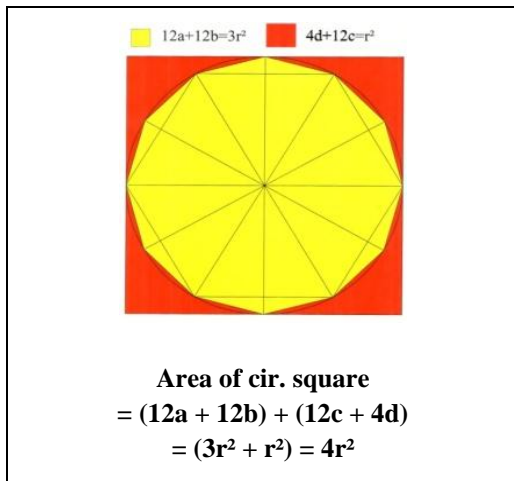
$$\text{Area of circle} = (12a + 12b + 12c)$$

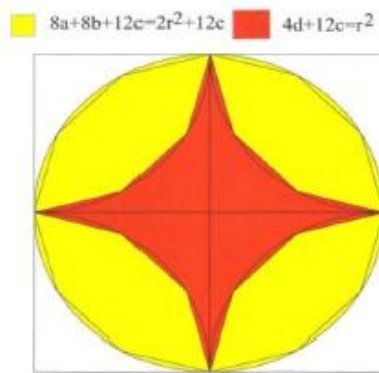
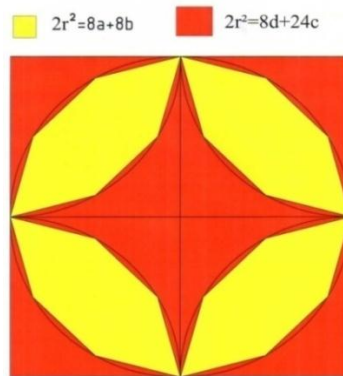
$$= (3r^2 + 12c)$$

$$= (4r^2 - 4d)$$

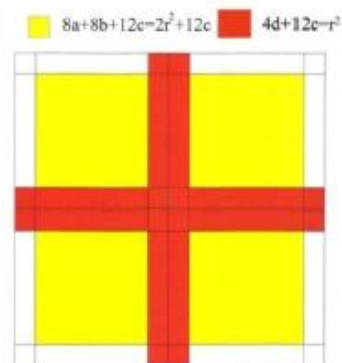
$$(12c + 4d = 1r^2) \quad (12c + 4d = 1r^2)/4$$

$$= (3c + d = 0.25r^2 = a + b)$$

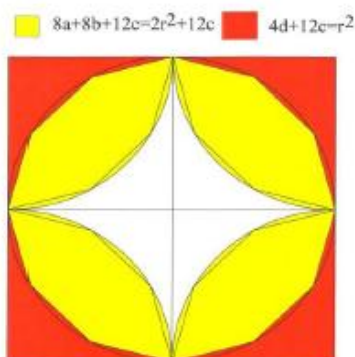




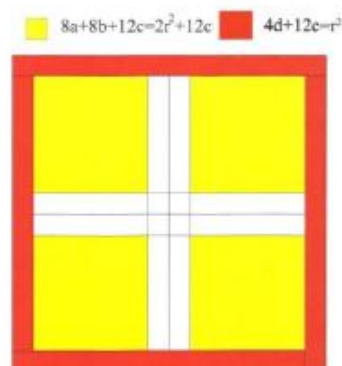
Area of circle



Area of circle



Area of circle

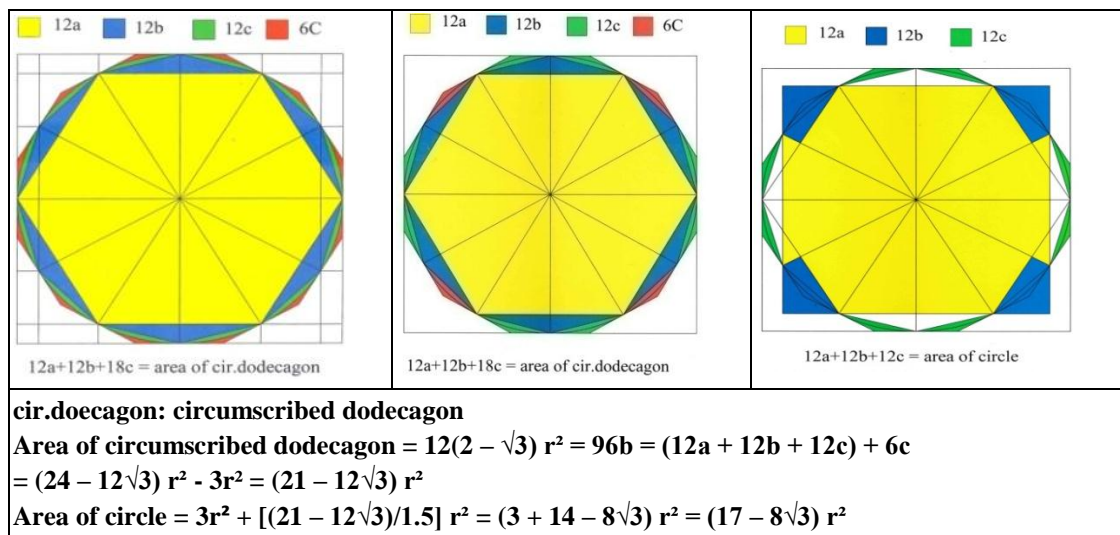
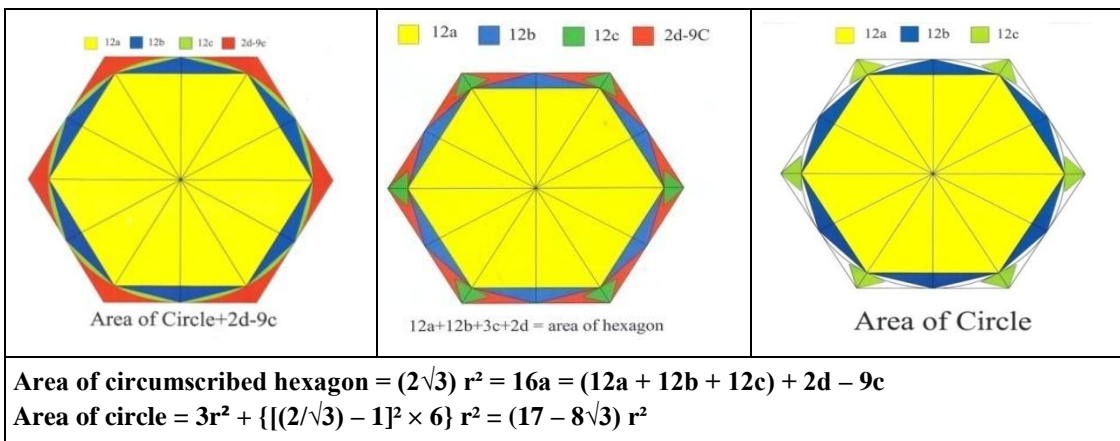
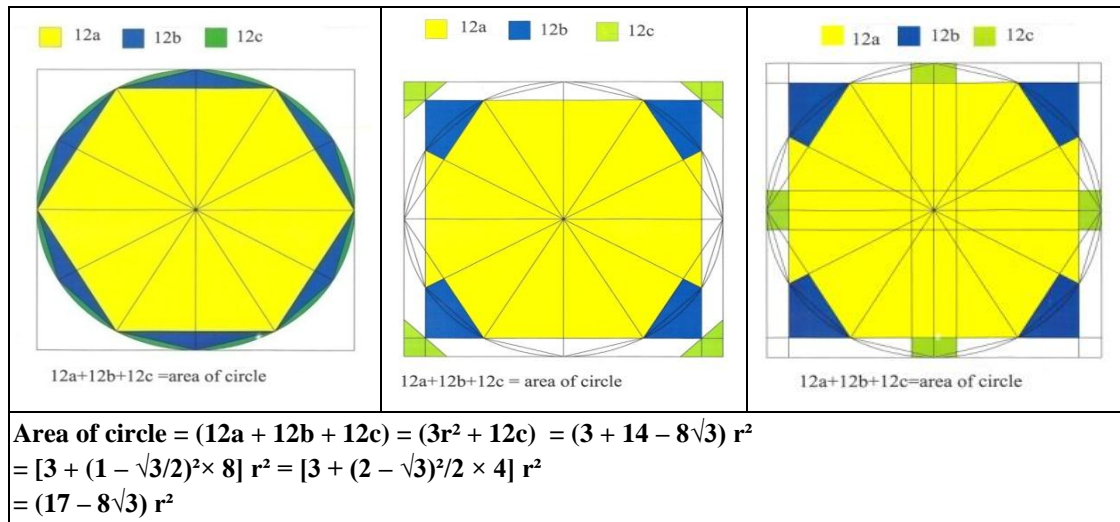


Area of circle

$$\text{Area of circle} = (8a + 8b + 12c) + (4d + 12c) = (2r^2 + 12c + 1r^2)$$

$$= [(\sqrt{3} - 1)^2 \times 4] + 1r^2 = (17 - 8\sqrt{3}) r^2$$

Proofs using various figures (circumscribed hexagon and dodecagon method)



Comparison between current and exact value of Pi (π)

Current value of Pi = 3.141592653589793238462643383279502...

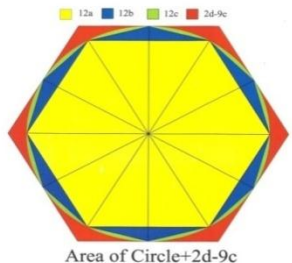
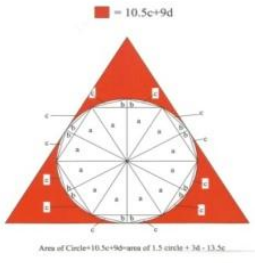
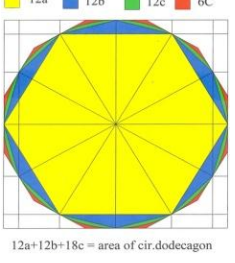
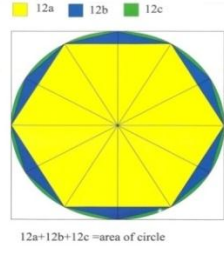
Exact value of Pi = $(17 - 8\sqrt{3})$ = 3.143593539448981651780429267953021...

Following table shows value of 12c and 4d based on current value of Pi and exact value of Pi.

$$12c = (\pi - 3) r^2, \quad 4d = (4 - \pi) r^2$$

Current value of Pi = 3.14159265358...	Exact value of Pi = $(17 - 8\sqrt{3})$ = 3.14359353944...
$12c = (\pi r^2 - 3r^2) = 0.1415926535897...r^2$	$12c = [(17 - 8\sqrt{3}) r^2 - 3r^2]$ = $(14 - 8\sqrt{3}) r^2$ = 0.14359353... r^2
$3c = 12c/4 = 0.035398163...r^2$	$3c = 12c/4 = (3.5 - 2\sqrt{3}) r^2$ = 0.03589837... r^2
$4d = (4r^2 - \pi r^2) = 0.858407346...r^2$	$4d = [4r^2 - (17 - 8\sqrt{3}) r^2]$ = $(8\sqrt{3} - 13) r^2$ = 0.85640646... r^2
$d = 4d/4 = 0.214601836...r^2$	$d = 4d/4 = (2\sqrt{3} - 3.25) r^2$ = 0.214101615... r^2

There are two values of 3c and d from above table. In order to decide correct value of Pi, below figures are studied with examples.

 <p>Area of Circle+2d-9c</p>	 <p>Area of Circle=10.5c+9d=area of 1.5 circle + 3d = 13.5c</p>	 <p>12a+12b+18c = area of cir.dodecagon</p>	 <p>12a+12b+12c=area of circle</p>
<p>Area of circumscribed hexagon = 16a = $(2\sqrt{3}) r^2$ = 3.464101615...r^2 (3.464101615...r^2 - 3.25r^2) = 0.214101615...r^2 = d i.e. Area of circumscribed hexagon = $(3.25r^2 + d)$</p>	<p>Area of circumscribed triangle = 24a = $(3\sqrt{3}) r^2$ = 5.196152423...r^2, (5.25 - 5.196152423...) r^2 = 0.053847577...r^2 = 4.5c i.e. Area of cir. triangle = $(5.25r^2 - 4.5c)$</p>	<p>Area of cir. dodecagon = 96b = $12(2 - \sqrt{3}) r^2$ = (3.215390309...r^2 - 3r^2) = 0.215390309...r^2 = 6(3c = 0.035898384...)r^2 i.e. Area of cir. Dodd. = $(3r^2 + 18c)$</p>	<p>Area of circle = πr^2 = $(3r^2 + 12c)$ = $(4r^2 - 4d)$</p>

Above equations in terms of 3c and d satisfies following exact equations which are universally accepted.

Area of circumscribed (6 hexagon + 1 dodecagon)

$$\begin{aligned} &= 6(2\sqrt{3}) r^2 + 1(24 - 12\sqrt{3}) r^2 = 24r^2 \\ &= 6(3.25r^2 + d) + (3r^2 + 18c) \\ &= (19.5r^2 + 6d) + (3r^2 + 18c) \\ &= 22.5r^2 + 6(d + 3c) \quad (a + b = 3c + d = 0.25r^2) \\ &= (22.5r^2 + 1.5r^2) = 24r^2 \end{aligned}$$

Area of circumscribed (4 triangle + 1 dodecagon)

$$\begin{aligned} &= 4(3\sqrt{3}) r^2 + 1(24 - 12\sqrt{3}) r^2 = 24r^2 \\ &= 4(5.25r^2 - 4.5c) + (3r^2 + 18c) \\ &= (21r^2 - 18c) + (3r^2 + 18c) \\ &= 24r^2 \end{aligned}$$

Area of circumscribed (2 triangle + 3 hexagon + 1 dodecagon)

$$\begin{aligned} &= 2(3\sqrt{3}) r^2 + 3(2\sqrt{3}) r^2 + 1(24 - 12\sqrt{3}) r^2 = 24r^2 \\ &= [2(5.25 - 4.5c) + 3(3.25r^2 + d) + (3r^2 + 18c)] \\ &= [(10.5 - 9c) + (9.75r^2 + 3d) + (3r^2 + 18c)] \\ &= 23.25r^2 + 3(d + 3c) \quad (a + b = 3c + d = 0.25r^2) \\ &= (23.25r^2 + 0.75r^2) = 24r^2 \end{aligned}$$

Area of circumscribed (6 hexagon + 1.5 circle)

$$\begin{aligned} &= 6(3.25r^2 + d) + 1.5(4r^2 - 4d) \\ &= (19.5r^2 + 6d) + (6r^2 - 6d) \\ &= 25.5r^2 \\ &\text{area of circle} = (25.5 r^2 - 6 \text{ cir. hexagon})/1.5 \\ &= [25.5 - 6(2\sqrt{3})]/1.5 = (17 - 8\sqrt{3}) \end{aligned}$$

Area of circumscribed (2 triangle + 1 hexagon + 1 circle)

$$\begin{aligned} &= 2(5.25r^2 - 4.5c) + 1(3.25r^2 + d) + (3r^2 + 12c) \\ &= [(10.5r^2 - 9c) + (3.25r^2 + d) + (3r^2 + 12c)] \\ &= (16.75r^2 + d + 3c) = 16.75r^2 + 0.25r^2 = 17r^2 \\ &\text{area of circle} = 17r^2 - \text{Area of circumscribed (2 triangle + 1 hexagon)} \\ &= [17r^2 - 2(3\sqrt{3}) r^2 + (2\sqrt{3}) r^2] \\ &= (17 - 8\sqrt{3}) r^2 \end{aligned}$$

Area of circumscribed (6 hexagon + 17 dodecagon)

$$\begin{aligned}
&= 6(2\sqrt{3}) r^2 + 17(24 - 12\sqrt{3}) r^2 = (12\sqrt{3}) r^2 + (408 - 204\sqrt{3}) r^2 = (408 - 192\sqrt{3}) r^2 \\
&= 6(3.25r^2 + d) + 17(3r^2 + 18c) \\
&= (19.5r^2 + 6d + 51r^2 + 306c) \\
&= 70.5r^2 + (6d + 18c) + 288c \quad (a + b = 3c + d = 0.25r^2) \\
&= (70.5r^2 + 1.5r^2 + 288c) \\
&= (72r^2 + 288c) \\
&= 24(3r^2 + 12c) = 24\pi r^2 \\
\pi r^2 &= (408 - 192\sqrt{3}) r^2 / 24 \\
&= (17 - 8\sqrt{3}) r^2
\end{aligned}$$

As mentioned in above section, further proof of equations is given below

Area of circumscribed hexagon = $(3.25 + d) = [3r^2 + (d + 3c) + d] = (3r^2 + 2d + 3c)$

(Area of circumscribed hexagon – area of circle)

$$= (3r^2 + 2d + 3c) - (3r^2 + 12c) = (2d - 9c)$$

i.e. **Area of circumscribed hexagon** = $(\pi r^2 + 2d - 9c)$

Area of circumscribed triangle = $(5.25 - 4.5c) = [3r^2 + 9(d + 3c) - 4.5c]$

$$= (3r^2 + 9d + 27c - 4.5c)$$

(area of circumscribed triangle – area of circle)

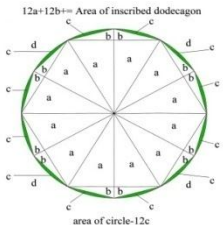
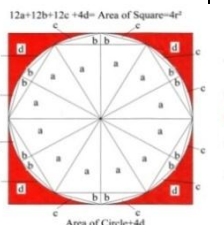
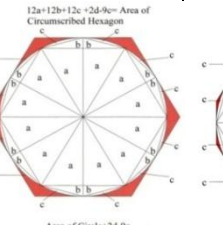
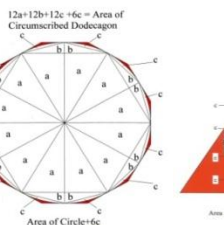
$$= (3r^2 + 9d + 22.5c) - (3r^2 + 12c) = (9d + 10.5c)$$

i.e. **area of circumscribed triangle** = $(\pi r^2 + 9d + 10.5c)$

Area of circumscribed dodecagon = $(3r^2 + 18c)$

(Area of circumscribed dodecagon – area of circle) = $(3r^2 + 18c) - (3r^2 + 12c) = 6c$

i.e. **Area of circumscribed dodecagon** = $(\pi r^2 + 6c)$

Exact area		Derived area		
 <p>12a²+12b²+12c²+4d²= Area of Square=4r²</p>		 <p>12a²+12b²+12c²+2d²= Area of Circumscribed Hexagon</p>	 <p>12a²+12b²+12c²+6c = Area of Circumscribed Dodecagon</p>	 <p>12a²+12b²+12c²+6c = Area of Circumscribed Dodecagon</p>
Area of inscribed dodecagon = $3r^2$ = $(\pi r^2 - 12c)$	Area of circumscribed square = $4r^2$ = $(\pi r^2 + 4d)$	Area of circumscribed hexagon = $(2\sqrt{3}) r^2$ = $(\pi r^2 + 2d - 9c)$	Area of circumscribed Dodecagon = $12(2 - \sqrt{3})r^2$ = $(\pi r^2 + 6c)$	Area of circumscribed triangle = $3(\sqrt{3})r^2$ = $(\pi r^2 + 9d + 10.5c)$

Proof of above equations

Area of circumscribed (6 hexagons + 1 dodecagon)

$$\begin{aligned} &= 6(2\sqrt{3}) r^2 + 1(24 - 12\sqrt{3}) r^2 = 24r^2 \\ &= 6(\pi r^2 + 2d - 9c) + 1(\pi r^2 + 6c) \\ &= 7(\pi r^2 + 12d - 48c) \\ &= 4(\pi r^2 - 12c) + 3(\pi r^2 + 4d) \\ &= \text{area of } 4(3r^2) + 3(4r^2) = 24r^2 \\ &= [\text{Area of (4) inscribed dodecagon + (3) circumscribed Square}] \end{aligned}$$

Area of circumscribed (2 triangle + 3 hexagon + 1 dodecagon)

$$\begin{aligned} &= 2(3\sqrt{3}) r^2 + 3(2\sqrt{3}) r^2 + 1(24 - 12\sqrt{3}) r^2 = 24r^2 \\ &= 2(\pi r^2 + 9d + 10.5c) + 3(\pi r^2 + 2d - 9c) + 1(\pi r^2 + 6c) \\ &= [6\pi r^2 + (18d + 21c) + (6d - 27c) + 6c] \\ &= (6\pi r^2 + 24d) \\ &= 6(\pi r^2 + 4d) \\ &= 6(4r^2) \\ &= \text{area of 6 circumscribed square} \end{aligned}$$

If we use the approximate value of Pi (3.14159...) then above equation does not satisfy. But if we use algebraic value of Pi $(17 - 8\sqrt{3})$ then we get exact results. Above equations are verified by using old value of Pi but answer doesn't match. With the presented innovative method we can solve infinite examples similar to above.

Note x, y is any number

Area of circumscribed [(x + y) dodecagon + (2x + 6y) hexagon]

$$= \text{area of [(x + 4y) inscribed dodecagon + (x + 3y) circumscribed square + x circle]}$$

Area of circumscribed [(x + y) dodecagon + (2x + 3y) hexagon + 2y triangle]

$$= \text{area of [(x) inscribed dodecagon + (x + 6y) circumscribed square + x circle]}$$

Area of circumscribed (103 dodecagon + 413 hexagons + 134 triangles)

$$\begin{aligned} &= 103(24 - 12\sqrt{3}) r^2 + 413(2\sqrt{3}) r^2 + 134(3\sqrt{3}) r^2 = (2472 - 8\sqrt{3}) r^2 \\ &= 103(\pi r^2 + 6c) + 413(\pi r^2 + 2d - 9c) + 134(\pi r^2 + 9d + 10.5c) \\ &= (103\pi r^2 + 618c) + (413\pi r^2 + 826d - 3717c) + (134\pi r^2 + 1206d + 1407c) \end{aligned}$$

$$\begin{aligned}
&= (650\pi r^2 - 1692c + 2032d) = 141(\pi r^2 - 12c) + 508(\pi r^2 + 4d) + 1(\pi r^2) \\
&= 141(3r^2) + 508(4r^2) + 1(\pi r^2) \\
&= [(423 + 2032 + (17 - 8\sqrt{3}))] r^2 = (2472 - 8\sqrt{3}) r^2 \\
&= \text{area of 141 inscribed dodecagon} + 508 \text{ circumscribed square} + 1 \text{ circle}
\end{aligned}$$

Supportive work

Exact equations	Derived equations
Area of ins. dodecagon = $3r^2 = (\pi r^2 - 12c)$	Area of inscribed square $= 2r^2 = \pi r^2 - (4d + 24c)$
Area of cir. square = $4r^2 = (\pi r^2 + 4d)$	Area of inscribed triangle $= (0.75\sqrt{3}) r^2 = \pi r^2 - (6.75d + 33.375c)$
	Area of inscribed hexagon = $(1.5\sqrt{3}) r^2 = \pi r^2 - (1.5d + 18.75c)$
	Area of circumscribed hexagon $= (2\sqrt{3}) r^2 = \pi r^2 + (2d - 9c)$
	Area of circumscribed triangle $= (3\sqrt{3}) r^2 = \pi r^2 + (9d + 10.5c)$
	Area of circumscribed dodecagon $= (24 - 12\sqrt{3}) r^2 = (\pi r^2 + 6c)$

Using above equations in following table. We get appropriate answer.

$$\begin{aligned}
\text{Sum of each row} &= \dots r^2 = (\dots \pi r^2 + \dots d - \dots c) = \dots (\pi r^2 + 4d) + \dots (\pi r^2 - 12c) \\
&= (\dots 4r^2 + \dots 3r^2)
\end{aligned}$$

Area	(Inscribed			+ Circumscribed)			=	
Figures	Square	Triangle	Hexagon	Hexagon	Triangle	Dodd	Ins. Dodd	Square
Value	$= 2r^2$	$(0.75\sqrt{3}) r^2$	$(1.5\sqrt{3}) r^2$	$(2\sqrt{3}) r^2$	$(3\sqrt{3}) r^2$	$12(2 - \sqrt{3}) r^2$	$= (\dots 3r^2$	$+ \dots 4r^2)$
$26r^2$	$= (1$			$+ 6$		$+ 1)$	$= (6$	$+ 2)$
$26r^2$	$= (1$	$+ 2$	$+ 1$	$+ 3$	$+ 1$	$+ 1)$	$= (10$	$-1)$
$28r^2$	$= (2$	$+ 2$	$+ 1$		$+ 3$	$+ 1)$	$= (8$	$+ 1)$
$30r^2$	$= (3$			$+ 3$	$+ 2$	$+ 1)$	$= (6$	$+ 3)$
$32r^2$	$= (4$	$+ 2$	$+ 1$	$+ 3$	$+ 1$	$+ 1)$	$= (16$	$-4)$

50r ²	= (1	+ 2	+ 3	+ 3	+ 4	+ 2)	= (10	+ 5)
54r ²	= (3	+ 2	+ 3	+ 3	+ 4	+ 2)	= (14	+ 3)
78r ²	= (3	+ 2	+ 7	+ 3	+ 6	+ 3)	= (18	+ 6)
82r ²	= (5	+ 4	+ 2	+ 6	+ 6	+ 3)	= (22	+ 4)
106r ²	= (5	+ 6	+ 5	+ 6	+ 8	+ 4)	= (30	+ 4)
110r ²	= (7	+ 6	+ 3	+ 6	+ 9	+ 4)	= (30	+ 5)
138r ²	= (9	+ 4	+ 4	+ 9	+ 11	+ 5)	= (30	+ 12)
164r ²	= (10	+ 4	+ 8	+ 12	+ 11	+ 6)	= (40	+ 11)
188r ²	= (10	+ 10	+ 11	+ 12	+ 12	+ 7)	= (60	+ 2)
200r ²	= (4	+ 8	+ 6	+ 15	+ 17	+ 8)	= (32	+ 26)
222r ²	= (3	+ 6	+ 3	+ 18	+ 21	+ 9)	= (18	+ 42)
256r ²	= (8	+ 4	+ 10	+ 9	+ 28	+ 10)	= (20	+ 49)
274r ²	= (5	+ 12	+ 14	+ 6	+ 30	+ 11)	= (38	+ 40)
334r ²	= (11	+ 18	+ 29	+ 21	+ 19	+ 13)	= (110	+ 1)
486r ²	= (3	+ 10	+ 9	+ 21	+ 59	+ 20)	= (2	+ 120)
754r ²	= (17	+ 42	+ 71	+ 45	+ 44	+ 30)	= (242	+ 7)
782r ²	= (7	+ 16	+ 22	+ 27	+ 95	+ 32)	= (14	+ 185)
1118r ²	= (19	+ 38	+ 43	+ 57	+ 111	+ 45)	= (134	+ 179)
1300r ²	= (38	+ 50	+ 73	+ 108	+ 83	+ 51)	= (312	+ 91)
1642r ²	= (17	+ 46	+ 59	+ 105	+ 157	+ 67)	= (162	+ 289)
2000r ²	= (4	+ 62	+ 67	+ 147	+ 185	+ 83)	= (192	+ 356)
5000r ²	= (40	+ 86	+ 131	+ 249	+ 567	+ 205)	= 112	+ 1166)
17000r ²	= (136	+ 234	+ 379	+ 840	+ 1980	+ 697)	= (64	+ 4202)
46000r ²	= (1484	+ 878	+ 1147	+ 2307	+ 4841	+ 1793)	= (3800	+ 8650)
100000r ²	= (2396	+ 1694	+ 2783	+ 5781	+ 10199	+ 3967)	= (7280	+ 19540)
123456r ²	= (4236	+ 3792	+ 5867	+ 9357	+ 9047	+ 4791)	= (24864	+ 12216)

CONCLUSION

Current value of pi does not give exact computational results, but from above equations it is proved that the derived value of pi gives exact results.

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