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On Nano αg^*s Closed Sets in Topological Spaces

[1]M. Dharani, [2]V. Senthilkumaran and [3]Y. Palaniappan

[1]M.Phil Scholar, Aringar Anna Government Arts college, Musiri-621211, Tamilnadu, India.

[2]Associate Professor of Mathematics, Aringar Anna Government Arts college, Musiri-621211, Tamilnadu, India.

[3]Associate Professor of Mathematics (Retd), Aringar Anna Government Arts college, Musiri-621211, Tamilnadu, India.

Abstract

In this paper, we introduce and study a new class of nano closed sets called nano αg^*s closed sets. Its relation to various other nano closed sets are obtained.

Keywords: Nano closed sets, Nano αg^*s closed sets

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1. INTRODUCTION

Lellis Thivagar etal [2] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called as nano open sets. Nano has origin in the Greek word 'Nanos' which means 'dwarf' in its modern scientific sense. The concerned topology is called nano topology as atmost in can have five elements. Njastad [6] and Mashour etal [5] investigated the concept α of open and α closed sets respectively in topological spaces. The generalized closed (briefly g closed) sets were analyzed by Levine [4]. Arya and Nour [1] introduced and studied weaker forms of closed sets namely, generalized semiclosed (briefly gs closed) sets using open sets. T.D. Rayanagouder [7] introduced a new class of closed sets, called αg^*semiclosed (briefly αg^*s closed) sets using gs open sets in topological spaces.

In this paper, we study nano αg^{*}s closed sets and its relation to various other nano closed sets.

2. PRELIMINARIES

Definition 2.1: [2] Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and R be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let X$\subseteq \mathcal{U}$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \underset{x\epsilon\mathcal{U}}{U}\{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X)= \underset{x\epsilon\mathcal{U}}{U}\{R(x): R(x) \cap X \neq \emptyset\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X)=U_R(X) - L_R(X)$.

Definition 2.2:[2] let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and τ_R(X)= $\{ \mathcal{U}, \varphi, L_R(X), U_R(X), B_R(X)\}$, where X$\subseteq$ $\mathcal{U}$. τ_R(X) satisfies the following axioms.

(i) $\mathcal{U}$ and $\varphi\epsilon\tau_R(X)$.

(ii) The union of the elements of any subcollection of τ_R(X) is in τ_R(X).

(iii) The intersection of the elements of any finite subcollection of τ_RX) is in τ_R(X). That is, τ_R(X) forms a topology on $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_R$(X)) as the nano topological space. The elements of τ_R(X) are called as nano-open sets. A set A is said to be nano closed if its complement is nano-open.

Definition 2.3: [2] If $(\mathcal{U}, \tau_R$(X)) is a nano topological space with respect to X where X$\subseteq$ $\mathcal{U}$ and if A$\subseteq \mathcal{U}$, then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A). That is, Nint(A) is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest nano closed set containing A.

Definition 2.4: A nano subset A of a nano topological space $(\mathcal{U}, \tau_R$(X)) is called

(i) Nano pre closed if Ncl Nint(A) $\subseteq A$

(ii) Nano semi closed if Nint Ncl(A) $\subseteq A$

(iii) Nano α closed if Ncl Nint Ncl(A) $\subseteq A$

(iv) Nano semi pre closed if Nint Ncl Nint(A) $\subseteq A$

(v) Nano regular closed if Ncl Nint(A) $=A$

For a nano subset A of $(\mathcal{U}, \tau_R$(X)) the intersection of all nano pre closed. (nano semi closed, nano α closed, nano semi pre closed) sets of $(\mathcal{U}, \tau_R$(X)) containing A is called nano pre closure of A (nano semi closure, nano α closure, nano semi pre closure of A) and is denoted by Npcl(A) (Nscl(A), Nαcl(A), Nspcl(A)).

Definition 2.5: A nano subset A of a nono topological space $(\mathcal{U}, \tau_R$(X)) is called a

1) Nano generalized closed (briefly Ng closed) if Ncl(A) $\subseteq U, whenever$ A$\subseteq U$ and U is nano open in $\mathcal{U}$.
2) Nano generalized semi closed (briefly Ngs closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
3) Nano α generalized regular closed (briefly Nαgr closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano regular open in $\mathcal{U}$.
4) Nano α generalized semi closed (briefly Nαgs closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano semi open in $\mathcal{U}$.
5) Nano α generalized closed (briefly Nαg closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
6) Nano generalized semi pre closed (briefly Ngsp closed) if Nspcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
7) Nano generalized pre closed (briefly Ngp closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano open in $\mathcal{U}$.
8) Nano g^{*} pre closed (briefly Ng^{*}p closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Ng open in $\mathcal{U}$.
9) Nano generalized pre regular closed (briefly Ngpr closed) if Npcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano regular open in $\mathcal{U}$.
10) Nano semi generalized closed (briefly Nsg closed) if Nscl(A) $\subseteq U$ whenever A$\subseteq U$ and U is nano semi open in $\mathcal{U}$.
11) Nano $g^{\#}\alpha$ closed (briefly N$g^{\#}\alpha$ closed) if Nαcl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Ng open in $\mathcal{U}$.
12) Nano $g^{\#}s$ closed (briefly N$g^{\#}s$ closed) if N$s$cl(A) $\subseteq U$ whenever A$\subseteq U$ and U is Nαg open in $\mathcal{U}$.

The complements of the above mentioned nano closed sets are respective nano open sets.

3. Nαg^*s CLOSED SETS

In this section, we introduce a new class of nano sets, called Nαg^*s closed sets in topological space and investigate some of their properties.

Definition 3.1: A nano subset A of a nano topological space $(\mathcal{U}, \tau_R(\mathrm{X}))$ is said to be nano αg^* semi closed (briefly Nαg^*s closed) if Nαcl$\subseteq U$ whenever A$\subseteq U$ and U is Ngs open in $\mathcal{U}$.

Theorem 3.2: Every nano closed set is Nαg^*s closed but not conversely.

Proof: Let A be a nano closed set in$(\mathcal{U}, \tau_R(\mathrm{X}))$. Note that N$\alpha cl(A) \subseteq cl(A)$ is always true and N$cl(A)$=A as A is nano closed. So if A$\subseteq G$. where G is Ngs open set in $(\mathcal{U}, \tau_R(\mathrm{X}))$, then N$\alpha cl(A) \subseteq G$. Hence A is N$\alpha g^*$s closed set in $(\mathcal{U}, \tau_R(\mathrm{X}))$.

Example 3.3: Let $\mathcal{U}= \{a, b, c\}$, $\mathcal{U}/R=\{\{a\}, \{b, c\}\}$, X = $\{a\}$, τ_R(X) = $\{ \mathcal{U}, \varphi, \{a\}\}$.

The subsets $\{b\}$, and $\{c\}$ are Nαg^*s closed sets but not nano closed sets in $(\mathcal{U}, \tau_R(\mathrm{X}))$.

Theorem 3.4: Every Nαg^*s – closed set is Ng^*s – closed set but not conversely.

Proof: Let A be Nαg^*s – closed set in $(\mathcal{U}, \tau_R(\mathrm{X}))$. Let A$\subseteq U$, where U is Ng- open and so it is Ngs- open set. Then Nαcl(A) $\subseteq U$. Note that N$s$cl(A) $\subseteq$ N$\alpha cl(A)$ is always true. Therefore Nscl(A) $\subseteq U$. Hence A is Ng^*s -closed set in $(\mathcal{U}, \tau_R(\mathrm{X}))$.

Example 3.5: Let $\mathcal{U}= \{a, b, c, d\}, \mathcal{U}/R =\{\{a\}, \{c\}, \{b, d\}\}$, X = $\{a, b\}$,

$\tau_R(\mathrm{X}) = \{ \mathcal{U}, \varphi, \{a\}, \{a, b, d\}, \{b, d\}\}$

$\{a\}$ is Ng^*s closed but not Nαg^*s closed.

Theorem 3.6: Every Nαg^*s – closed set is Nαgr– closed set but not conversely.

Proof: Let A be a Nαg^*s – closed set in $(\mathcal{U}, \tau_R(\mathrm{X}))$. Let G be a nano regular -open set and so it is Ngs- open set such that A$\subseteq G$. As A is Nαg^*s closed we have N$\alpha cl(A) \subseteq G$. Therefore N$\alpha cl(A) \subseteq G$. Hence A is Nαgr –closed set in $(\mathcal{U}, \tau_R(\mathrm{X}))$.

Example 3.7: Refer Example: 3.5

$\{a, b\}$ is Nαgr closed but not Nαg^*s closed.

Theorem 3.8: Every Nαg^*s - closed set is Nαgs-closed set but not conversely.

Proof: Let A be a Nαg^*s- closed set in $(\mathcal{U}, \tau_R(\mathrm{X}))$. Let A$\subseteq$ U, where U is nano semi-open, it is Ngs- open set. Then Nαcl(A) $\subseteq$U. Hence A is Nαgs-closed set in $(\mathcal{U}, \tau_R(\mathrm{X}))$.

Example 3.9: Refer Example: 3.5

$\{b, c\}$ is Nαgs closed but not Nαg^{*}s closed.

Theorem 3.10: Every Nαg^{*}s - closed set is Nαg-closed set but not conversely.

Proof: Since every Nαgs- closed set is Nαg- closed in $(\mathcal{U}, \tau_R$(X)). By theorem 3.8, every Nαg^{*}s - closed set is Nαgs-closed set in $(\mathcal{U}, \tau_R$(X)). Hence every Nαg^{*}s - nano closed set is Nαg-closed set in $(\mathcal{U}, \tau_R$(X)).

Example 3.11: Refer Example: 3.5

$\{b, c\}$ is Nαg closed but not Nαg^{*}s closed.

Theorem 3.12: Every Nα- closed set is Nαg^{*}s -closed set but not conversely.

Proof: Trivial

Example 3.13: Let $\mathcal{U}=\{a, b, c\}$, $\mathcal{U}/R=\{\{a, b\},\{c\}\}$, X = $\{a, b\}$, τ_R(X) = $\{\mathcal{U}, \varphi, \{a, b\}\}$

$\{a, c\}$ is Nαg^{*}s closed but not Nα closed.

Theorem 3.14: Every Nαg^{*}s - closed set is Ngs-closed set but not conversely.

Proof: Every Nαgs- closed set is Ngs- closed in $(\mathcal{U}, \tau_R$(X)). By theorem 3.8, every Nαg^{*}s - closed set is Nαgs-closed set in $(\mathcal{U}, \tau_R$(X)). Hence every Nαg^{*}s - nano closed set is Ngs-closed set in $(\mathcal{U}, \tau_R$(X)).

Example 3.15: Refer Example: 3.5

$\{a\}$ is Ngs closed but not Nαg^{*}s closed.

Theorem 3.16: Every Nαg^{*}s - closed set is Ngsp-closed set but not conversely.

Proof: Every Ngs- closed set is Ngsp- closed in $(\mathcal{U}, \tau_R$(X). By theorem 3.14, every Nαg^{*}s - closed set is Ngs-closed set. Hence Nαg^{*}s - closed set is Ngsp-closed set.

Example 3.17: Refer Example: 3.5

$\{b, c\}$ is Ngsp closed but not Nαg^{*}s closed.

Theorem 3.18: Every Nαg^{*}s - closed set is Ngp-closed set but not conversely.

Proof: Let A be a Nαg^{*}s – closed set in $(\mathcal{U}, \tau_R$(X)). Let G be nano open set and so it is Ngs- open set such that A⊆ G. Then Nαcl(A) ⊆G. But Npcl(A) ⊆Nαcl(A) is always true. Therefore Npcl(A) ⊆G. Hence A is Ngp-closed set $(\mathcal{U}, \tau_R$(X)).

Example 3.19: Refer Example: 3.5

$\{b\}$ is Ngp closed but not Nαg^{*}s closed.

Theorem 3.20: Every Nαg^{*}s - closed set is Ngpr-closed set but not conversely.

Proof: Every Ngp- closed set is Ngpr- closed in $(\mathcal{U}, \tau_R$(X)). By theorem 3.18, every Nαg^{*}s -closed set is Ngp-closed set in $(\mathcal{U}, \tau_R$(X)). Hence every Nαg^{*}s - closed set is Ngpr-closed set in $(\mathcal{U}, \tau_R$(X)).

Example 3.21: Refer Example: 3.5

$\{b\}$ is Ngpr closed but not Nαg^*s closed.

Theorem 3.22: Every Nαg^*s - closed set is Ng^*p-closed set but not conversely.

Proof: Let A be a Nαg^*s –closed set in $(\mathcal{U}, \tau_R$(X)). Let G be a Ng-open set and so it is Ngs- open set such that A⊆ G. Then Nαcl(A) ⊆G. Note that Npcl(A) ⊆ Nαcl(A) is always true. Therefore Npcl(A) ⊆G. Hence A is g^*p -closed set in $(\mathcal{U}, \tau_R$(X)).

Example 3.23: Refer Example: 3.5

$\{b\}$ is Ng^*p closed but not Nαg^*s closed.

Theorem 3.24: Every Nαg^*s - closed set is Nsg-closed set but not conversely.

Proof: Let A be a Nαg^*s – closed set in $(\mathcal{U}, \tau_R$(X)). Let G be a nano semi-open set and so it is Ngs- open set such that A⊆ G. Then Nαcl(A) ⊆G. Note that Nscl(A) ⊆ Nαcl(A) is always true. Therefore Nscl(A) ⊆G. Hence A is Nsg -closed set in $(\mathcal{U}, \tau_R$(X)).

Example 3.25 Refer Example: 3.5

$\{a\}$ is Nsg closed but not Nαg^*s closed.

Theorem 3.26: Every Nαg^*s - closed set is $Ng^\#\alpha$ -closed set but not conversely.

Proof: Let A be a Nαg^*s – closed set in $(\mathcal{U}, \tau_R$(X)). Let G be a Ng-open set and so it is Ngs- open set such that A⊆ G. Then Nαcl(A) ⊆G. Hence A is $Ng^\#\alpha$-closed set in $(\mathcal{U}, \tau_R$(X)).

Example 3.27: Refer Example: 3.5

$\{b, c\}$ is N$g^\#\alpha$ closed but not Nαg^*s closed.

Theorem 3.28: Every Nαg^*s - closed set is N$g^\# s$ -closed set but not conversely.

Proof: Let A be a Nαg^*s – closed set in $(\mathcal{U}, \tau_R$(X)). Let G be a Nαg-open set and so it is Ngs- open set such that A⊆ G. Then Nαcl(A) ⊆G. But Nscl(A) ⊆ Nαcl(A) is always true. Therefore Nscl(A) ⊆G. Hence A is N$g^\# s$ -closed set in $(\mathcal{U}, \tau_R$(X)).

Example 3.29: Refer Example: 3.5

$\{a\}$ is N$g^\# s$ closed but not Nαg^*s closed.

Remark 3.30: The concepts of nano semi-closed sets and Nαg^*s – closed sets are independent of each other as seen from the following examples.

Example 3.31: Refer Example: 3.13

$\{a, c\}$ is Nαg^*s closed but not nano semi closed.

Example 3.32: Refer Example: 3.5

$\{a\}$ is nano semi closed but not Nαg^*s closed.

Theorem 3.33: If A and B are Nαg^{*}s - closed sets, then AUB is Nαg^{*}s - nano closed set in $(\mathcal{U}, \tau_{\mathcal{R}}$(X)).

Proof: If AUB$\subseteq G$ and G is Ngs- open, then A$\subseteq G$ and B$\subseteq G$. Since A and B are Nαg^{*}s - closed sets, Nαcl(A) ⊆G and Nαcl(B) ⊆G and hence G ⊇ Nαcl(A) U Nαcl(B) = Nαcl(AUB). Thus AUB is Nαg^{*}s closed set in $(\mathcal{U}, \tau_R$(X)).

Theorem 3.34: A subset A is an Nαg^{*}s - closed set in $(\mathcal{U}, \tau_R$(X)) if and only if N$\alpha cl(A) - A$ contains no non-empty Ngs- closed set in $(\mathcal{U}, \tau_R$(X)).

Proof: Let F be a Ngs-closed set contained in N$\alpha cl(A) - A$. Then $A^c \subseteq F^c$ and F^c is a Ngs-open set of $(\mathcal{U}, \tau_R$(X)). Since A is Nαg^{*}s - closed set, Nαcl(A) $\subseteq F^c$. This implies F$\subseteq X -$ Nαcl(A) . Then F$\subseteq (X -$ Nαcl(A)) ∩ (Nαcl(A) $- A$). F$\subseteq \big(X -$ Nαcl(A)$\big)$ ∩ Nαcl(A) $= \varphi$ Therefore F=φ.

Conversely, suppose that Nαcl(A) − A contain no non empty Ngs- closed set in $(\mathcal{U}, \tau_R$(X)). Let G be a Ngs- open set such that A$\subseteq G$. If Nαcl(A) $\not\subset G$, then Nαcl(A) ∩ G^c is a non empty Ngs- closed set of Nαcl(A) $- A$, which is a contradiction. Therefore N$\alpha cl(A)$ ⊆G and hence A is an Nαg^{*}s- closed set in $(\mathcal{U}, \tau_R$(X)).

Theorem 3.35: If a subset A of a nano topological space $(\mathcal{U}, \tau_R$(X)) is Nαg^{*}s - closed such that A $\subseteq B \subseteq$ Nαcl(A), then B is also Nαg^{*}s - nano closed.

Proof: Let U be a Ngs-open set in X such that B$\subseteq U$, then A$\subseteq U$. Since A is Nαg^{*}s-closed, Nαcl(A) ⊆U. By hypothesis, $B \subseteq$ Nαcl(A) and hence Nαcl(B) ⊆ N$\alpha cl\big($Nαcl(A)$\big) =$ Nαcl(A) $\subseteq U$. Consequently, Nαcl(B) $\subseteq U$. Therefore B is also Nαg^{*}s - closed set in $(\mathcal{U}, \tau_R$(X)).

Theorem 3.36: If A is Ngs- open and Nαg^{*}s - closed set, then A is Nα- closed set.

Proof: Let A $\subseteq A$, where A is Ngs-open. Then Nαcl(A) ⊆A as A is Nαg^{*}s - closed set in $(\mathcal{U}, \tau_{\mathcal{R}}$(X)). But A$\subseteq$ Nαcl(A) is always true. Therefore A= Nαcl(A). Hence A is $N\alpha -$ closed set in $(\mathcal{U}, \tau_R$(X)).

Theorem 3.37: If A $\subseteq Y$ ⊆X and suppose that A is Nαg^{*}s - closed set in X, then A is Nαg^{*}s - closed relative to Y.

Proof: Given that A $\subseteq Y$ ⊆X and A is Nαg^{*}s - closed set in $(\mathcal{U}, \tau_R$(X)). To prove that A is Nαg^{*}s -closed relative to Y. Let A $\subseteq Y \cap G$, where G is nano open and so Ngs-open in $(\mathcal{U}, \tau_R$(X)). Since A is an Nαg^{*}s - closed set in X, A $\subseteq G$ which implies that Nαcl(A) ⊆G. That is Y∩ Nαcl(A) $\subseteq Y \cap G$ *w*here Y∩ Nαcl(A) is the Nα −closure of A in Y. Thus A is Nαg^{*}s - closed relative to Y.

Definition 3.38: A subset A of a nano topological space $(\mathcal{U}, \tau_R$(X)) is called Nαg^{*}-semi open (briefly Nαg^{*}s - open) set if its compliment A^c is Nαg^{*}s - nano closed.

Theorem 3.39: In a nano topological space $(\mathcal{U}, \tau_R$(X)). We have

1) Every nano pen set is Nαg^*s - open set.
2) Every Nαg^*s - open set is Ng^*S- open set
3) Every Nαg^*s - open set is Nαgr- open set
4) Every Nαg^*s - open set is Nαgs- open set
5) Every Nαg^*s - open set is Nαg - open set
6) Every Nα - open set is Nαg^*s - open set
7) Every Nαg^*s - open set is Ngs- open set
8) Every Nαg^*s - open set is Ngsp- open set
9) Every Nαg^*s - open set is Ngp- open set
10) Every Nαg^*s - open set is Ngpr- open set
11) Every Nαg^*s - open set is Ng^*p- open set
12) Every Nαg^*s - open set is Nsg- open set
13) Every Nαg^*s - open set is N$g^{\#}\alpha$ - open set
14) Every Nαg^*s - open set is N$g^{\#}s$- open set

Proof: obvious.

Theorem 3.40: A subset A of a nano topological space X is Nαg^*s - open if and only if F $\subseteq N\alpha int(A)$ wherever F $\subseteq A$ and F is Ngs -closed.

Proof: Assume that A is Nαg^*s - open. Then A^c is Nαg^*s closed. Let F be a Ngs-closed set in X containing in A. Then F^c is Ngs- open set containing $A^c\ in\ (\mathcal{U}, \tau_R$(X)). Since A^c is Nαg^*s- closed, Nαcl(A) $\subseteq F^c$.Taking complements on both sides, we have F $\subseteq N\alpha int(A)$.

Conversely, assume that F is contained in $N\alpha int(A), wh$enever F $\subseteq A$ and F is Ngs-closed set in $(\mathcal{U}, \tau_R$(X)). Let G be a Ngs- open set containing A^c.Then G^c is N$\alpha cl(\ A^c\ )$.Taking complements on both sides, N$\alpha cl(\ A^c\ ) \subseteq G$. Hence A^c is Nαg^*s - closed. Therefore A is Nαg^*s - open.

Theorem 3.41: If $N\alpha int(A) \subseteq B \subseteq A$ and if A is Nαg^*s - open set, then B is Nαg^*s - open in $(\mathcal{U}, \tau_R$(X)).

Proof: We have $N\alpha int(A) \subseteq B \subseteq A$. Then $A^c \subseteq B^c \subseteq$ N$\alpha cl(\ A^c\ )$and A^c is Nαg^*s – closed set. By theorem 3.35, B^c is Nαg^*s -closed. Hence B is Nαg^*s - open.

Theorem 3.42: The intersection of two Nαg^*s - open sets is again an Nαg^*s - open set.

Proof: The proof follows from the theorem 3.33.

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