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A New and Simple Way to Prove Pythagorean Theorem

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Abstract

The Pythagoean Theorem is the most famous and the best known theorem in the world which brought extraordinary development, not only in mathematics, but also in non-mathematical fields. There seems to be about more than 370 proofs of this theorem exists, from Pythagoras' own proof in 6th century B.C. to modern proofs of this theorem.

This paper provides a new way to prove Pythagorean theorem algebraically using known results. It is very useful to prove Pythagorean theorem as it is one of the most important theorems in mathematics.

So, this paper contributes by providing a new and simple proof of Pythagorean theorem which even a school student can understand.

Keywords: Pythagorean Theorem, Proof, Similarity of Triangles.

INTRODUCTION

There are many proofs available for Pythagorean theorem, from Pythagoras' own proof in 6th century B.C., through Euclid's proof, Thabit ibn Qurra's proof in 9th century, the Indian mathematician Bhaskara's proof in 12th century, James Grafield's proof in 19th century to modern proofs of this theorem.

The book: *The Pythagorean Proposition* [1] written by E. S. Loomis, first edition published in 1928, is a major collection of proofs of Pythagoras theorem. This book has been collected as many as 370 different proofs of Pythagorean Theorem

The aim of this paper is to provide a new and simple way to prove Pythagorean Theorem using properties of similar of triangles.

PYTHAGOREAN THEOREM

The Pythagorean Theorem, first proved by Pythagoras of Samos who lived in 6th century B.C., is the fundamental relation in Euclidean geometry among the three sides of a right triangle in a plane. The statement of this theorm is as follows:

" In a space which satisfies the axioms of plane Euclidean geometry, the square of the hypotenuse of a right triangle is equal to the sum of the squares of its two other sides." [2]

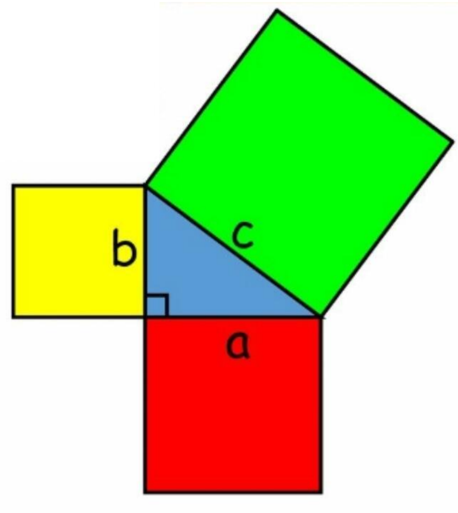

Figure 1

Mathematically, we can express the Pythagorean theorem as

$$a^2 + b^2 = c^2$$

where c is the length of hypotenuse of a right triangle and a and b are the length of its other two sides.

From figure 1,

Red Area + *Yellow Area* = *Green Area*

In order to prove Pythagorean theorem we will use the following known results:

→ Corresponding sides of similar triangles are in the same ratio.

→ Altitude on hypotenuse of a right triangle divides the triangle into two smaller triangles which are similar to original triangle.

Now we will prove this theorem algebraically.

PROOF OF PYTHAGOREAN THEOREM

Let us consider a right ΔABC where $\angle B$ is right angle and $AB = a, BC = b$ and $AC = c$.

We have to prove that $a^2 + b^2 = c^2$.

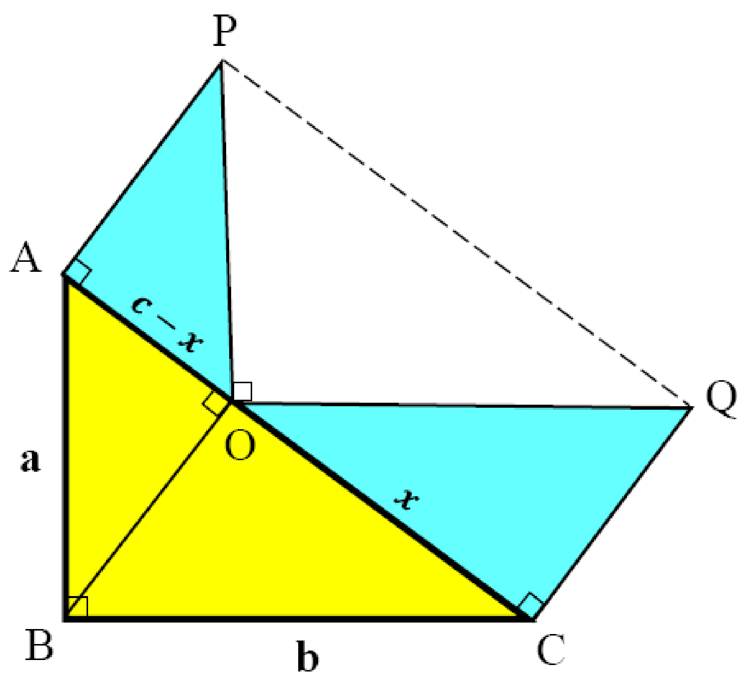

Figure 2.

From Figure 2, in ΔABC, OB is the altitude on AC. Let us construct ΔOAP which is congruent to ΔAOB and also construct ΔQCO which is congruent to ΔBOC as shown in Figure 2.

Let $CO = x$.

$\therefore OA = c - x$

By construction, as $\Delta OAP \cong \Delta AOB$,

$$AB = OP = a$$

and $$AP = OB$$

$\therefore ABOP$ is a parallelogram.

Also, as $\Delta QCO \cong \Delta BOC$,

$$BC = QO = b$$

and $$OB = CQ$$

$\therefore BCQO$ is a parallelogram.

From above discussion, we can conclude that

$$AP = OB = CQ \tag{1}$$

We know that in a right triangle, the altitude on hypotenuse divides the triangle into two smaller triangles which are similar to original triangle.

So we can conclude that

$$\Delta ABC \sim \Delta AOB \sim \Delta BOC \sim \Delta \text{OAP} \sim \Delta \text{QCO} \tag{2}$$

As $\Delta ABC \sim \Delta AOB$ and $\Delta ABC \sim \Delta BOC$,

$$\frac{AB}{BC} = \frac{AO}{OB}$$

and $$\frac{AB}{BC} = \frac{BO}{OC}$$

From the above two equations,

$$\frac{AO}{OB} = \frac{BO}{OC}$$

i.e. $$OB^2 = AO \cdot OC$$

or $$OB^2 = x(c - x)$$

or $$OB = \sqrt{x(c - x)}$$

From (1) and the above equation,

$$OB = AP = CQ = \sqrt{x(c - x)} \tag{3}$$

From (1) and (2),

$AP = CQ$ and $\angle PAO = \angle QCO = 90°$

$\therefore ACQP$ is a rectangle.

Now, $\angle POA + \angle POQ + \angle QOC = 180°$

But $\angle POA$ and $\angle QOC$ are complementary angles. From (2)

$\Rightarrow \quad \angle POQ = 90°$

i.e. ΔPOQ is a right triangle.

Now, from figure 2,

$ar(ACQP) = ar(PAO) + ar(QCO) + ar(POQ)$

$$\therefore AC \cdot CQ = \frac{PA \cdot AO}{2} + \frac{QC \cdot CO}{2} + \frac{PO \cdot OQ}{2}$$

$$\therefore \ c\sqrt{x(c-x)} = \frac{(\sqrt{x(c-x)})(c-x)}{2} + \frac{x\sqrt{x(c-x)}}{2} + \frac{ab}{2}$$

On dividing both sides of the equation by $\sqrt{x(c-x)}$, we get

$$c = \frac{c-x}{2} + \frac{x}{2} + \frac{ab}{2\sqrt{x(c-x)}}$$

$$\therefore c = \frac{c}{2} + \frac{ab}{2\sqrt{x(c-x)}}$$

$$\therefore \ \sqrt{x(c-x)} = \frac{ab}{c}$$

But $\sqrt{x(c-x)} = CQ$ From (3)

$$So, \ \ CQ = \frac{ab}{c}$$

As $\Delta ABC \sim \Delta QCO$,

$$\frac{OC}{CQ} = \frac{BC}{AB}$$

$$\therefore \frac{OC}{\frac{ab}{c}} = \frac{b}{a}$$

$$\therefore OC = \frac{b^2}{c} \tag{4}$$

As $\Delta ABC \sim \Delta OAP$,

$$\frac{OP}{AO} = \frac{AC}{AB}$$

$$\therefore \frac{a}{AO} = \frac{c}{a}$$

$$\therefore AO = \frac{a^2}{c} \tag{5}$$

From figure 2,

$$AO = AC - OC$$

$$\therefore AO = c - \frac{b^2}{c} \qquad \text{From (4)}$$

$$\therefore AO = \frac{c^2 - b^2}{c} \tag{6}$$

On comparing (5) and (6), we get

$$\frac{a^2}{c} = \frac{c^2 - b^2}{c}$$

$\Rightarrow \quad a^2 + b^2 = c^2$

or $\quad \boldsymbol{AB^2 + BC^2 = AC^2}$

This proves the Pythagorean theorem.

It does not matter whether $a < b, a = b$ or $a > b$, this new proof of Pythagorean theorem can be used for any right triangle.

RESULTS

By deriving one of the most important theorems in mathematics, the Pythagorean theorem, this paper adds a new proof of this theorem in the total number of proofs available in the world.

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