

Plane Wave Propagation at Viscoelastic Solid and Fluid Saturated Incompressible Porous Solid Interface

Neelam Kumari

*Assistant Professor,
Department of Mathematics, Ch. Devi Lal University, Sirsa, 125055, India.*

Abstract

In this paper, the reflection and transmission phenomenon at a plane interface between viscoelastic solid half space and fluid saturated porous half space has been analyzed. P-wave or SV-wave is considered to be incident on the plane interface through fluid saturated porous solid half space. The amplitude ratios of various reflected and transmitted waves to that of incident wave are derived. These amplitude ratios have been computed numerically for a specific model and results thus obtained are depicted graphically with angle of incidence of incident wave. It is found that these amplitude ratios depend on angle of incidence of the incident wave and material properties of the medium. A particular case of reflection at free surface of fluid saturated porous half space has been deduced and discussed. A special case of empty porous solid is obtained and discussed from the present investigation. Also, the viscous effect of linear viscoelastic solid has been observed on various amplitude ratios.

1. Introduction

The state of the deep interior of the earth cannot be explained by assuming the earth to be an elastic solid. Keeping this fact in mind several problems of reflection and refraction in a linear viscoelastic solid have been discussed by many researchers like Cooper and Reiss (1965), Cooper (1967) etc.

The classical theory is inadequate to describe the mechanical behaviour of fluid saturated porous material especially when the material contains liquid filled pores. Due to the different motions of the solid and liquid phases and different material properties and the complicated structures of pores, the mechanical behaviour of a fluid saturated porous medium is very complex and difficult. So many researchers tried to overcome this difficulty from time to time.

Based on the work of Fillunger model (1913), Bowen (1980) and de Boer and Ehlers

(1990a, 1990b) developed an interesting theory for porous medium having all constituents to be incompressible. There are sufficient reasons for considering the fluid saturated porous constituents as incompressible. For example, in the composition of soil both the solid constituents and liquid constituents are incompressible. Therefore, the assumption of incompressible constituents meet the properties appearing in many branches of engineering and avoids the introduction of many complicated material parameters as considered in the Biot theory because Biot's theory was based on the assumption of compressible constituents. Based on this theory, many researchers like de Boer and Liu (1994, 1995), de Boer and Didwania (2004), Tajuddin and Hussaini (2006), Kumar and Hundal (2007), Kumar et. al. (2011) etc. studied some problems of wave propagation in fluid saturated porous media.

Using the theory of de Boer and Ehlers (1990) for fluid saturated porous medium and Kumar and Singh (2000) theory for viscoelastic solid, the reflection and transmission of longitudinal wave (P-wave) or transverse wave (SV-wave) at an interface between linear viscoelastic solid half space and fluid saturated porous half space is investigated. The reflection coefficients of reflected waves at the free surface have also been obtained. A special case when fluid saturated porous half space reduces to empty porous solid has been deduced and discussed. Amplitudes ratios for various reflected and transmitted waves are computed for a particular model and depicted graphically and discussed accordingly. The model considered is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model will be of great use in the fields related to earth sciences.

2. Basic equations

The equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces are given by de Boer and Ehlers (1990) as

$$\nabla \cdot (\eta^S \dot{\mathbf{u}}^S + \eta^F \dot{\mathbf{u}}^F) = 0, \quad (1)$$

$$(\lambda^S + \mu^S) \nabla (\nabla \cdot \mathbf{u}^S) + \mu^S \nabla^2 \mathbf{u}^S - \eta^S \nabla p - \rho^S \ddot{\mathbf{u}}^S + S_v (\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) = 0, \quad (2)$$

$$\eta^F \nabla p + \rho^F \ddot{\mathbf{u}}^F + S_v (\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S) = 0, \quad (3)$$

$$\mathbf{T}_E^S = 2\mu^S \mathbf{E}^S + \lambda^S (\mathbf{E}^S \cdot \mathbf{I}) \mathbf{I}, \quad (4)$$

$$\mathbf{E}^S = \frac{1}{2} (\text{grad } \mathbf{u}^S + \text{grad}^T \mathbf{u}^S), \quad (5)$$

where $\mathbf{u}^i, \dot{\mathbf{u}}^i, \ddot{\mathbf{u}}^i, i = F, S$ denote the displacement, velocity and acceleration of fluid and solid phases, respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^S and ρ^F are the densities of the solid and fluid constituents, respectively. $\mathbf{T}_E^S$ is the effective stress in the solid phase and $\mathbf{E}^S$ is the linearized langrangian strain tensor. λ^S and μ^S are the macroscopic Lamé's parameters of the porous solid and η^S and η^F are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \quad (6)$$

In the case of isotropic permeability, the tensor $\mathbf{S}_v$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers (1990) as

$$\mathbf{S}_v = \frac{(\eta^F)^2 \gamma^{FR}}{K} \mathbf{I}, \quad (7)$$

where γ^{FR} is the specific weight of the fluid and K is the Darcy's permeability coefficient of the porous medium and $\mathbf{I}$ stands for unit vector.

Assuming the displacement vector $\mathbf{u}^i$ ($i = F, S$) as

$$\mathbf{u}^i = (\mathbf{u}^i, 0, w^i), \text{ where } i = F, S, \quad (8)$$

and hence equations (1)-(3) give the equations of motion for fluid saturated incompressible porous medium in the component form as

$$(\lambda^S + \mu^S) \frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^S - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t} \right] = 0, \quad (9)$$

$$(\lambda^S + \mu^S) \frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t} \right] = 0, \quad (10)$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t} \right] = 0, \quad (11)$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t} \right] = 0, \quad (12)$$

$$\eta^S \left[\frac{\partial^2 u^S}{\partial x \partial t} + \frac{\partial^2 w^S}{\partial z \partial t} \right] + \eta^F \left[\frac{\partial^2 u^F}{\partial x \partial t} + \frac{\partial^2 w^F}{\partial z \partial t} \right] = 0, \quad (13)$$

where

$$\theta^S = \frac{\partial(u^S)}{\partial x} + \frac{\partial(w^S)}{\partial z}. \quad (14)$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \quad (15)$$

Using the Helmholtz decomposition of displacement vector, the displacement components u^i and w^i are related to the potential functions ϕ^i and ψ^i as

$$u^i = \frac{\partial \phi^i}{\partial x} + \frac{\partial \psi^i}{\partial z}, w^i = \frac{\partial \phi^i}{\partial z} - \frac{\partial \psi^i}{\partial x}, i = F, S. \quad (16)$$

Using, (16) in eqs. (9)-(13), we obtain the following equations:

$$\nabla^2 \phi^S - \frac{1}{C^2} \frac{\partial^2 \phi^S}{\partial t^2} - \frac{S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2} \frac{\partial \phi^S}{\partial t} = 0, \quad (17)$$

$$\phi^F = -\frac{\eta^S}{\eta^F} \phi^S, \quad (18)$$

$$\mu^S \nabla^2 \psi^S - \rho^S \frac{\partial^2 \psi^S}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t} \right] = 0, \quad (19)$$

$$\rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t} \right] = 0, \quad (20)$$

$$(\eta^F)^2 p - \eta^S \rho^F \frac{\partial^2 \phi^S}{\partial t^2} - S_v \frac{\partial \phi^S}{\partial t} = 0, \quad (21)$$

where

$$C = \sqrt{\frac{(\eta^F)^2 (\lambda^S + 2\mu^S)}{(\eta^F)^2 \rho^S + (\eta^S)^2 \rho^F}}. \quad (22)$$

Also, the normal and tangential stresses in the solid phase takes the form,

$$t_{zz}^S = \lambda^S \left(\frac{\partial^2 \phi^S}{\partial x^2} + \frac{\partial^2 \phi^S}{\partial z^2} \right) + 2\mu^S \left(\frac{\partial^2 \phi^S}{\partial z^2} - \frac{\partial^2 \psi^S}{\partial x \partial z} \right), \quad (23)$$

$$t_{zx}^S = \mu^S \left(2 \frac{\partial^2 \phi^S}{\partial x \partial z} + \frac{\partial^2 \psi^S}{\partial z^2} - \frac{\partial^2 \psi^S}{\partial x^2} \right). \quad (24)$$

The time harmonic solution of the system of equations (17)-(21) can be considered as

$$(\phi^S, \phi^F, \psi^S, \psi^F, p) = (\phi_1^S, \phi_1^F, \psi_1^S, \psi_1^F, p_1) \exp(i\omega t), \quad (25)$$

where ω is the complex circular frequency.

Making use of (25) in equations (17)-(21), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{C_1^2} - \frac{i\omega S_v}{(\lambda^S + 2\mu^S)(\eta^F)^2} \right] \phi_1^S = 0, \quad (26)$$

$$[\mu^S \nabla^2 + \rho^S \omega^2 - i\omega S_v] \psi_1^S = -i\omega S_v \psi_1^F, \quad (27)$$

$$[-\omega^2 \rho^F + i\omega S_v] \psi_1^F - i\omega S_v \psi_1^S = 0, \quad (28)$$

$$(\eta^F)^2 p_1 + \eta^S \rho^F \omega^2 \phi_1^S - i\omega S_v \phi_1^S = 0, \quad (29)$$

$$\phi_1^F = -\frac{\eta^S}{\eta^F} \phi_1^S. \quad (30)$$

Equation (26) represents the propagation of a longitudinal wave with velocity V_1 , given by

$$V_1^2 = \frac{1}{G_1}, \quad (31)$$

$$\text{where } G_1 = \left[\frac{1}{C_1^2} - \frac{iS_v}{\omega(\lambda^S + 2\mu^S)(\eta^F)^2} \right]. \quad (32)$$

From equations (27) and (28), we obtain

$$\left[\nabla^2 + \frac{\omega^2}{V_2^2} \right] \psi_1^S = 0. \quad (33)$$

Equation (33) corresponds to the propagation of a transverse wave with velocity V_2 , given by

$$V_2^2 = \frac{1}{G_2},$$

where

$$G_2 = \left\{ \frac{\rho^s}{\mu^s} - \frac{iS_v}{\mu^s \omega} - \frac{S_v^2}{\mu^s (-\rho^s \omega^2 + i\omega S_v)} \right\}, \quad (34)$$

For M_2 (linear viscoelastic solid medium)

Following Borchardt (1973), the equation governing the small motions in a linear viscoelastic solid may be written as

$$(\mathbf{K}' + 4\mathbf{M}'/3)\nabla(\nabla \cdot \mathbf{u}') - \mathbf{M}'\nabla \times (\nabla \times \mathbf{u}') = \rho_1 \ddot{\mathbf{u}}', \quad (35)$$

where symbols $\mathbf{K}'$ is the complex bulk modulus, $\mathbf{M}'$ is the shear modulus, ρ_1 is the density of linear viscoelastic solid and $\mathbf{u}'$ is the displacement vector. Superposed dots on right hand side of equation (35) stand for second partial derivative with respect to time.

The stresses in the linear viscoelastic solid are given by

$$\sigma_{kl}' = (\mathbf{K}' - 2\mathbf{M}'/3)\theta \delta_{kl} + 2\mathbf{M}' e_{kl}, \quad (36)$$

where

$$e_{kl} = \frac{1}{2} \left(\frac{\partial u_k'}{\partial x_l} + \frac{\partial u_l'}{\partial x_k} \right), \theta = \nabla \cdot \mathbf{u}', \quad (37)$$

Using Helmholtz's theorem

$$\mathbf{u}' = \nabla \phi' + \nabla \times \psi', \nabla \cdot \psi' = 0, \quad (38)$$

We can show that ϕ' and ψ' satisfy

$$\alpha^2 \nabla^2 \phi' = \ddot{\phi}' \text{ and } \beta^2 \nabla^2 \psi' = \ddot{\psi}', \quad (39)$$

$$\text{where } \alpha^2 = \left(\mathbf{K}' + \frac{4\mathbf{M}'}{3} \right) / \rho_1, \beta^2 = \mathbf{M}' / \rho_1, \quad (40)$$

and

$$\psi' = -(\psi')_y, \quad (41)$$

$$u' = \frac{\partial \phi'}{\partial x} + \frac{\partial \psi'}{\partial z}, w' = \frac{\partial \phi'}{\partial z} - \frac{\partial \psi'}{\partial x}. \quad (42)$$

3. Formulation of the problem and its solution

We consider a two dimensional problem by taking the z-axis pointing into lower half-space and the plane interface $z=0$ separating the linear viscoelastic solid half space M_2 [$z < 0$] and fluid saturated porous medium M_1 [$z > 0$] (see figure1). Consider a longitudinal wave (P-wave) or transverse wave (SV-wave) propagating through the fluid saturated porous half space medium M_1 and incident at the plane $z=0$ and making an angle θ_0 with normal to the surface. Corresponding to each incident wave (P-wave or SV-wave), we get two reflected waves P-wave and SV-wave in the medium M_1 and two transmitted waves P-wave and SV-wave in medium M_2 .

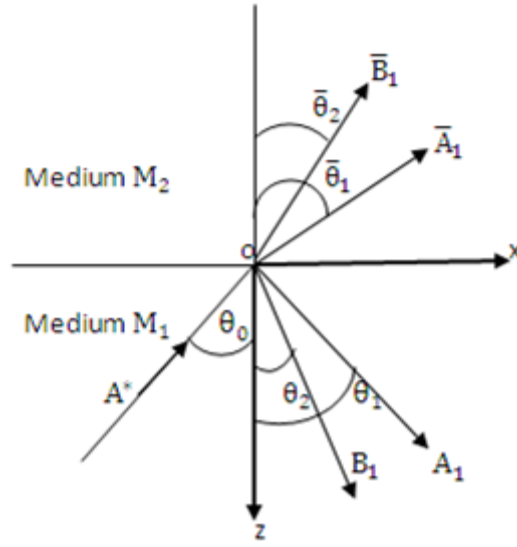


Fig. 1 Geometry of the problem

In medium M_1

The potential function satisfying the equations (17)-(21) can be taken as

$$\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\} [A_{01} \exp\{ik_1(x \sin\theta_0 - z \cos\theta_0) + i\omega_1 t\} + A_1 \exp\{ik_1(x \sin\theta_1 + z \cos\theta_1) + i\omega_1 t\}], \quad (43)$$

$$\{\psi^S, \psi^F\} = \{1, m_3\} [B_{01} \exp\{ik_2(x \sin\theta_0 - z \cos\theta_0) + i\omega_2 t\} + B_1 \exp\{ik_2(x \sin\theta_2 + z \cos\theta_2) + i\omega_2 t\}], \quad (44)$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}, m_2 = -\left[\frac{\eta^S \omega_1^2 \rho^F - i\omega_1 S_v}{(\eta^F)^2} \right], m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2^2 \rho^F}, \quad (45)$$

where A_{01} and B_{01} are amplitudes of the incident P-wave and SV-wave, respectively and A_1 , B_1 are amplitudes of the reflected P-wave and SV-wave respectively and to be determined from boundary conditions.

In medium M_2

The potential function satisfying the equation (39) can be taken as

$$\phi' = \bar{A}_1 \exp\{i\bar{k}_1(x \sin\bar{\theta}_1 - z \cos\bar{\theta}_1) + i\bar{\omega}_1 t\}, \quad (46)$$

$$\psi' = \bar{B}_1 \exp\{i\bar{k}_2(x \sin\bar{\theta}_2 - z \cos\bar{\theta}_2) + i\bar{\omega}_2 t\}, \quad (47)$$

where $\bar{k}_1$ and $\bar{k}_2$ are wave numbers of refracted P-wave and SV-wave, respectively. $\bar{A}_1$ and $\bar{B}_1$ are amplitudes of refracted P-wave and SV-wave and are unknown to be determined from boundary conditions.

4. Boundary conditions

The appropriate boundary conditions at the interface $z=0$ are the continuity of displacement and stresses. Mathematically, these boundary conditions can be expressed as:

$$t_{zz}^s - p = \sigma_{zz}, t_{zx}^s = \sigma_{zx}, u^s = u', w^s = w', (48)$$

In order to satisfy the boundary conditions, the extension of the Snell's law will be

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\bar{\theta}_1}{\alpha} = \frac{\sin\bar{\theta}_2}{\beta}, (49)$$

Also

$$k_1 V_1 = k_2 V_2 = \bar{k}_1 \alpha = \bar{k}_2 \beta = \omega, \text{ at } z = 0, (50)$$

For P-wave,

$$V_0 = V_1, \theta_0 = \theta_1, (51)$$

For SV-wave,

$$V_0 = V_2, \theta_0 = \theta_2, (52)$$

For incident longitudinal wave at the interface $z=0$, putting $\mathbf{B}_{01} = \mathbf{0}$ in equation (44) and for incident transverse wave putting $\mathbf{A}_{01} = \mathbf{0}$ in equation (43). Substituting the expressions of potentials given by (43)-(44) and (46)-(47) in equations (16), (23)-(24) and (36)-(37) and using equations (48)-(52), we get a system of four non homogeneous which can be written as

$$\sum_{j=0}^4 a_{ij} Z_j = Y_i, (i = 1, 2, 3, 4) (53)$$

where

$$Z_1 = \frac{A_1}{A^*}, Z_2 = \frac{A_2}{A^*}, Z_3 = \frac{\bar{A}_1}{A^*}, Z_4 = \frac{\bar{B}_1}{A^*} (54)$$

Also a_{ij} in non dimensional form can be written as

$$\begin{aligned} a_{11} &= \frac{\lambda^s}{\mu^s} + 2\cos^2\theta_1 + \frac{m_2}{\mu^s k_1^2}, a_{12} = -2\sin\theta_2 \cos\theta_2 \frac{k_2^2}{k_1^2}, a_{13} = \frac{\bar{k}_1^2}{k_1^2 \mu^s} \left[-K^* + \frac{2}{3}M^*(\sin^2\bar{\theta}_1 - 2\cos^2\bar{\theta}_1) \right], \\ a_{14} &= -\frac{M^* \bar{k}_2^2}{k_1^2 \mu^s} \sin 2\bar{\theta}_1, a_{21} = 2\sin\theta_1 \cos\theta_1, a_{22} = \frac{k_2^2}{k_1^2} (\cos^2\theta_2 - \sin^2\theta_2), a_{23} = \frac{M^* \bar{k}_1^2}{k_1^2 \mu^s} \sin 2\bar{\theta}_1, \\ a_{24} &= -\frac{M^* \bar{k}_2^2}{k_1^2 \mu^s} \cos 2\bar{\theta}_2, a_{31} = \sin\theta_1, a_{32} = \frac{k_2}{k_1} \cos\theta_2, a_{33} = -\frac{\bar{k}_1}{k_1} \sin\bar{\theta}_1, a_{34} = \frac{\bar{k}_2}{k_1} \cos\bar{\theta}_2, \\ a_{41} &= \cos\theta_1, a_{42} = -\frac{k_2}{k_1} \sin\theta_2, a_{43} = \frac{\bar{k}_1}{k_1} \cos\bar{\theta}_1, a_{44} = \frac{\bar{k}_2}{k_1} \sin\bar{\theta}_2, (55) \end{aligned}$$

For incident longitudinal wave:

$$A^s = A_{01}, B_{01} = 0, Y_1 = -a_{11}, Y_2 = a_{21}, Y_3 = -a_{31}, Y_4 = a_{41}, \quad (56)$$

For incident transverse wave:

$$A^s = B_{01}, A_{01} = 0, Y_1 = a_{12}, Y_2 = -a_{22}, Y_3 = a_{32}, Y_4 = -a_{42}, \quad (57)$$

5. Particular cases:

CASE-1

If pores are absent or gas is filled in the pores then ρ^F is very small as compared to ρ^S and can be neglected, so the relation (22) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}. \quad (58)$$

and the coefficients a_{11} in (55) changes to

$$a_{11} = \frac{\lambda^S}{\mu^S} + 2\cos^2\theta_1, \quad (59)$$

and the remaining coefficients in (55) remain same. In this situation the problem reduces to the problem of viscous liquid half space over empty porous solid half space.

CASE-2

When upper half space is not present in the given formulation.

Considering a fluid saturated incompressible porous medium with free boundary surface, i. e. upper half space is not present in the given formulation. A plane wave (P-wave) or SV-wave propagating through the fluid saturated incompressible porous medium making an angle θ_0 with z-axis at the free surface $z=0$. Corresponding to each incident wave we get two reflected waves. Boundary conditions for this case reduces to

$$t_{zz} = t_{zz}^s - p, t_{zx} = t_{zx}^s, \quad (60)$$

And hence we obtain a system of three non-homogeneous equations which can be written as

$$\sum_{j=1}^2 a_{ij} Z_j = Y_i, (i = 1, 2) \quad (61)$$

where $a_{11}, a_{12}, a_{21}, a_{22}$, are given by equation (55)

CASE-3

When linear viscoelastic solid half space (medium M_2) reduces to elastic solid half space

6. Numerical results and discussion

The theoretical results obtained above indicate that the amplitude ratios Z_i ($i = 1, 2, 3$) depend on the angle of incidence of incident wave. In order to study in more detail the behaviour of various amplitude ratios on the angle of incidence, we have computed them numerically by taking the following values relevant elastic parameters.

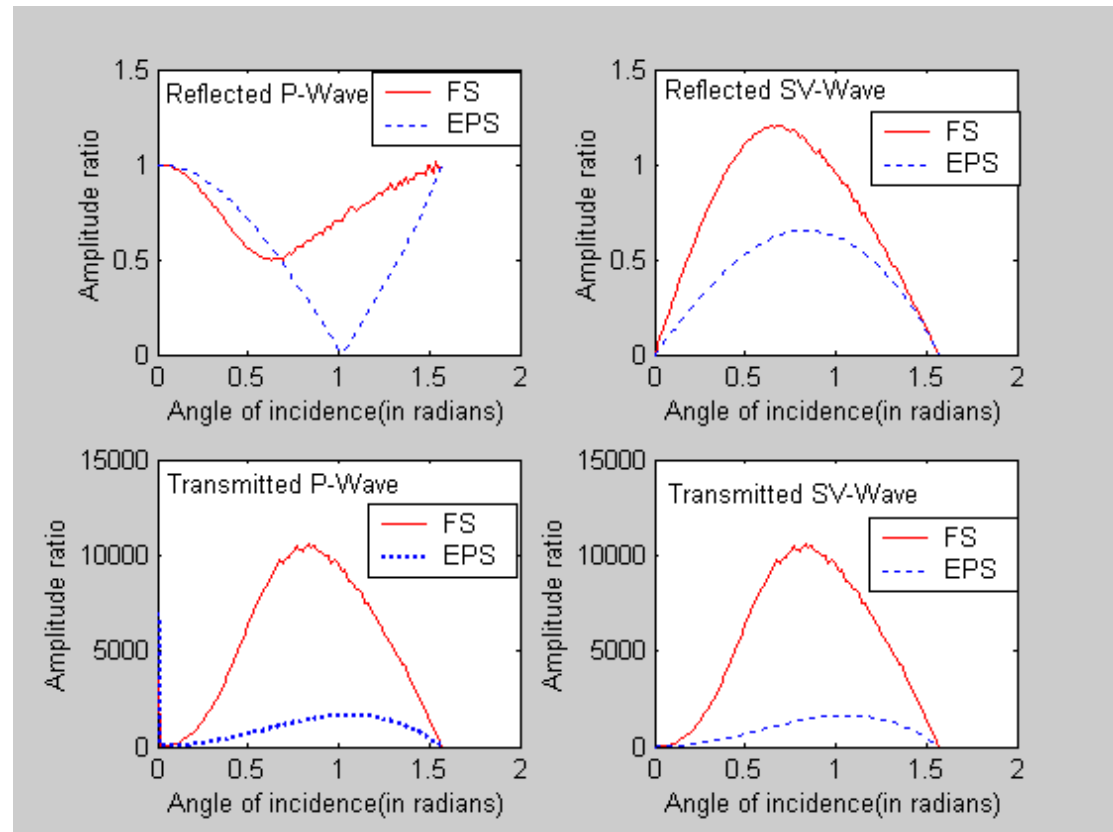
In medium M_1 , the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu (1993) as

$$\eta^s = 0.67, \eta^f = 0.33, \rho^s = 1.34 \text{ Mg/m}^3, \rho^f = 0.33 \text{ Mg/m}^3, \lambda^s = 5.5833 \text{ MN/m}^2, \\ K^f = 0.01 \text{ m/s}, \gamma^{FR} = 10.00 \text{ KN/m}^3, \mu^s = 8.3750 \text{ N/m}^2, \omega^* = 10/\text{s}, (62)$$

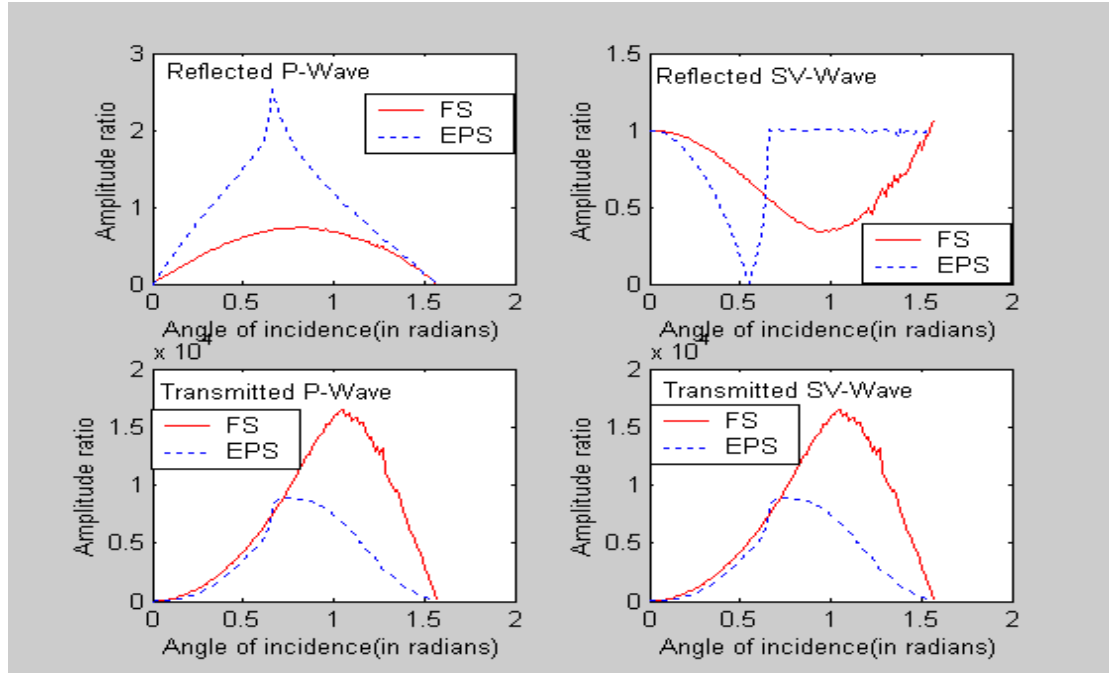
In medium M_2 ,

Following Silva (1976), the physical parameters representing the crust as a linear viscoelastic solid are as follows

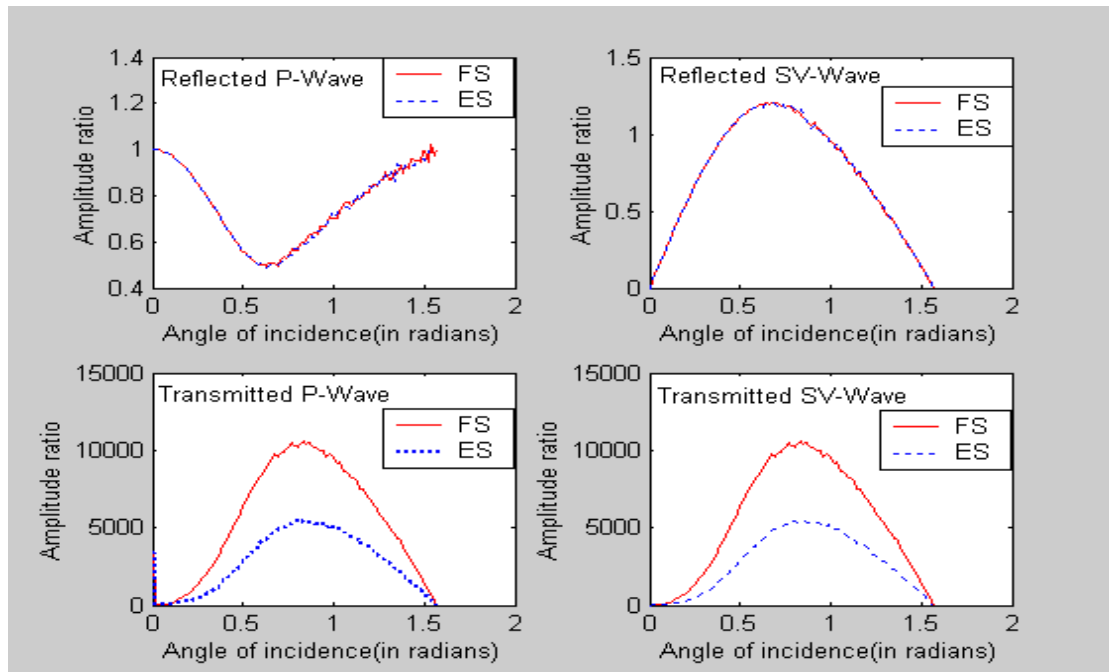
$$Q_p = 100, Q_s = 45, \rho_1 = 2.6 \text{ gm/cm}^3, V_p = 6.1 \text{ km/s}, V_s = 3.5 \text{ km/s} (63)$$



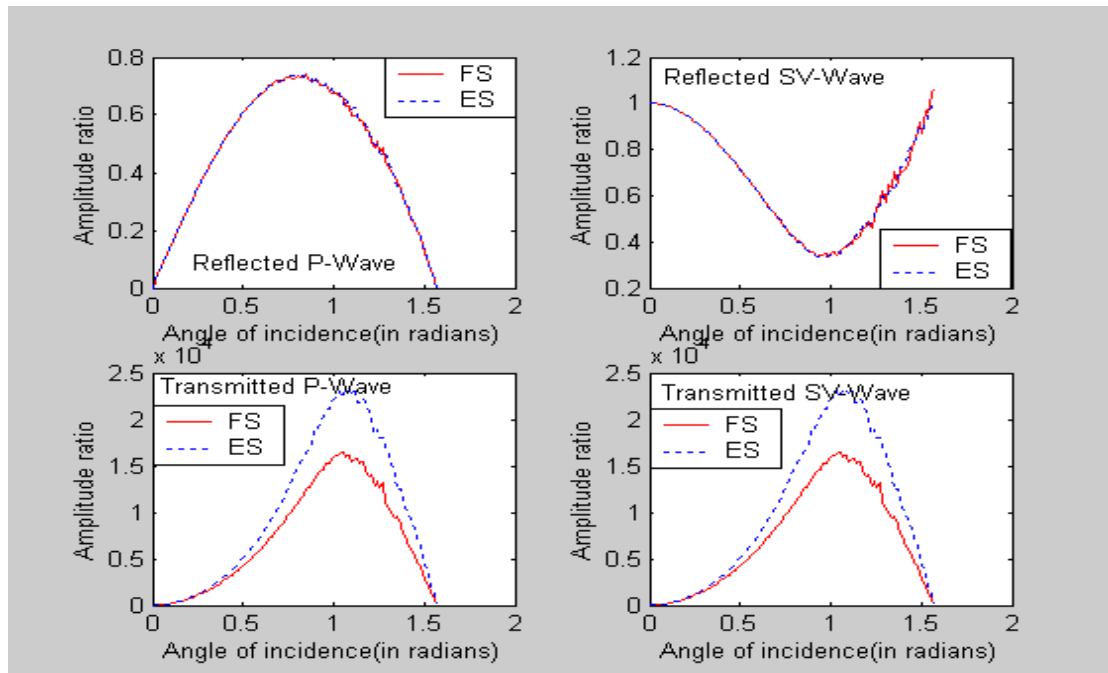
Figs. 2-5. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave showing effect of fluid in pores.



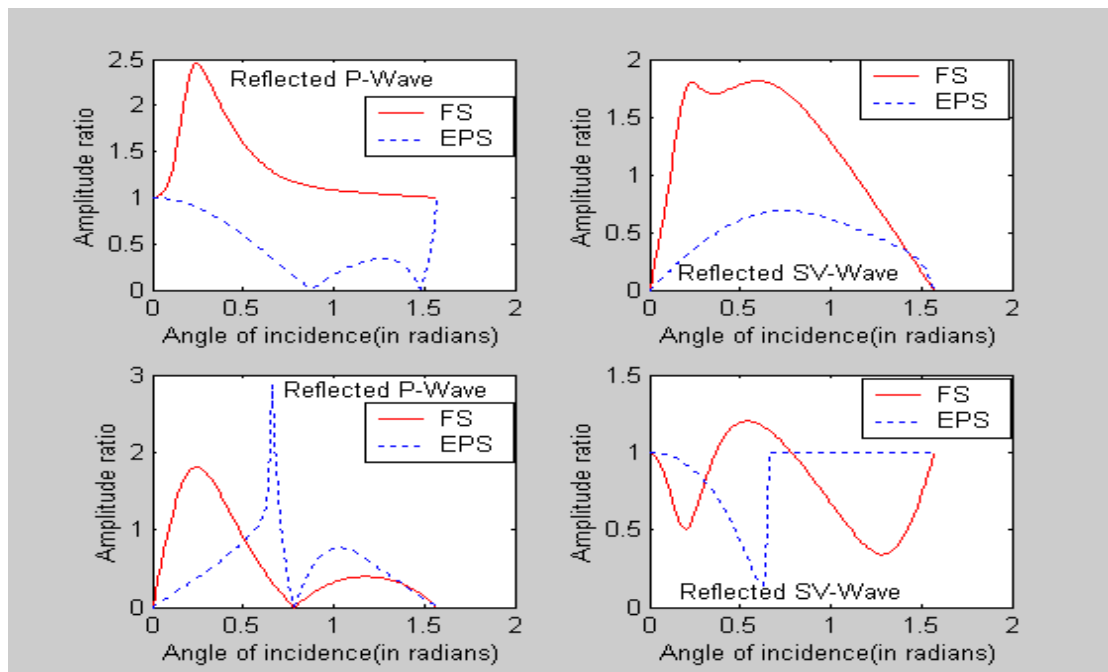
Figs. 6-9. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of P-wave showing effect of viscosity of viscoelastic solid.



Figs. 10-13. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave showing effect of fluid in pores.



Figs. 14-17. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave, transmitted P-wave and transmitted SV-wave with angle of incidence of SV-wave showing effect of viscosity of viscoelastic solid.



Figs. 18-21. Variation of the amplitude ratios of reflected P-wave, reflected SV-wave with angle of incidence of P and SV-wave showing effect of fluid in pores in case of free surface.

7. Conclusion

The phenomenon of reflection and transmission of incident elastic waves at the interface between a linear viscoelastic solid half space and fluid saturated porous half space has been studied when P-wave or SV-wave is incident. It is observed that

- i. The amplitudes ratios of various reflected and refracted waves depend on the angle of incidence of the incident wave and material properties.
- ii. The effect of fluid filled in the pores of incompressible fluid saturated porous medium is significant on amplitudes ratios for refracted waves but on the amplitudes ratios for reflected waves, it has less effect.
- iii. If we neglect the viscous effect of viscoelastic solid then the variations in amplitudes ratios for reflected is not much but for amplitudes ratios for refracted waves has been affected significantly.
- iv. A significant difference in the values of amplitudes ratios for reflected waves is noticed in the case of free surface boundary. The model presented in this paper is one of the more realistic forms of the earth models. It may be of some use in engineering, seismology and geophysics etc.

9. References

- [1] Bowen, R. M., Incompressible porous media models by use of the theory of mixtures, *J. Int. J. Engg. Sci.* 18, 1129-1148, (1980).
- [2] Cooper, Henery F. and Reiss, Edward F., Propagation and Reflection of Viscoelastic Waves, *J. Acoust. Soc. Am.*, 38, 24, 1965.
- [3] Cooper, Henery F., Reflection and Transmission of Oblique Plane Waves at a Plane Interface between Viscoelastic Media, *J. Acoust. Soc. Am.* 42, 1064, 1967.
- [4] de Boer, and Didwania, A. K., Two phase flow and capillarity phenomenon in porous solid-A Continuum Thermomechanical Approach, *Transport in Porous Media (TIPM)*, 56, 137-170, 2004.
- [5] de Boer, R. and Ehlers, W., Uplift, friction and capillarity-three fundamental effects for liquid-saturated porous solids. *Int. J. Solid Structures B* 26, 43-57, (1990).
- [6] de Boer, R. and Ehlers, W., The development of the concept of effective stress, *Acta Mechanica A* 83, 77-92, (1990).
- [7] de Boer, R., Ehlers, W. and Liu, Z., One-dimensional transient wave propagation in fluid-saturated incompressible porous media, *Archive of Applied Mechanics*, 63 (1), 59-72, 1993.
- [8] de Boer, R. and Liu, Z., Plane waves in a semi-infinite fluid saturated porous medium, *Transport in Porous Media*, 16 (2), 147-173, 1994.
- [9] de Boer, R. and Liu, Z. ; Propagation of acceleration waves in incompressible liquid-saturated porous solid, *Transport in porous Media (TIPM)*, 21, 163-173, 1995.
- [10] Fillunger, P., *Der Auftrieb in Talsperren*. Osterr. Wochenschrift fur den offenl. Baudienst, Franz Deuticke, Wien, 1913.

- [11] Kumar, R. and Hundal, B. S. ; Surface wave propagation in fluid-saturated incompressible porous medium, *Sadhana*, 32 (3), 155-166, 2007.
- [12] Kumar, R., Miglani, A. and Kumar, S., Reflection and Transmission of plane waves between two different fluid saturated porous half spaces, *Bull. Pol. Ac., Tech.*, 227-234, 59 (2), 2011.
- [13] Kumar, R., Singh, B., Reflection of plane waves at a planar viscoelastic micropolar interface, *Indian J. Pure and Applied Mathematics*, 31 (3), 287-303, 2000.
- [14] Silva, W., Body waves in a layered anelastic solid, *Bulletin of the Seismological Society of America*, 66 (5), 1539-1554, 1976.
- [15] Tajuddin, M. and Hussaini, S. J., Reflection of plane waves at boundaries of a liquid filled poroelastic half-space, *J. Applied Geophysics* 58, 59-86, (2006).

