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Cone b-Metric Spaces and Common Fixed Point Theorems of Generalized Contraction mappings

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Abstract

The aim of this article is to establish and extend some common fixed point results for generalized contraction mapping in complete cone b- metric spaces. Our presented theorems are generalizations of the results by Kurre, R. et al. [12].

Keywords: Fixed Point, contraction mappings, Complete Cone Metric Space, Complete cone b- metric space.

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I. INTRODUCTION

There are many researchers who have worked on the fixed point theory of contractive mappings (see, for example, [1, 2]). In [2], The police Mathematician Banach, S. (1922) demonstrated a crucial result of contraction mapping, which became known as the Banach contraction principle. Many writers have improved, expanded, and generalized the results of. Banach, S. [2] in many directions.

Recently, in 2011, Hussein and Shah [5] introduced the concept of cone b-metric spaces as a generalization of b-metric spaces and cone metric spaces. They established some topological properties in such spaces and improved some recent results about KKM mappings in the setting of a cone b-metric space. In 2013, Shi and Xu [6] proved common fixed point theorems for two weakly compatible self-mappings in cone b-metric spaces. In 2013, Huang and Xu [7], presented some new examples in cone b-metric spaces and proved some fixed point theorems of contractive mappings without the assumption of normality in cone b-metric spaces. In

[8], George, R. and. Fisher, B. (2013) obtained a common fixed point theorem of Taskovic type for three mappings in non-normal cone b-metric spaces, which will extend and generalize recent results of Huang and Xu [7]. In 2014, Tiwari, S.K. et al. [9], generalized and proved common fixed point theorems for self-mapping satisfying a general contractive condition on complete cone b-metric spaces of the results [6]. In cone b-metric spaces, [10] extended [5] and proved common fixed point theorems. In the sequel, Tiwari, S.K. and Kurre, R.[11], a generalized fixed point theory of cone b-metric spaces. In 2019, Kurre, R. [12] generalized the results of Saluja, G.S. [3] and Kumar, P. and Ansari, K. Z. [4].
The purpose of this article is to generalize and extend the fixed point theorem of generalized contraction mapping in cone b-metric space. Our results extend and improve the results of Kurre, R. [12].

II. PRELIMINARY NOTES

First, we recall the definition of cone metric spaces and some properties of theirs [13].
Definition: 2.1 [10]. Let E be a real Banach space and P a subset of E. Then P is called a cone if and only if:

1. P is closed, non-empty and $P \neq \{0\}$;
2. $a, b \in R$, $a, b \geq 0$ $x, y \in \boldsymbol{P} \Rightarrow ax+by \in \boldsymbol{P}$;
3. $x \in$ P and $-x \in$ P => $x = 0$.

For given a cone $P \subset E$, we define a Partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in \boldsymbol{P}$. We shall write $x \ll y$ to denote $x \leq y$ but $x \neq y$ to denote $y - x \in p^0$, where p^0 stands for the interior of *P*.
The cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$, $0 \leq x \leq y$ implies $\|x\| \leq K\|y\|$.The least positive number K satisfying the above is called the normal constant of *P*. The least positive number satisfying the above is called the normal constant *P*.
In the following, we always suppose that E is a Banach space, P is a cone in E with int $P \neq \emptyset$ and $\leq$ is a partial ordering with respect to *P*.

Definition 2.2[13]: Let *X* be a non – empty set. Suppose the mapping $d: X \times X \rightarrow E$ satisfies the following condition:

1. $0 < d(x, y)$ for all $x, y \in$ X and $d(x, y) = 0$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$ for all $x, y \in X$;
3. $d(x, y) \leq d(x, z) + d(z, y)$ for $all\ x, y, z \in X$.

Then *d* is called a cone metric on *X*, and (*X*, *d*) is called a cone metric space.

Example 2.3 [13]: Let $E = R^2$, $P = \{(x, y) \in E: x, y \geq 0\}$, $X = R$ and $d: X \times X \rightarrow E$, on defined by $d(x, y) = (|x - y|, \propto |x - y|)$ where $\propto \geq 0$ is a constant. Then (X, d) is a cone metric space.

Example: 2.4. Let E= l^1, P $= \{\{x_n\}n \geq 1 \in E: x_n \geq 0, for\ all\ n\}$ (X, d) a metric space and $d: X \times X \to E$, defined by $d(x, y) = \left\{\frac{d(x,y)}{2^n}\right\}$ $n \geq 1$.Then (X, d) is a cone metric space.

Definition 2.5[5]: Let X be a non – empty set. Suppose the mapping $d: X \times X \to E$ satisfies the following condition:

1. $\theta < d\,(x, y)$ for all $x, y \in X$ with $x \neq y$ and $d\,(x, y) = 0$ if and only if $x = y$;
2. $d\,(x, y) = d\,(y, x)$ for all $x, y \in X$;
3. $d\,(x, y) \leq s[\,d\,(x, z) + d\,(z, y)]\ for\ all\ x, y, z \in X.$

Then d is called a cone b- metric on X, and the pair (X, d) is called a cone b- metric space. It is obvious that cone b- metric spaces generalize b-metric spaces and cone metric spaces.

Example 2.5 [7]: Let $E = R^2$, $P = \{(x, y) \in E: x, y \geq 0\}$, $X = R$ and $d: X \times X \to E$, on defined by $d(x, y) = (|\,x - y\,|^p, \propto |x - y|^p)$ where $\propto \geq 0$ and $p > 1$are two constant. Then (X, d) is a cone b- metric space but not a cone metric spaces. In fact, we only need to prove (iii) in Definition 2.5 as follows:
Let $x, y, z \in X$. Set $u = x - z, v = z - y$, so $x - y = u + v$. from the inequality
$(a + b)^p \leq (2 \max\{a, b\})^p \leq 2^p(a^p + b^p)$ for all$a, b \geq 0$,
We have

$$\begin{aligned} |x - y|^p &= |u + v|^p \\ &\leq (|u| + |v|)^p \\ &\leq (|u|^p + |v|^p) \\ &= 2^p(|x - z|^p + |z - y|^p), \end{aligned}$$

This implies that $d\,(x, y) \leq s[\,d\,(x, z) + d\,(z, y)]$ with $s = 2^p > 1$. But $|x - y|^p$ $=|x - z|^p + |z - y|^p$, is impossible for all $x > z > y$. Indeed, taking account of the inequality $(a + b)^p > a^p + b^p$ for all $a, b > 0$,we arrive at

$$\begin{aligned} |x - y|^p &= |u + v|^p \leq (u + v)^p \\ &> u^p + v^p = (x - z)^p + (z - z)^p \end{aligned}$$

$= |x - z|^p + |z - y|^p$, for all $x > z > y$. thus, (iii) definition 2.5 is not satisfied, $i.e\ (X, d)$ is not a cone metric space. $lim\ n \to \infty\ xn = x\ or\ xn \to x, (n \to \infty)$.

Definition 2.6 [5]:Let (Ω, d) be a cone b- metric space, $\xi \in \Omega$ and $\{\xi_n\}$ a sequence in Ω. Then,

1. $\{\xi_n\}$ converges toξ whenever for every $c \in E$ with $\theta \ll c$, there is a natural number N such that $d\,(\xi_n, \xi \ll c$ for $all\ n \geq N$. We denote this by $\lim_{n\to\infty} \xi_n = \xi\ or\ \xi_n \to \xi, (n \to \infty)$.
2. $\{\xi_n\}$ is said to be a Cauchy sequence if for every $c \in E$ with $0 \ll c$, there is a natural number N such that $d(\xi_n, \xi_m) \ll c$ for all $n, m \geq N$.
3. (Ω, d) is called a complete cone metric space if every Cauchy sequence in X is Convergent.

Lemma 2.7[12]

1. Let P be a cone and $\{a_n\}$ be a sequence in E. If $c \in intP$ and $\theta \leq a_n \to \theta$ as $(n \to \infty)$, then there exist N such that for all $n > N$, we have $a_n \leq c$.
2. Let $x, y, z \in E$, if $x \leq y$ and $y \leq z$ then $x \ll z$.
3. Let P be a cone and $a \leq b + c$ for each $c \in intP$, then $a \leq b$.

Lemma 2.8[5] Let P be a cone and $\theta \leq u \leq c$ for each $c \in intP$, then $u = \theta$.

Lemma 2.9[14] Let P be a cone. If $u \in P$ and $u \leq Ku$ for some $0 \leq k \leq 1$then $u = \theta$.

III. Main Results

In this section we shall extend and generalize the results of R. Kurre, et al.[12] and obtain some common fixed point theorems of generalized contraction mappings in the framework of cone b-metric spaces.

Theorem 3.1: Let (Ω, d) be a complete cone b- metric space with the coefficient $s \geq 1$. Suppose $F, G: \Omega \to \Omega$ be a self mappings satisfying the generalized contraction mapping

$$d\big(F_\alpha(\xi), G_\beta(\phi)\big) \leq \lambda_1 d(\xi, \phi) + \lambda_2 d(\xi, F_\alpha(\xi)) + \lambda_3 d(\phi, G_\beta(\phi)) + \lambda_4 [d\big(\xi, G_\beta(\phi)\big) + d(\phi, F_\alpha(\xi))] \tag{3.1.1}$$

for all $\xi, \phi \in \Omega$, where $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \in [0,1)$ are constants such that $\lambda_1 + \lambda_2 + \lambda_3 + 2s\lambda_4 < 1$.

Then F and G have a unique common fixed point $\xi^* \in \Omega$. And for any $x \in X$,iterative sequence $\{F_\alpha{}^{2k+1}x\}$ and $\{G_\beta{}^{2k+2}x\}$ converges to the common fixed point.

Proof: Let ξ_0 be an arbitrary point in Ω. We define the iterative sequence $\{\xi_{2n}\}$ and $\{\xi_{2n+1}\}$ by

$$\xi_{2k+1} = F_\alpha \xi_{2k} = F_\alpha{}^{2k} x_0 \ldots \tag{3.1.2}$$

and

$$\xi_{2k+2} = G_\beta \xi_{2k+1} = G_\beta{}^{2k+1} x_0 \tag{3.1.3}$$

Then from (3.1.1), we have

$$\begin{aligned} d\big(\xi_{2k+1}, \xi_{2k}\big) &= d(F_\alpha \xi_{2k}, G_\beta \xi_{2k-1}) \\ &\leq \lambda_1 d(\xi_{2k}, \xi_{2k-1}) + \lambda_2 d(\xi_{2k}, F_\alpha \xi_{2k}) + \lambda_3 d(\xi_{2k-1}, G_\beta \xi_{2k-1}) \\ &\quad + \lambda_4 [d\big(\xi_{2k}, G_\beta \xi_{2k-1}\big) + d(\xi_{2k-1}, F_\alpha \xi_{2k})]. \\ &\leq \lambda_1 d(\xi_{2k}, \xi_{2k-1}) + \lambda_2 d(\xi_{2k}, \xi_{2k+1}) + \lambda_3 d(\xi_{2k-1}, \xi_{2k}) \\ &\quad + \lambda_4 [d(\xi_{2k}, \xi_{2k}) + d(\xi_{2k-1}, \xi_{2k+1})]. \\ &\leq \lambda_1 d(\xi_{2k}, \xi_{2k-1}) + \lambda_2 d(\xi_{2k}, \xi_{2k+1}) + \lambda_3 d(\xi_{2k-1}, \xi_{2k}) \\ &\quad + \lambda_4 d(\xi_{2k-1}, \xi_{2k+1}). \\ &\leq \lambda_1 d(\xi_{2k}, \xi_{2k-1}) + \lambda_2 d(\xi_{2k}, \xi_{2k+1}) + \lambda_3 d(\xi_{2k-1}, \xi_{2k}) + s\lambda_4 d(\xi_{2k}, \xi_{2k-1}) \\ &\quad + s\lambda_4 d(\xi_{2k}, \xi_{2k+1}) \end{aligned}$$

$$\leq (\lambda_1 + \lambda_3 + s\lambda_4)d(\xi_{2k}, \xi_{2k-1}) + (\lambda_2 + s\lambda_4)d(\xi_{2k}, \xi_{2k+1})$$

This implies that

$$d(\xi_{2k+1,}\xi_{2k}) \leq \frac{(\lambda_1 + \lambda_3 + s\lambda_4)}{1-(\lambda_2 + s\lambda_4)} d(\xi_{2k}, \xi_{2k-1})$$

$\leq rd(\xi_{2k}, \xi_{2k-1}), \ldots\ldots\ldots$ (3.1.4)

where $r = \frac{(\lambda_1+\lambda_3+s\lambda_4)}{1-(\lambda_2+s\lambda_4)} \leq 1$.As $\lambda_1 + \lambda_2 + \lambda_3 + 2\lambda_4 < 1$.Similarly, we obtain

$d(\xi_{2k+1,}\xi_{2k}) \leq rd(\xi_{2k-1}, \xi_{2k-2}) \ldots\ldots$ (3.1.5)

Using (3.1.3) in (3.1.2), we get

$d(\xi_{2k+1,}\xi_{2k}) \leq r^2 d(\xi_{2k-1}, \xi_{2k-2}) \ldots\ldots\ldots\ldots$ (3.1.6)

Continuing this process, we obtain

$d(\xi_{2k+1,}\xi_{2k}) \leq r^n d(\xi_1, \xi_0) \ldots\ldots$ (3.1.7)

For any $k \geq 1, p \geq 1$, we have

$$d(\xi_{2k}, \xi_{2k+2p}) \leq s[d(\xi_{2k}, \xi_{2k+1}) + d(\xi_{2k+1}, \xi_{2k+2p})]$$
$$\leq sd(\xi_{2k}, \xi_{2k+1}) + s^2 d(\xi_{2k+1}, \xi_{2k+2}) + s^3 d(\xi_{2k+2}, \xi_{2k+3})$$
$$+\ldots\ldots\ldots+s^{2p-1}d(\xi_{2k+2p-2}, \xi_{2k+2p-1}) + s^{2p-1}d(\xi_{2k+2p-1}, \xi_{2k+2p})$$
$$\leq sr^{2k}d(\xi_1, \xi_0) + s^2 r^{2k+1}d(\xi_1, \xi_0) + s^3 r^{2k+2}d(\xi_1, \xi_0)$$
$$+\ldots\ldots\ldots+s^{2p}r^{2k+2p-1}d(\xi_1, \xi_0).$$
$$= sr^{2k}[1 + (sr) + (sr)^2 + (sr)^3 \ldots\ldots\ldots\ldots + (sr)^{2p-1}]d(\xi_1, \xi_0)$$
$$\leq \frac{sr^{2k}}{1-sr} d(\xi_1, \xi_0).$$

Let $0 \ll c$ be given. Notice that $\frac{sr^{2m}}{1-sr} d(\xi_1, \xi_0) \to 0$ as $m \to \infty$ for any p. Making full use of lemma 2.7(i), we find $m_0 \in N$ such that

$$\frac{sr^{2k}}{1-sr} d(\xi_1, \xi_0) \ll c, \text{ for each } k \geq k_0.$$

Thus, $d(\xi_{2k}, \xi_{2k+2p}) \leq \frac{sr^{2k}}{1-sr} d(\xi_1, \xi_0) \ll c$, for all $k \geq 1, p > 1$. So, by lemma 2.7(ii) $\{\xi_{2k}\}$ is a Cauchy sequence in (Ω, d). Since (Ω, d) is a complete cone b- metric space, there exist $\xi^* \in X$ such that $\xi_{2k} \to \xi^*$ as $k \to \infty$ Taken $k_0 \in N$ such that $d(\xi_{2k}, \xi^*) \ll \frac{r(1- s(\lambda_2+\lambda_4))}{s(1+\lambda_1+\lambda_4)}$ for all $k \geq k_0$.Hence

$$d(F_\alpha\xi^*, \xi^*) \leq s[d(F_\alpha\xi^*, F\xi_{2k}) + d(F_\alpha\xi_{2k}, \xi^*)]$$
$$= sd(F_\alpha\xi^*, F_\alpha\xi_{2k}) + sd(F_\alpha \xi_{2k}, \xi^*)$$
$$= sd(F_\alpha\xi^*, F_\alpha\xi_{2k}) + sd(\xi_{2k+1}, \xi^*)$$
$$\leq s[\lambda_1 d(\xi^*, \xi_{2k}) + \lambda_2 d(\xi^*, F_\alpha\xi^*) + \lambda_3 sd(\xi_{2k,}F_\alpha\xi_{2k})$$
$$+\lambda_4 \{d(\xi^*, F_\alpha\xi_{2k}) + d(\xi_{2k}, F_\alpha\xi^*)\}] + sd(\xi_{2k+1,}\xi^*).$$
$$\leq s[\lambda_1 d(\xi, \xi_{\ 2k}) + \lambda_2 d(\xi^*, F_\alpha\xi^*) + \lambda_3 d(\xi_{2k,}\xi_{2k+1})$$
$$+\lambda_4\{d(\xi^*, \xi_{2k+1}) + d(\xi_{2k}, F_\alpha \xi^*)\}] + sd(\xi_{\ 2k+1,} \xi^*).$$
$$\leq s[\lambda_1 d(\xi^*, \xi_{2k}) + \lambda_2 d(\xi^*, F_\alpha \xi^*) + s\lambda_3\{d(\xi_{2k,}\xi^*) + d(\xi^*, \xi_{2k+1})\}$$
$$+\lambda_4\{d(\xi^*, \xi_{2k+1}) + sd(\xi_{2k}, \xi^*) + sd(\xi^*, F_\alpha\xi^*)\}] + sd(\xi_{2k+1,} \xi^*).$$
$$= s[\lambda_1 d(\xi^*, \xi_{2k}) + \lambda_2 d(\xi^*, F_\alpha\xi^*) + s\lambda_3\{d(\xi_{2k,}\xi^*) + d(\xi^*, \xi_{2k+1})\}$$
$$+\lambda_4\{d(\xi^*, \xi_{2k+1}) + sd(\xi_{2k}, \xi^*) + sd(\xi^*, F_\alpha\xi^*)\}] + sd(\xi_{2k+1,} \xi^*)$$

This implies that

$$d(F_\alpha\xi^*, \xi^*) \leq s(\lambda_2 + s\lambda_4)d(F_\alpha\xi^*, \xi^*) + s(\lambda_1 + s\lambda_3 + s\lambda_4)d(\xi_{2k}, \xi^*)$$

$$+ s(1+s\lambda_3+\lambda_4)\, d(\xi^*,\xi_{2k+1}).$$
$$1\text{-}s(\lambda_2+s\lambda_4)d(F_\alpha\xi^*,\xi^*) \le s(\lambda_1+s\lambda_3+s\lambda_4)d(\xi_{2k},\xi^*) + s(1+s\lambda_3+\lambda_4)\, d(\xi^*,\xi_{2k+1})$$

So, $d(F_\alpha\xi^*,\xi^*) \le \frac{s(\lambda_1+s\lambda_3+s\lambda_4)}{1-s(\lambda_2+s\lambda_4)}\, d(\xi_{2k},\xi^*) + \frac{s(1+s\lambda_3+\lambda_4)}{1-s(\lambda_2+s\lambda_4)}\, d(\xi_{2k,}\xi_{2k+1}) \ll c$ for each $k \ge k_0$. Then by lemma 2.8 we deduce that $d(F_\alpha\xi^*,\xi^*)=0, i.e., F_\alpha\xi^* = \xi^*$.That is ξ^* is a fixed point of F. Similarly, we can prove that, $G_\beta\xi^* = \xi^*$. That is ξ^* is a fixed point of G. Therefore, $F_\alpha\xi^* = \xi^* = G_\beta\xi^*$. Hence, ξ^* is a common fixed point of F and G.

Now to prove its uniqueness, If ξ^{**} is another common fixed point of F and G such that $F\xi^{**} = \xi^{**} = G\xi^{**}$, then by the given condition (3.1.1), we have

$$d(\xi^*,\xi^{**}) = d(F_\alpha\xi^*, G_\beta\xi^{**})$$
$$\le \lambda_1 d(\xi^*,\xi^{**}) + \lambda_2 d(\xi^*,F_\alpha\xi^*) + \lambda_3 d(\xi^{**},G_\beta\xi^{**})$$
$$+\lambda_4\,[d(\xi^*,G_\beta\xi^{**}) + d(\xi^{**},F_\alpha\,\xi^*)]$$
$$\le (\lambda_1+2s\lambda_4)(\xi^*,\xi^{**}).$$

By lemma 2.9, $\xi^* = \xi^{**}$.Therefore, ξ^* is unique common fixed point of F and G.This completes the proof of the theorem.

From theorem 3.1, we obtain the following results as corollaries.

Corollary 3.2: Let (Ω, d) be a complete cone b- metric space with the coefficient $s \ge 1$. Suppose $F, G\colon \Omega \to \Omega$ be a self mappings satisfying contractive map

$$d(F_\alpha(\xi), G_\beta(\phi)) \le \lambda d(\xi,\phi)$$

for all $\xi,\phi \in \Omega$, where $\lambda \in [0,1/s]$ is a constant.Then Then F and G have a unique common fixed point $\xi^* \in \Omega$. And for any $x \in X$,iterative sequence $\{{F_\alpha}^{2k+1}x\}$ and $\{{G_\beta}^{2k+2}x\}$ converges to the common fixed point.

Proof: The proof of the corollary 3.2 is immediately follows from theorem 3.1 by taking $\lambda_2=\lambda_3=\lambda_4=0$ and $\lambda_1=\lambda$. This completes the proof.

Corollary 3.3: Let (Ω, d) be a complete cone b- metric space with the coefficient $s \ge 1$. Suppose $F, G\colon \Omega \to \Omega$ be a self mappings satisfying the generalized Contraction map:

$$d(F_\alpha(\xi), G_\beta(\phi)) \le \lambda[d(\xi, F_\alpha(\xi)) + d(\phi, G_\beta(\phi))]$$

for all $\xi,\phi \in \Omega$, where $\lambda \in [0,1/2s]$ is a constant. Then F and G have a unique common fixed point $\xi^* \in \Omega$. And for any $x \in X$,iterative sequence $\{{F_\alpha}^{2k+1}x\}$ and $\{{G_\beta}^{2k+2}x\}$ converges to the common fixed point.

Proof: The proof of the corollary 3.3 is immediately follows from theorem 3.1 by taking $\lambda_1=\lambda_4=0$ and $\lambda_2=\lambda_3=\lambda$. This completes the proof.

Corollary 3.4: Let (Ω, d) be a complete cone b- metric space with the coefficient $s \ge 1$. Suppose $F, G\colon \Omega \to \Omega$ be a self mappings satisfying the generalized Contraction map:

$$d(F_\alpha(\xi), G_\beta(\phi)) \leq \lambda[d\left(\xi, G_\beta(\phi)\right) + d(\phi, F_\alpha(\xi))]$$

for all $\xi, \phi \in \Omega$, where $\lambda \in [0,1/2s]$ is a constant. Then F and G have a unique common fixed point $\xi^* \in \Omega$. And for any $x \in X$, iterative sequence $\{{F_\alpha}^{2k+1}x\}$ and $\{{G_\beta}^{2k+2}x\}$ converges to the common fixed point.

Proof: The proof of the corollary 3.4 is immediately follows from theorem 3.1 by taking $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and $\lambda_4 = \lambda$.This completes the proof.

Theorem 3.5: Let (Ω, d) be a complete cone b- metric space with the coefficient $s \geq 1$. Suppose $F, G: \Omega \to \Omega$ be any two self mappings satisfying the generalized contraction mapping

$d(F_\alpha(\xi), G_\beta(\phi)) \leq \lambda_1[d(\xi, \phi) + d(\xi, F_\alpha(\xi)) + d(\phi, G_\beta(\phi))]$

$+ \lambda_2[d(\xi, G_\beta(\phi)) + d(\phi, F_\alpha(\xi))]$.... .... (3.5.1)

for all $\xi, \phi \in \Omega$, where $\lambda_1, \lambda_2, \in [0,1)$are constants such that $2(\lambda_1 + \lambda_2 s) < 1$. Then Then F and G have a unique common fixed point $\xi^* \in \Omega$. And for any $\xi \in \Omega$,iterative sequence $\{{F_\alpha}^{2l+1}\xi\}$ and $\{{G_\beta}^{2l+2}\xi\}$ converges to the common fixed point.

Proof: Let ξ_0 be an arbitrary point in Ω. We define the iterative sequence $\{\xi_{2n}\}$ and $\{\xi_{2n+1}\}$ by

$\xi_{2l+1} = F_\alpha \xi_{2l} = {F_\alpha}^{2l} x_0$......... (3.5.2)

and

$\xi_{2l+2} = G_\beta \xi_{2l+1} = {G_\beta}^{2l+1} x_0$...... (3.5.3)

Put $\xi = \xi_{2l}$ and $\phi = \xi_{2l-1}$ in (3.5.1) we get

$$\begin{aligned} d(\xi_{2l+1,}\xi_{2l}) &= d(F_\alpha \xi_{2k,} G_\beta \xi_{2k-1}) \\ &\leq \lambda_1[d(\xi_{2l}, \xi_{2l-1}) + d(\xi_{2l}, F_\alpha \xi_{2l}) + d(\xi_{2l-1}, G_\beta \xi_{2l-1})] \\ &\quad + \lambda_2[(d(\xi_{2l}, G_\beta \xi_{2l-1}) + d(\xi_{2l-1}, F_\alpha \xi_{2l})] \\ &\leq \lambda_1[d(\xi_{2l}, \xi_{2l-1}) + d(\xi_{2l}, \xi_{2l+1}) + d(\xi_{2l-1}, \xi_{2l})] \\ &\quad + \lambda_2[(d(\xi_{2l}, \xi_{2l}) + d(\xi_{2l-1}, \xi_{2l+1})]. \\ &= \lambda_1[d(\xi_{2l}, \xi_{2l-1}) + d(\xi_{2l}, \xi_{2l+1}) + d(\xi_{2l-1}, \xi_{2l})] \\ &\quad + s\lambda_2[d(\xi_{2l-1}, \xi_{2l}) + d(\xi_{2l}, \xi_{2l+1})] \\ &\leq (2\lambda_1 + s\lambda_2) d(\xi_{2l}, \xi_{2l-1}) + (\lambda_1 + s\lambda_2) d(\xi_{2l}, \xi_{2l+1}) \end{aligned}$$

This implies that

$$d(\xi_{2l+1,}\xi_{2l}) \leq \frac{(2\lambda_1 + +s\lambda_2)}{1 - (\lambda_1 + s\lambda_2)} d(\xi_{2l}, \xi_{2l-1})$$

$\leq hd(\xi_{2l}, \xi_{2i-1})$, ... (3.5.4)

where $h = \frac{(2\lambda_1 + +s\lambda_2)}{1-(\lambda_1 + s\lambda_2)} \leq 1$.As $2(\lambda_1 + \lambda_2 s) < 1$, we obtain that $h < 1$. Similarly, we obtain

$d(\xi_{2l}, \xi_{2l-1}) \leq hd(\xi_{2l-1}, \xi_{2l-2})$ .... (3.5.5)

Using (3.5.5) in (3.5.4), we get

$d(\xi_{2l+1,}\xi_{2i}) \leq h^2 d(\xi_{2l-1}, \xi_{2l-2})$............ (3.5.6)

Continuing this process, we obtain

$d(\xi_{2l+1,}\xi_{2l}) \leq h^n d(\xi_1, \xi_0)$...... (3.5.7)

For any $l \geq 1, p \geq 1$, we have

$$d(\xi_{2l}, \xi_{2l+p}) \leq s[d(\xi_{2l}, \xi_{2l+1}) + d(\xi_{2l+1}, \xi_{2l+p})]$$
$$\leq sd(\xi_{2l}, \xi_{2l+1}) + s^2 d(\xi_{2l+1}, \xi_{2l+2}) + s^3 d(\xi_{2l+2}, \xi_{2l+3})$$
$$+\ldots\ldots\ldots + s^{2p-1} d(\xi_{2l+2p-2}, \xi_{2l+2p-1}) + s^{2p-1} d(\xi_{2l+2p-1}, \xi_{2l+2p}).$$
$$\leq sh^{2l} d(\xi_1, \xi_0) + s^2 h^{2l+1} d(\xi_1, \xi_0) + s^3 h^{2l+2} d(\xi_1, \xi_0)$$
$$+\ldots\ldots\ldots + s^{2p} h^{2l+2p-1} d(\xi_1, \xi_0).$$
$$= sh^{2l}[1 + (sh) + (sh)^2 + (sh)^3 \ldots\ldots\ldots + (sh)^{2p-1}] d(\xi_1, \xi_0).$$
$$\leq \frac{sh^{2l}}{1-sh} d(\xi_1, \xi_0).$$

Let $0 \ll r$ be given. Notice that $\frac{sh^{2l}}{1-sh} d(\xi_1, \xi_0) \to 0$ as $l \to \infty$ for any p. Making full use of lemma 2.7(i), we find $l_0 \in N$ such that $\frac{sh^{2l}}{1-sh} d(\xi_1, \xi_0) \ll \epsilon$, for each $l \geq l_0$. Thus, $d(\xi_{2l}, \xi_{2l+p}) \ll \epsilon$ for all $l \geq 1, p > 1$. So, by lemma 2.7(ii), $\{\xi_{2n}\}$ is a Cauchy sequence in (Ω, d). Since (Ω, d) is a complete cone b- metric space, there exist $u \in X$ such that $\xi_{2l} \to u$, as $l \to \infty$. Taken $l_0 \in N$ such that $d(\xi_{2l,} u) \ll \frac{r(1-s(\lambda_2+s\lambda_2)}{s(2\lambda_1+\lambda_4)}$ for all $l \geq l_0$. Hence

$$d(F_\alpha u, u) \leq s[d(F_\alpha u, F_\alpha \xi_{2l}) + d(F_\alpha \xi_{2l}, u)]$$
$$= sd(F_\alpha u, \xi_{2l}), + sd(F_\alpha \xi_{2l}, u)$$
$$\leq s\lambda_1[d(u, \xi_{2l}) + d(u, F_\alpha u) + d(\xi_{2l,} F_\alpha \xi_{2l})] + \lambda_2[d(u, F_\alpha \xi_{2l}) + d(\xi_{2l}, F_\alpha u)]$$
$$+ sd(\xi_{2l+1,}, u).$$
$$\leq s[\lambda_1\{d(u, \xi_{2l}) + d(u, F_\alpha u) + d(\xi_{2l,} \xi_{2l-1}) + \lambda_2 s\{d(u, \xi_{2l+1}) + d(\xi_{2l}, F_\alpha u)\}]$$
$$+ sd(\xi_{2l+1,}, u)$$
$$= s[\lambda_1\{d(u, \xi_{2l}) + d(u, F_\alpha u) + d(\xi_{2l,} u) + d(u, \xi_{2l+1})\} + \lambda_2\{d(u, \xi_{2l+1}) +$$
$$(sd(\xi_{2l}, u) + sd(u, F_\alpha u))\}]$$
$$+ sd(\xi_{2l+1,,} u).$$

This implies that

$$d(F_\alpha u, u) \leq s(\lambda_1 + s\lambda_2) d(F_\alpha u, u) + s(2\lambda_1 + \lambda_2) d(u, \xi_{2l})$$
$$+ s(\lambda_1 + \lambda_2 + 1) d(u, \xi_{2l+1})$$
$$1 - s(\lambda_2 + s\lambda_2) d(F_\alpha u, u) \leq s(2\lambda_1 + \lambda_2) d(u, \xi_{2l}) + s(\lambda_1 + \lambda_2 + 1) d(u, \xi_{2l+1}).$$

So $d(F_\alpha u, u) \leq \frac{s(2\lambda_1+\lambda_2)}{1-s(\lambda_2+s\lambda_2)} d(\xi_{2l}, u) + \frac{s(\lambda_1+\lambda_2+1)}{1-s(\lambda_2+s\lambda_2)} d(u, \xi_{2l+1}) \ll r$, for each $l \geq l_0$. Then by lemma 2.8 we deduce $d(F_\alpha u, u) = 0$, i.e., $Fu = u$. That is, u is a fixed point of F.

Similarly, we can prove that, $G_\beta u = u$. That is u is a fixed point of G. Therefore, $F_\alpha u = u = G_\beta u$. Hence, u is a common fixed point of F and G.

Now to prove its uniqueness, If u^* is another common fixed point of F and G such that $Fu^* = u^* = Gu^*$, then by the given condition (3.5.1), we have

$$d(u, u^*) = d(F_\alpha u, G_\beta u^*)$$
$$\leq \lambda_1[d(u, u^*) + d(u, F_\alpha u) + d(u^*, G_\beta u^*) + \lambda_2 [d(u, G_\beta u^*) + d(u^*, F_\alpha u)].$$
$$\leq (\lambda_1 + 2s\lambda_4) d(u, u^*).$$

By lemma 2.9, $u = u^*$.Therefore, u is unique common fixed point of F and G.This completes the proof of the theorem.

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Theorem 3.6: Let (Ω, d) be a complete cone b- metric space with the coefficient $s \geq 1$. Suppose $T, F: \Omega \rightarrow \Omega$ be a self mappings satisfying the following contraction mappings

$$d\left(T_\alpha{}^a(\xi), F^b{}_\beta(\phi)\right) \leq \lambda_1[d\left(\xi, T_\alpha{}^a(\xi)\right) + d(\phi, F^b{}_\beta(\phi))] + \lambda_2\left[d\left(\xi, F^b{}_\beta(\phi)\right) + d\left(\phi, T_\alpha{}^a(\xi)\right)\right]$$
$$+ \lambda_3 max[d\left(\xi, T_\alpha{}^a(\xi)\right), d(\phi, F^b{}_\beta(\phi)), d\left(\xi, F^b{}_\beta(\phi)\right)] + \lambda_4[d(\xi, \phi) + d\left(\phi, T_\alpha{}^a(\xi)\right)] \ldots\ldots\ldots (3.6.1)$$

for all $x, y \in \Omega$, and $a, b \geq 0$ where $\lambda_1, \lambda_2, \lambda_3 \in [0,1)$ are constants such that $2(\lambda_1 + s\lambda_2 + s\,\lambda_3) + s\,\lambda_4 < 1$. Then F has a unique fixed point in X. Furthermore, the iterative sequences Then F and G have a unique common fixed point $\xi^* \in \Omega$. And for any $\xi \in \Omega$,iterative sequence $\left\{F_\alpha{}^{2i+1}\xi\right\}$ and $\left\{G_\beta{}^{2i+2}\xi\right\}$ converges to the common fixed point.

Proof: Let ξ_0 be an arbitrary point in Ω. We define the iterative sequence $\{\xi_{2n}\}$ and $\{\xi_{2n+1}\}$ by

$$\xi_{2i+1} = T_\alpha^a \xi_{2i} = T_\alpha^{a^{2l}} \xi_0 \ldots\ldots\ldots \quad (3.6.2)$$

and

$$\xi_{2i+2} = F_\beta^b \xi_{2i+1} = F_\beta^{b^{2l+1}} \xi_0 \ldots\ldots \quad (3.6.3)$$

Put $\xi = \xi_{2l}$ and $\phi = \xi_{2l-1}$ in (3.6.1) we get

$$d\left(\xi_{2i+1,,}\xi_{2i}\right) = d(T_\alpha^a \xi_{2i}, F_\beta^b \xi_{2i-1})$$
$$\leq \lambda_1[d(\xi_{2i}, T_\alpha^a \xi_{2i}) + d\left(\xi_{2i-1}, F_\beta^b \xi_{2i-1}\right)] + \lambda_2[d\left(\xi_{2i}, F_\beta^b \xi_{2i-1}\right) + d(\xi_{2i-1}, T_\alpha^a \xi_{2i})]$$
$$+\lambda_3 \max[\, d(\xi_{2i}, T_\alpha^a \xi_{2i}) + d\left(\xi_{2i-1}, F_\beta^b \xi_{2i-1}\right), d\left(\xi_{2i}, F_\beta^b \xi_{2i-1}\right)]$$
$$+\lambda_4[d(\xi_{2i}, \xi_{2i-1}) +, d(\xi_{2i-1}, T_\alpha^a \xi_{2i})].$$
$$\leq \lambda_1[d(\xi_{2i}, \xi_{2i+1}) + d(\xi_{2i-1}, \xi_{2i})] + \lambda_2[d(\xi_{2i}, \xi_{2i}) + d(\xi_{2i-1}, \xi_{2i+1})]$$
$$+\lambda_3 \max[\, d(\xi_{2i}, \xi_{2i+1}), d(\xi_{2i-1}, \xi_{2i}), d(\xi_{2i}, \xi_{2i})\,] + \lambda_4[\, d(\xi_{2i}, \xi_{2i-1}) + d(\xi_{2i-1}, \xi_{2i+1})].$$
$$= \lambda_1[d(\xi_{2i}, \xi_{2i+1}) + d(\xi_{2i-1}, \xi_{2i})] + \lambda_2 d(\xi_{2i-1}, \xi_{2i+1})$$
$$+ \lambda_3 \max[d(\xi_{2i}, \xi_{2i+1}), d(\xi_{2i-1}, \xi_{2i})\,]$$
$$+\lambda_4[\, d(\xi_{2i}, \xi_{2i-1}) + d(\xi_{2i-1}, \xi_{2i+1})].$$
$$= \lambda_1[d(\xi_{2i}, \xi_{2i+1}) + d(\xi_{2i-1}, \xi_{2i})\,)] + s\lambda_2[d(\xi_{2i-1}, \xi_{2i}) + d(\xi_{2i}, \xi_{2i+1})]$$
$$+\lambda_3 \max[\, d(\xi_{2i}, \xi_{2i+1}), d(\xi_{2i-1}, \xi_{2i})] + \lambda_4[\, d(\xi_{2i}, \xi_{2i-1}) + s\{d(\xi_{2i-1}, \xi_{2i}) + d(\xi_{2i}, \xi_{2i+1})\}]$$
$$\leq [\lambda_1 + s\lambda_2 + \lambda_3 + (1 + s)\lambda_4] d(\xi_{2i-1}, \xi_{2i}\,) + (\lambda_1 + s\lambda_2 + \lambda_3 + s\lambda_4) d\left(\xi_{2i+1,}\xi_{2i}\right).$$

Therefore,

$$1-[\lambda_1 + s\lambda_2 + \lambda_3 + s\lambda_4]\, d\left(\xi_{2i+1,}\xi_{2i}\right) \leq [\lambda_1 + s\lambda_2 + \lambda_3 + (1 + s)\lambda_4] d(\xi_{2i-1}, \xi_{2i}\,)$$

Hence,

$$d\left(\xi_{2i+1,}\xi_{2i}\right) \leq \frac{[\lambda_1 + s\lambda_2 + \lambda_3 + (1+s)\lambda_4]}{1-[\lambda_1 + s\lambda_2 + \lambda_3 + s\lambda_4]}\, d(\xi_{2i-1}, \xi_{2i}\,)$$

$$\leq hd(\xi_{2i},\xi_{2i-1}), \text{(3.6.4)}$$

where $h = \frac{[\lambda_1+s\lambda_2+\lambda_3+(1+s)\lambda_4]}{1-[\lambda_1+s\lambda_2+\lambda_3+s\lambda_4]} \leq 1$ .As $2(\lambda_1+s\lambda_2+s\,\lambda_3)+s\lambda_4<1$ we obtain that $h<1$. Similarly, we obtain

$hd(\xi_{2i},\xi_{2i-1}) \leq hd(\xi_{2i-1},\xi_{2i-2})$ …… (3.6.5)

Using (3.6.5) in (36.4), we get

$d(\xi_{2i+1,}\xi_{2i}) \leq h^2 d(\xi_{2i-1},\xi_{2i-2})$………. (3.6.6)

Continuing this process, we obtain

$d(\xi_{2i+1,}\xi_{2i}) \leq h^n d(\xi_1,\xi_0)$ (3.6.7)

For any $m \geq 1, p \geq 1$, we have

$$\begin{aligned}
d(\xi_{2m},\xi_{2m+2p}) &\leq s[d(\xi_{2m},\xi_{2m+1})+d(\xi_{2m+1},\xi_{2m+2p})]\\
&\leq sd(\xi_{2m},\xi_{2m+1})+s^2d(\xi_{2m+1},\xi_{2m+2})+s^3d(\xi_{2m+2},\xi_{2m+3})\\
&+\ldots\ldots+s^{2p-1}d(\xi_{2m+2p-2},\xi_{2m+2p-1})+s^{p-1}d(\xi_{2m+2p-1},\xi_{2m+2p})\\
&\leq sh^{2m}d(\xi_1,\xi_0)+s^2h^{2m+1}d(\xi_1,\xi_0)+s^3h^{2m+2}d(\xi_1,\xi_0)\\
&+\ldots\ldots+s^ph^{2m+2p-1}d(\xi_1,\xi_0)\\
&= sh^{2m}[1+(sh)+(sh)^2+(sh)^3.\ldots\ldots\ldots.+(sh)^{2p-1}]d(\xi_1,\xi_0).\\
&\leq \frac{sh^{2m}}{1-sh}\, d(\xi_1,\xi_0).
\end{aligned}$$

Let $0 \ll \epsilon$ be given. Notice that $\frac{sh^{2m}}{1-sh}\, d(\xi_1,\xi_0) \to 0$ as $m \to \infty$ for any p. Making full use of lemma 2.7(i), we find $m_0 \in N$ such that $\frac{sh^m}{1-sh}\, d(\xi_1,\xi_0) \ll \epsilon$, for each $m \geq m_0$. Thu, $d(\xi_{2m},\xi_{2m+2p})) \leq \epsilon$ that $\frac{sh^m}{1-sh}\, d(x_1,x_0) \ll \epsilon$ for all $m \geq 1, p > 1$. So, by lemma 2.7(ii) $\{\xi_{2n}\}$ is a Cauchy sequence in (Ω,d). Since (Ω,d) is a complete cone b-metric space, there exist $v \in X$ such that $\xi_{2n} \to v$ as $n \to \infty$ Taken $n_0 \in N$ such that $d(\xi_{2n,}v) \ll \frac{\epsilon\,[\,1-s\lambda_1+s^2\lambda_2+s^2\lambda_4)]}{s^2\,(\lambda_1+\lambda_2+\lambda_3+\lambda_4)}$ for all $n \geq n_0$.Hence

$$\begin{aligned}
d(T_\alpha^a v,v) &\leq s[d(T_\alpha^a v,T_\alpha^a\xi_{2i})+d(T_\alpha^a\xi_{2i},v)]\\
&= sd(T_\alpha^a v,T_\alpha^a\xi_{2i}),+sd(T_\alpha^a\xi_{2i},v)\\
&\leq s[\lambda_1\{d(v,T_\alpha^a v)+d(\xi_{2i,}T_\alpha^a\xi_{2i})\}+\lambda_2\{d(v,T_\alpha^a\xi_{2i})+d(\xi_{2i},T_\alpha^a v)\}\\
&+\lambda_3 max\{d(v,T_\alpha^a v),d(\xi_{2i},T_\alpha^a\xi_{2i}),d(v,T_\alpha^a\xi_{2i})\}+\lambda_4\{d(v,\xi_{2i})+d(\xi_{2i},T_\alpha^a v))\}\\
&+sd(\xi_{2i+1,},v).\\
&\leq s[\lambda_1\{d(v,T_\alpha^a v)+d(\xi_{2i,}\xi_{2i+1})\}+\lambda_2\{d(v,\xi_{2i+1})+d(\xi_{2i},T_\alpha^a v)\}\\
&+\lambda_3 max\{d(v,T_\alpha^a v),d(\xi_{2i,}\xi_{2i+1}),d(v,\xi_{2i+1})\}+\lambda_4\{d(v,\xi_{2i})+d(\xi_{2i},T_\alpha^a v)\\
&+sd(\xi_{2i+1,},v).\\
&\leq (\lambda_1+s^2\lambda_2+s^2\lambda_4)\,d(T_\alpha^a v,v)+s^2\,(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\,d(\xi_{2i},v)\\
&+\{s(1+\lambda_2)+s^2(\lambda_1+\lambda_3)\}\,d(v,\xi_{2i+1})
\end{aligned}$$

Implies that

$$[\,1-s\lambda_1+s^2\lambda_2+s^2\lambda_4)]d(T_\alpha^a v,v) \leq s^2\,(\lambda_1+\lambda_2+\lambda_3+\lambda_4)\,d(\xi_{2i},v)+\{s(1+\lambda_2)+s^2(\lambda_1+\lambda_3)\}\,d(v,\xi_{2i+1})$$

So, $d(T_\alpha^a v,v) \leq \frac{s^2\,(\lambda_1+\lambda_2+\lambda_3+\lambda_4)}{[\,1-s\lambda_1+s^2\lambda_2+s^2\lambda_4)]}\,d(\xi_{2i},v)+\frac{\{s(1+\lambda_2)+s^2(\lambda_1+\lambda_3)\}}{[\,1-s\lambda_1+s^2\lambda_2+s^2\lambda_4)]}\,d(v,\xi_{2i+1}) \ll \epsilon$ for each $n \geq n_0$. Then by lemma 2.8 we deduce $d(T_\alpha^a v,v)=0, i.e., T_\alpha^a v=v$.That is v is a fixed point of T.

Similarly, we can prove that, ${F^b}_\beta(v) = v$. That is v is a fixed point of F. Therefore, $T^a_\alpha v = v = {F^b}_\beta(v)$. Hence, v is a common fixed point of T and F.
Now to prove its uniqueness, If v^* is another common fixed point of F and G such that $Tv^* = v^* = Fv^*$, then by the given condition (3.5.1), we have

$$d(v,v^*) = d(Tv,Fv^*)$$
$$\leq \lambda_1[d(v,Tv) + d(v^*,Fv^*)\ ] + \lambda_2[d(v,Fv^*) + d(v^*Tv)\ \}$$
$$+\lambda_3 max\{d(v,Tv), d(v^*,Fv^*), d(v,Fv^*)\} + \lambda_4\ [d(v,v^*) + d(v^*,Tv)\ ]$$
$$\leq [2s(\lambda_2 + \lambda_4) + \lambda_3]\ d(v,v^*)$$
$$\leq 2(\lambda_1 + s\lambda_2 + s\ \lambda_3) + s\ \lambda_4]\ d(v,v^*)$$

Owing to $0 \leq [2(\lambda_1 + s\lambda_2 + s\ \lambda_3) + s\ \lambda_4\ ] < 1$. Then by lemma 2.9, $v = v^*$. Therefore, v is unique common fixed point of T and F.

IV. CONCLUSION

The main results are a few valuable additions to the available references for cone b-metric spaces and some fixed-point theorems for contrasting mappings in the configuration of cone b-metric spaces. The results presented here generalise and complement some of the earlier work presented in the existing literature by Kurre, R. et al. [12].

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