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Information Radius Via Verma Information Measure in Intuitionistic Fuzzy Environment

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Abstract

In the present communication we introduce the new measure of information radius in intuitionistic fuzzy environment corresponding to Verma information measure.

Key words: Intuitionistic fuzzy environment, Measure of information, Information radius, Directed-divergence etc.

1. Introduction

Fuzzy set theory was proposed by Zadeh, Lotfi A. [10] in 1965 as an extension of the classical notion of a set (Zadeh, 1965). With the proposed methodology, Zadeh introduced a mathematical method with which decision-making using fuzzy descriptions of some information becomes possible. The basis of this theory is the *fuzzy set*, which is a set that does not have clearly defined limits and can contain elements only at some degree; in other words, elements can have a certain degree of membership. Hence, suitable functions are used namely, membership functions that determine the membership degree of each element in a fuzzy set. If we consider an input variable x with a field of definition S, the fuzzy set A in S is defined as: If A be the subset of universe of discourse $i.e.\ X = \{x_1, \dots \dots, x_n\}$ then, A is defined as,

$$A = \{x_i/\mu_A(x_i): i = 1,2, \dots \dots \dots \dots, n\}.$$

Where $\mu_A(x_i)$ is a membership function and having the following properties: (i) If $\mu_A(x_i) = 0$, x_i does not belong to A and there is no ambiguity. (ii) If $\mu_A(x_i) = 1$, x_i belong to A and there is no ambiguity. (iii) If $\mu_A(x_i) = 0.5$, there is maximum ambiguity whether x_i belong to A or not.

Intuitionistic fuzzy sets are sets whose elements have membership grades and non-membership grades. The intuitionistic fuzzy set generalizes fuzzy set, since the indicator function of fuzzy set is a special case of the membership function and non-membership function of intuitionistic fuzzy set.

It is an uncertainty based model proposed by Atanassov, K. T. [1, 2] in 1986, which extends the notion of fuzzy sets by relaxing the constraint in fuzzy sets that the non-membership value is one's complement of the membership value of every element. According to him, if F be a fixed set then an intuitionistic fuzzy set S in F is an object having the form

$$S = \{< x, \mu_S(x), \nu_S(x) >/x \in F\}.$$

Where the function $\mu_S(x)$ and $\nu_S(x)$ define the degree of membership and degree of non-membership of the element $x \in S$ to $S \subset F$ respectively. The function $\mu_S(x)$ and $\nu_S(x)$ satisfy the following condition.

$$(\forall\, x \in F)\ (0 \le \mu_S(x) + \nu_S(x) \le 1).$$

Obviously, fuzzy set has the form $\{< x, \mu_S(x), 1 - \mu_S(x) >/x \in F\}$.

A measure of fuzziness $f\big(\mu_S(x), \nu_S(x)\big)$ is an Intuitionistic fuzzy set should have atleast the following conditions:

(C_1) It should be continuous in this range of

$(0 \le \mu_S(x_i) + \nu_S(x_i) \le 1), (i = 1, \dots \dots, n).$

(C_2) It should be zero when $\mu_S(x_i) = 0$ and $\nu_S(x_i) = 0$.

(C_3) It should be not changed when any of $\mu_S(x_i)$ is changed into $\nu_S(x_i)$.

(C_4) It should be defined for all $\mu_S(x_i)$ and $\nu_S(x_i)$ $(i = 1, \dots \dots, n)$ in the range of

$(0 \le \mu_S(x_i) + \nu_S(x_i) \le 1), (i = 1, \dots \dots, n).$

(C_5) It should be maximum when $\mu_S(x_i) = \frac{1}{2}$ and $\nu_S(x_i) = \frac{1}{2}$ $(i = 1, \dots \dots, n)$.

(C_6) It should be increasing function of $\mu_S(x_i)$ when $0 \le \mu_S(x_i) \le \frac{1}{2}$ and decreasing function of $\mu_S(x_i)$ when $\frac{1}{2} \le \mu_S(x_i) \le 1$ and other variable are kept fixed. It should be decreasing function of $\nu_S(x_i)$ when $0 \le \nu_S(x_i) \le \frac{1}{2}$ and increasing function of $\nu_S(x_i)$ when $\frac{1}{2} \le \nu_S(x_i) \le 1$ and other variable are kept fixed.

(C_7) It should be concave function of $\mu_S(x_i)$, when $\nu_S(x_i)$ set as a constant.

If A and B are the two intuitionistic fuzzy sets. Then Shannon [6] measure of information in intuitionistic fuzzy environment is given by

$$S(A) - \sum_{i=1}^{n} \big[\mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i)\big] \tag{1.1}$$

and by the use of Shennon information measure, the concept of information radius in intuitionistic fuzzy environment is given by

$$R(A,B)=\boldsymbol{S}\left(\frac{A+B}{2}\right)-2^{-1}[\boldsymbol{S}(A)+\boldsymbol{S}(B)] \tag{1.2}$$

But, the concept of information radius was given by Kapur [5] in classical environment. The above equation (1.2) is known as information radius in intuitionistic fuzzy environment (Sibson [7]) or Jensen-Shannon directed divergence measure (Burbea and Rao [4]) in intuitionistic fuzzy environment. In the next section, we shall study the new information radius in intuitionistic fuzzy environment corresponding to Verma [8, 9] information measure.

2. Our Results

2.1 INFORMATION RADIUS CONNECTED WITH VERMA [8, 9] i.e. HYBRID BURG [3] INFORMATION MEASURE

Verma [8, 9] i.e. hybrid Burg [3] intuitionistic fuzzy information measure, if avoiding the constant term, is

$$V_a(A)=\sum_{i=1}^{n}\left[\ln\left(1+a\mu_A(x_i)\right)+\ln\left(1+a\nu_A(x_i)\right)\right]-\sum_{i=1}^{n}\left[\ln\mu_A(x_i)+\ln\nu_A(x_i)\right],$$
$$a>0 \tag{2.1.1}$$

Now, if A and B are the two intuitionistic fuzzy sets and $V_a(A)$ and $V_a(B)$ are their intuitionistic fuzzy entropies, then the probabilistic intuitionistic fuzzy information radius $R(A,B)$ is defined as

$$R(A,B)=V_a\left(\frac{A+B}{2}\right)-2^{-1}[V_a(A)+V_a(B)] \tag{2.1.2}$$

Using (2.1.1), equation (2.1.2) gives the following result

$$R(A,B)=\sum_{i=1}^{n}\left[\ln\left(1+a\mu_{\frac{A+B}{2}}(x_i)\right)+\ln\left(1+a\nu_{\frac{A+B}{2}}(x_i)\right)\right]-2^{-1}\sum_{i=1}^{n}\left[\ln\left(1+a\mu_A(x_i)\right)+\right.$$
$$\left.\ln\left(1+a\nu_A(x_i)\right)+\ln\left(1+a\mu_B(x_i)\right)+\ln\left(1+a\nu_B(x_i)\right)\right]$$
$$-\sum_{i=1}^{n}\left[\ln\mu_{\frac{A+B}{2}}(x_i)+\ln\nu_{\frac{A+B}{2}}(x_i)\right]$$
$$+2^{-1}\sum_{i=1}^{n}[\ln\mu_A(x_i)+\ln\nu_A(x_i)+\ln\mu_B(x_i)+\ln\nu_B(x_i)]$$
$$=2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\mu_{\frac{A+B}{2}}(x_i)\right)+\nu_{\frac{A+B}{2}}\ln\left(1+a\nu_{\frac{A+B}{2}}(x_i)\right)\right]$$

$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}\ln\left(1+a\mu_A(x_i)\right)+\nu_{\frac{A+B}{2}}\ln\left(1+a\nu_A(x_i)\right)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\mu_{\frac{A+B}{2}}(x_i)\right)+\nu_{\frac{A+B}{2}}\ln\left(1+a\nu_{\frac{A+B}{2}}(x_i)\right)\right]$$

$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}\ln\left(1+a\mu_B(x_i)\right)+\nu_{\frac{A+B}{2}}\ln\left(1+a\nu_B(x_i)\right)\right]$$

$$-\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\mu_{\frac{A+B}{2}}(x_i)+\nu_{\frac{A+B}{2}}(x_i)\ln\nu_{\frac{A+B}{2}}(x_i)+\mu_{\frac{A+B}{2}}\ln\mu_{\frac{A+B}{2}}(x_i)+\nu_{\frac{A+B}{2}}(x_i)\ln\nu_{\frac{A+B}{2}}(x_i)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}\ln\mu_A(x_i)+\nu_{\frac{A+B}{2}}(x_i)\ln\nu_A(x_i)\right.$$
$$\left.+\mu_{\frac{A+B}{2}}\ln\mu_B(x_i)+\nu_{\frac{A+B}{2}}(x_i)\ln\nu_B(x_i)\right]$$

$$=-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{1+a\mu_A(x_i)}{1+a\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}\ln\left(\frac{1+a\nu_A(x_i)}{1+a\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{\mu_A(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}\ln\left(\frac{\nu_A(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{1+a\mu_B(x_i)}{1+a\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}\ln\left(\frac{1+a\nu_B(x_i)}{1+a\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{\mu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}\ln\left(\frac{\nu_B(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

Ofcourse, without loss of generality, we can reduce the first two terms

$$=-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\mu_A(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\nu_A(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{\mu_A(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{\nu_A(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\mu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\nu_B(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{\mu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{\nu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)\right].$$

Thus, we achieve the result $R(A,B)=\frac{1}{2}D_a\left(A,\frac{A+B}{2}\right)+\frac{1}{2}D_a\left(B,\frac{A+B}{2}\right)$.

Where, $D_a(A,B)=-\sum_{i=1}^{n}\left[\mu_B(x_i)\ln\left(1+a.\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)+\nu_B(x_i)\ln\left(1+a.\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)\right]$

$$+\sum_{i=1}^{n}\left[\mu_B(x_i)\ln\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)+\nu_B(x_i)\ln\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)\right]. \quad (2.1.3)$$

is the intuitionistic fuzzy directed divergence corresponding to Verma [8, 9] information measure.

2.2 INFORMATION RADIUS CONNECTED WITH MODIFIED VERMA [8, 9] INFORMATION MEASURE

Modified Verma [8, 9] i.e. hybrid Shannon [6] intuitionistic fuzzy information measure, if avoiding the constant term, is

$$V_a(A)=\sum_{i=1}^{n}\left[\ln\left(1+a\mu_A(x_i)\right)+\ln\left(1+a\nu_A(x_i)\right)\right]-\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i)+\nu_A(x_i)\ln\nu_A(x_i)\right],$$

$$a>0 \quad (2.2.1)$$

Now, if A and B are the two intuitionistic fuzzy sets and $V_a(A)$ and $V_a(B)$ are their intuitionistic fuzzy entropies, then the probabilistic intuitionistic fuzzy information radius $R(A,B)$ is defined as

$$R(A,B)=V_a\left(\frac{A+B}{2}\right)-2^{-1}[V_a(A)+V_a(B)] \quad (2.2.2)$$

Using (2.2.1), equation (2.2.2) gives the following result

$$R(A,B)=\sum_{i=1}^{n}\left[\ln\left(1+a\mu_{\frac{A+B}{2}}(x_i)\right)+\ln\left(1+a\nu_{\frac{A+B}{2}}(x_i)\right)\right]$$
$$-2^{-1}\sum_{i=1}^{n}\left[\ln\left(1+a\mu_A(x_i)\right)+\ln\left(1+a\nu_A(x_i)\right)\right.$$
$$\left.+\ln\left(1+a\mu_B(x_i)\right)+\ln\left(1+a\nu_B(x_i)\right)\right]$$

$$-\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\mu_{\frac{A+B}{2}}(x_i)+\nu_{\frac{A+B}{2}}(x_i)\ln\nu_{\frac{A+B}{2}}(x_i)\right]$$
$$+2^{-1}\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i)+\nu_A(x_i)\ln\nu_A(x_i)\right.$$
$$\left.+\mu_B(x_i)\ln\mu_B(x_i)+\nu_B(x_i)\ln\nu_B(x_i)\right]$$

$$=2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\mu_{\frac{A+B}{2}}(x_i)\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\nu_{\frac{A+B}{2}}(x_i)\right)\right]$$
$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\mu_A(x_i)\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\nu_A(x_i)\right)\right]$$
$$+2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\mu_{\frac{A+B}{2}}(x_i)\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\nu_{\frac{A+B}{2}}(x_i)\right)\right]$$
$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\mu_B(x_i)\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a\nu_B(x_i)\right)\right]$$
$$-2^{-1}\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_{\frac{A+B}{2}}(x_i)+\nu_A(x_i)\ln\nu_{\frac{A+B}{2}}(x_i)\right.$$
$$\left.+\mu_B(x_i)\ln\mu_{\frac{A+B}{2}}(x_i)+\nu_B(x_i)\ln\nu_{\frac{A+B}{2}}(x_i)\right]$$
$$+2^{-1}\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i)\right.$$
$$\left.+\nu_A(x_i)\ln\nu_A(x_i)+\mu_B(x_i)\ln\mu_B(x_i)+\nu_B(x_i)\ln\nu_B(x_i)\right]\Big]$$

$$=-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{1+a\mu_A(x_i)}{1+a\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{1+a\nu_A(x_i)}{1+a\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$
$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{1+a\mu_B(x_i)}{1+a\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(\frac{1+a\nu_B(x_i)}{1+a\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$
$$+2^{-1}\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\left(\frac{\mu_A(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_A(x_i)\ln\left(\frac{\nu_A(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]+2^{-1}\sum_{i=1}^{n}\left[\mu_B(x_i)\ln\left(\frac{\mu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)\right.$$

$$\mu_B(x_i)\ln\left(\frac{\mu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)\Bigg]$$

Ofcourse, without loss of generality, we can reduce the first two terms

$$= -2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\mu_A(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\nu_A(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\left(\frac{\mu_A(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_A(x_i)\ln\left(\frac{\nu_A(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$-2^{-1}\sum_{i=1}^{n}\left[\mu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\mu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_{\frac{A+B}{2}}(x_i)\ln\left(1+a.\frac{\nu_B(x_i)}{\nu_{\frac{A+B}{2}}(x_i)}\right)\right]$$

$$+2^{-1}\sum_{i=1}^{n}\left[\mu_B(x_i)\ln\left(\frac{\mu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)+\nu_B(x_i)\ln\left(\frac{\nu_B(x_i)}{\mu_{\frac{A+B}{2}}(x_i)}\right)\right].$$

Thus, we achieve the result $R(A,B)=\frac{1}{2}D_a\left(A,\frac{A+B}{2}\right)+\frac{1}{2}D_a\left(B,\frac{A+B}{2}\right)$.

Where $D_a(A,B)=-\sum_{i=1}^{n}\left[\mu_B(x_i)\ln\left(1+a.\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)+\nu_B(x_i)\ln\left(1+a.\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)\right]$

$$+\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)+\nu_A(x_i)\ln\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)\right] \quad (2.2.3)$$

is the intuitionistic fuzzy directed divergence corresponding to modified Verma [8, 9] information measure. Note that, (2.1.3) and (2.2.3) are the convex functions. Thus we have the following properties: (i) $D(A,B)\geq 0$, (ii) $D(A,B)=0$ iff $\mu_A(x_i)=\mu_B(x_i)$ or $\nu_A(x_i)=\nu_B(x_i)\ \forall i$ and (iii) $D(A,B)$ is a convex function of $\mu_A(x_i),\mu_B(x_i),\nu_A(x_i)$ and $\nu_B(x_i)\ \forall i$. Hence, intuitionistic fuzzy directed divergence $D(A,B)$ introduced in (2.1.3) and(2.2.3) are the valid measure of divergence. Also, the measure $R(A,B)$ satisfies the following properties: (i) $R(A,B)\geq 0$, (ii) $R(A,B)=0$ iff $\mu_A(x_i)=\mu_B(x_i)$ or $\nu_A(x_i)=\nu_B(x_i)\ \forall i$ and (iii) $R(A,B)$, being the sum of intuitionistic fuzzy directed divergence is convex and (iv) $R(A,B)$ is symmetric in the sense that $R(A,B)=R(B,A)$. Thus the intuitionistic fuzzy R-divergence $R(A,B)$ is a valid measure of divergence.

Conclusion

In the above discussion, we introduce the new measures of information radius and it has great applications in bioinformatics and genome comparison, in protein surface comparison, in the social sciences, in the quantitative study of history, fire experiments and in machine learning.

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