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Exact Solution for Transient Heat Conduction through Long Fin

***Dr. Rahul Salhotra and Harbans Singh Ber**

Mechanical Engineering Department
National Institute of Technology, Raipur (C.G.)-492010, India
**Corresponding Author's E-mail: salhotra_rahul@rediffmail.com*

Abstract

This work emphasizes on the analysis of transient heat conduction through fins. The exact local and mean temperature distribution had been generated by numerical technique methods. Exact solutions are given for the transient temperature influx-base fins with the method of Green's functions (GF)in the form of infinite series for three different tip conditions. The time of convergence is improved by replacing the series part. For the long fin case, exact fin solution is presented in graphical forms. Programmed solutions are determined for analysis of exact fin theory. Dimensionless temperature distribution is also presented for exact fin theory.

Keywords: Fin, Transient Conduction, Extended Surface, Exact Solution

Nomenclatures

A_h	surface area of fin for convection (m^2)
α	thermal diffusivity (m^2s^{-1})
Bi	Biot number, h_i $(V/A_h)/K$
β_n	eigen value [eq. (1)]
B_2	Biot number, hL/k
θ	dimensionless temperature
G	Green's function
ξ, X	dimensionless x-coordinate
h	heat transfer coefficient (W $m^{-2}K^{-1}$)

τ, dt	dimensionless time
k	thermal conductivity (W $m^{-1}K^{-1}$)
L	length of fin (m)
N_n	norm [eq. (1)] (m)
m	fin parameter, (m^{-1})
M	dimensionless fin parameter = mL
q_o	heat flux (W m^{-2})
Q	input heat (W)
T	temperature (K)
t	time (s)
V	fin volume (m^3)

1. Introduction

Fins are the extended surfaces used for enhancing the dissipation of heat transfer rate. The transient response of fins is important in a wide range of engineering devices, automobiles and industrial sectors. Work under steady state conduction had been carried out extensively. Transient heat conduction analysis for the fins is being considered for simplifying heat transfer queries. Transient Closed form solutions had been derived earlier by various researchers.

Chapman [1] studied the transient behavior of an annular fin of uniform thickness subjected to a sudden step change in the base temperature. Donald son and Shouman[2] studied the transient temperature distribution in a con vecting straight fin of constant are a for two distinct cases, namely, a step changeinbasetemperature, and a step change in base heat flow rate.Suryanarayana[3,4] studied the transientresponse of straight fins of constant cross-sectionalarea. However, rather than using the separation of variablestechnique followed by [2],Mao and Rooke[5] used the Laplace transform methodtostudy straight fins with three different transients: a step changein base temperature; a step changein baseheat flux and a step change in fluid temperature. Transientfinsof constant cross-section have also been studied with the method of Green's functions [6], a flexibleandpowerfulapproach that areapplicabletoany combination of end conditions on the fin. Kim [7] developedan approximate solution to the transientheat transferin straight finsof constantcross-sectional areaandconstantphysical andthermal properties. Aziz and Na [8]consideredthe transient responseofa semi-infinite fin of uniformthickness, initially at the ambienttemperature, subjected to a step change in temperature at its base,with fincooling governed by a power-law type dependence on temperature difference.Aziz and Kraus [9]presenta variety of analytical results for transient fins, developed by separation of variable and Laplacetransformtechniques.

Campo and Salazar [10] explored the analogy between the transient conduction in a planar slabfor short times andthe steadystateconduction in a straight fin ofuniform cross-section. Saha andAcharya[11]given adetailed parametricanalysisof the unsteadythree-dimensional flow and heat transfer in a pin-fin heat exchanger. Several numerical studies oftransient fins combined with complicating factors,such as natural convection [12, 13], spatial arrays of fins [14, 15] and phase change materials [16] have been presented.Mutlu and Al-Shemmeri[17]studied alongitudinalarray of straight fins suddenly heated at the base. Theinstantaneousheattransfer coefficientwas found at one point on the fin as a ratio of the measured temperature tothemeasure heat flux.

Present article deals with exact temperature distribution of transient heat conduction through fins.

2. Governing Equations for Transient Conduction of Fins

Efficient solutions for the transient heat transfer in flux-base long fins are presented for the three different tip conditions and improvement of convergence of the series solutions. Representation of dimensionless exact fin solution for long fin is also given.

General expression for the transient heat transfer through flux base fins for the three different tip conditions is given by[18].

$$T(x,t) - T_e = \frac{q_o L}{k}\frac{L}{N_0}\frac{\left(1-e^{-m^2\alpha t}\right)}{m^2L^2} + \frac{q_o L}{k}\sum_{n=1}^{\infty}\frac{L}{N_n}\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2} \times \left[1 - exp[-(m^2L^2+\beta_n^2)\alpha t/L^2]\right] \quad (1)$$

Table 1shows the Eigen values for three different tip conditions.

Table 1.Eigen values for three different tip conditions

Case	$\boldsymbol{\frac{L}{N_n}}$	***β_n Or Eigen condition***
$X21$	2	$(n - 1/2)\pi$
$X22$	$2; n \neq 0$ $1; n = 0$	$n\pi$
$X23$	$\frac{2(\beta_n^2 + B_2^2)}{(\beta_n^2 + B_2^2 + B_2)}$	$\beta_n tan(\beta_n) = B_2$

For the temperature-end condition,

$$T(x, t) - T_e = 2\frac{q_o L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2} \times \left[1 - exp[-(m^2L^2+\beta_n^2)\alpha t/L^2]\right] \quad (2)$$

Where $\beta_n = (n-1/2)\pi$

For the insulated end condition,

$$T(x,t) - T_e = \frac{q_0 L}{k}\frac{\left(1-e^{-m^2\alpha t}\right)}{m^2L^2} + 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2} \times \left[1 - exp[-(m^2L^2 + \beta_n^2)\alpha t/L^2]\right] \quad (3)$$

Where $\beta_n = n\pi$

For convective end condition,

$$T(x,t) - T_e = 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\left(\frac{\beta_n^2 + B_2^2}{\beta_n^2+B_2^2+B_2}\right)\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2} \times \left[1 - exp[-(m^2L^2 + \beta_n^2)\alpha t/L^2]\right] \quad (4)$$

Where β satisfies $\beta_n \tan\beta_n = B_2$ & $B_2 = h_2L/k$.

For the improvement in series convergence of steady part, infinite series expression is being transformed into finite non series expression with the help of Green's function.

Using the method of Green's functions, the steady-fin temperature has the form [19].

$$T(x) - T_e = \frac{q_0}{k} G_{X2J}(x, x' = 0) \quad (5)$$

Green's function G_{X2J} for the steady-fin is given by [24].

$$G_{X2J}(x,x') = \frac{R\left(e^{-m(2L-|x-x'|)} + e^{-m(2L-x-x')}\right)}{D} + \left(e^{-m|x-x'|} + e^{-m(x+x')}\right)/D \quad (6)$$

Where $D = 2m(1 - R.e^{-2mL})$

Coefficient R is determined by the tip condition:

$$R = \begin{cases} -1 \ type1 \ at \ x = L \\ 1 \ type2 \ at \ x = L \\ \frac{mL - B_2}{mL - B_2} \ type3 \ at \ x = L \end{cases}$$

Where $B_2 = h_2L/k$.

The Green's function may be evaluated at $x' = 0$ and substituted into Eq.(5) to give:

$$T(x) - T_e = \frac{q_0 L}{k}\frac{\left(R.e^{-m(2L-x)} + e^{-mx}\right)}{mL(1-R.e^{-2mL})} \quad (7)$$

The closed-form steady solutions given by Eq.(7) is used in the transient-fin solutions to replace the slow converging series by replacing the *series steady term* with *non series steady term*. The improved-convergenceform of the transient temperature in flux-base fins are given by:

For the temperature tip condition,

$$T(x,t) - T_e = \frac{q_0 L}{k}\frac{\left(e^{-mx} - e^{-m(2L-x)}\right)}{mL(1+ e^{-2mL})} - 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2} \times \left[exp[-(m^2L^2 + \beta_n^2)\alpha t/L^2]\right] \quad (8)$$

Where $\beta_n = (n-1/2)\pi$

For the insulated end condition,

$$T(x,t) - T_e = \frac{q_0 L}{k}\frac{(e^{-m(2L-x)}+e^{-mx})}{mL((1-e^{-2mL})} - \frac{q_0 L}{k}\frac{e^{-m^2\alpha t}}{m^2L^2} - 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{\beta_n x}{L}\right)}{m^2L^2+\beta_n^2}$$
$$\times \left[exp[-(m^2L^2 + \beta_n^2)\alpha t/L^2]\right] \quad (9)$$

Where $\beta_n = n\pi$

For convective tip condition,

$$T(x,t) - T_e = \frac{q_0 L}{k}\frac{\left(\frac{mL-B_2}{mL+B_2}e^{-m(2L-x)}+e^{-mx}\right)}{mL\left(1-\frac{mL-B_2}{mL+B_2}e^{-2mL}\right)} - 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos\left(\frac{\beta_n x}{L}\right)}{m^2L^2+\beta_n^2}\left(\frac{\beta_n^2+B_2^2}{\beta_n^2+B_2^2+B_2}\right)$$
$$(10) \times \left[exp[-(m^2L^2 + \beta_n^2)\alpha t/L^2]\right]$$

Where β satisfies $\beta_n \tan\beta_n = B_2$& $B_2 = h_2L/k$

It is instructive to examine above three temperature solutions as a group, contains a steady termanda transient series term. However, the insulated-tip solution uniquely contains an additional a non-series transient term.

Dimensionless Exact Fin Solution for Long Fin

The exact temperature expression for long fincasecontains twoterms: aseries steady term anda series transient term. Theseriescontainsanexponential factor with argumentm^{2}L^2+β_n^2.By comparing these arguments, it is clear thatastimeincreases the seriestermwill decaymorerapidly.

The numerical results are presented with the following dimensionless variables:

$$\theta = \frac{T-T_e}{q_0L/_k}\xi = X = x/L\tau = dt = \alpha t/L^2 M = \sqrt{Bi}\left(\frac{L}{V/_{A_h}}\right)Bi = \frac{h\left(V/_{A_h}\right)}{k} \quad (11)$$

Where

θ = dimensionless temperature

ξ = dimensionless location

τ = dimensionless time

M = fin parameter

Bi = biot number

The dimensionless exact fin temperature for long fin is given by:

$$\theta = 2\sum_{n=1}^{\infty}\frac{\cos\,(n-\frac{1}{2})\pi\xi}{M^2+\,(n-\frac{1}{2})^2\pi^2} \times \left[1 - exp[-(M^2 + (n-1/2)^2\pi^2)\tau]\right] \quad (12)$$

3. Result and Discussions

Graphical results for the exact fin theory are as follows.Fig.3.1 to 3.3 shows the temperature distribution curve for exact fin theory, taking M = 0.2 & τ = 0, 0.5 & 1. Curve does not change much for large dimensionless time when M is small. Temperature decreases when location changes from zero to one. Base temperature increases as the time increases.

Fig 3.4 to 3.6 shows the temperature distribution curve for exact fin theory, taking M = 1 & τ = 0, 0.5 & 1. Curve shows that the base temperature increases with increase in time. For low value of M temperature distribution curve remains same.

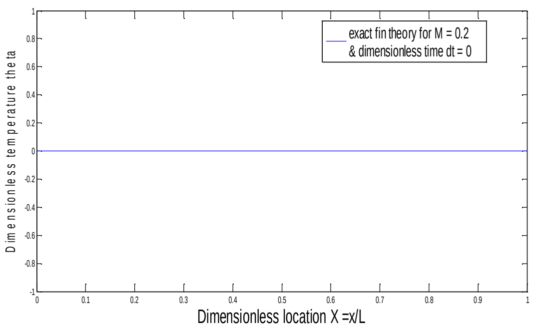

Fig.3.1 Temperature history of exact fin theory for M = 0.2 & dimensionless time dt = 0

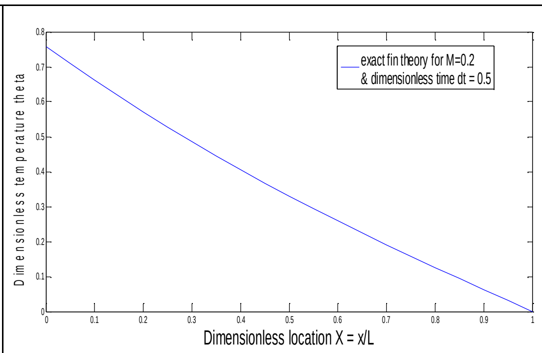

Fig.3.2 Temperature history of exact fin theory for M = 0.2 & dimensionless time dt = 0.5

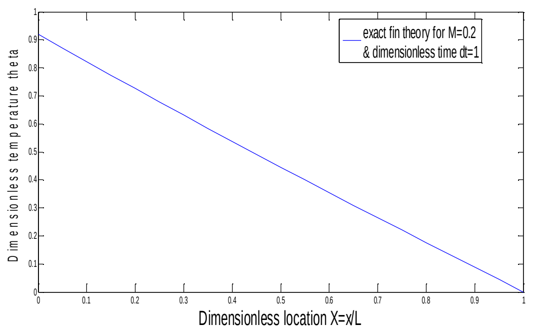

Fig.3.3 Temperature history of exact fin theory for M = 0.2 & dimensionless time dt = 1

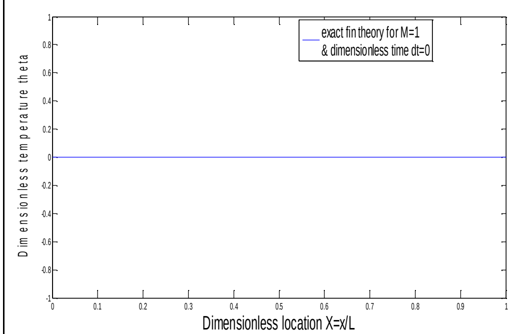

Fig.3.4 Temperature history of exact fin theory for M = 1 & dimensionless time dt = 0

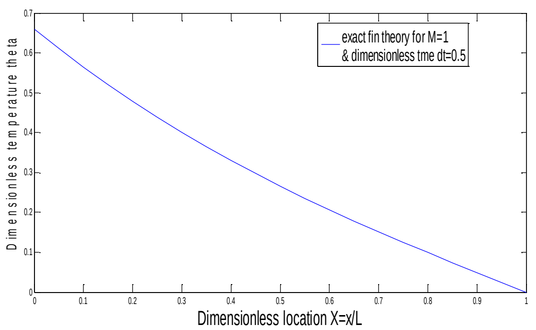

Fig.3.5 Temperature history of exact fin theory for M = 1 & dimensionless time dt = 0.5

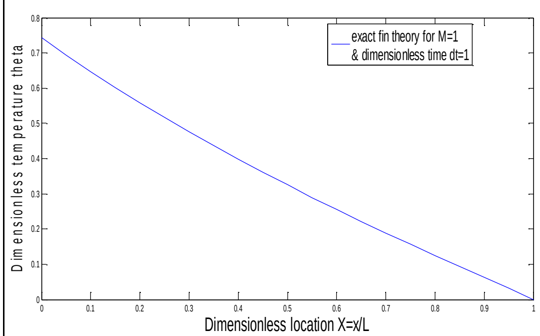

Fig.3.6 Temperature history of exact fin theory for M = 1 & dimensionless time dt = 1

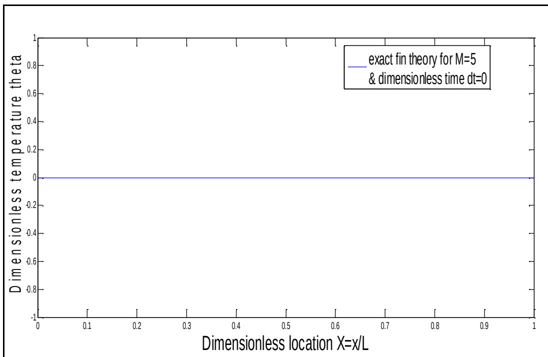

Fig.3.7 Temperature history of exact fin theory for M = 5 & dimensionless time dt = 0

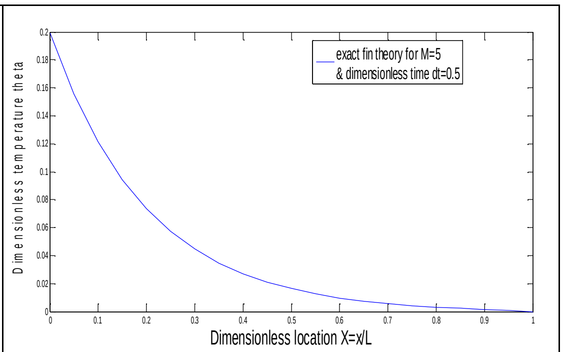

Fig.3.8 Temperature history of exact fin theory for M = 5 & dimensionless time dt = 0.5

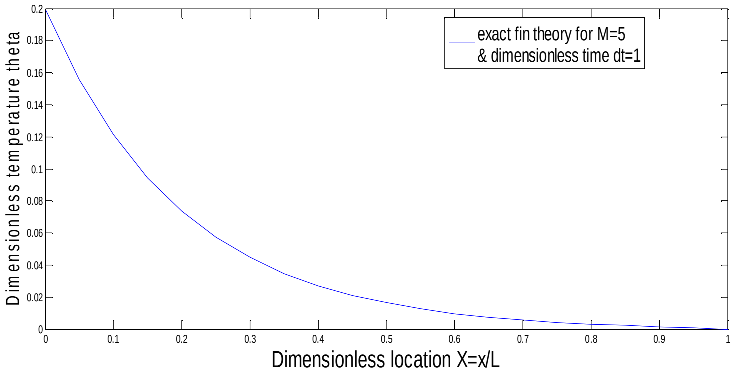

Fig.3.9 Temperature history of exact fin theory for M = 5 & dimensionless time dt = 1

Fig. 3.7 to 3.9 shows the temperature distribution curve for exact fin theory, taking M = 5 & τ = 0, 0.5 & 1. Curve attains steady state for large value of M. Curve does not change much for large dimensionless time.

For the exact fin theory, temperature was computed in the range as: [$0 < x=L < 1$ and $0 < \alpha t/L^2 < 1$] for fin parameter values M = 0.2, 1.0 and 5. This program was coded in Matlab.

4. Conclusion

The temperature distribution for long fin has been taken into account for exact fin theory. Following are the conclusions.

Complicated exact transient solutions can be simplified for temperature distribution analysis through Greens function method.

Convergence of the solution can be improved by replacing the series steady term with non- series finite steady term.

A unique solution is produced containing non dimensionless parameter for long fin case.

Larger value of M increases the transient response of fin.

The exact fin model can be a simple way to find temperature distribution associated with heat loss.

References

[1] A.J. Chapman, Transient heat conduction in annular fins of uniform thickness, Chem. Eng. Symp. 55 (29) (1959) 195–201.

[2] A.B. Donaldson, A.R. Shouman, Unsteady-state temperature distribution in a convecting fin of constant area, Appl. Sci. Res. 26 (1–2) (1972) 75–85.

[3] N.V. Suryanarayana, Transient response of straight fins, J. Heat Transf. 97 (1975) 417–423.

[4] N.V. Suryanarayana, Transient response of straight fins part II, J. Heat Transf. 98 (1976) 324–326.

[5] J. Mao, S. Rooke, Transient analysis of extended surfaces with convective tip, Int. Commun. Heat Mass Transf. 21 (1994) 85–94.

[6] J.V. Beck, K.D. Cole, A. Haji-Sheikh, B. Litkouhi, Heat Conduction Using Green's Functio- ns Hemisphere, New York, 1992.

[7] R.H. Kim, The Kantorovich method in the variational formulation to an unsteady heat condu-ctionLett. Heat Mass Transf. 3 (1) (1976) 73–80.

[8] A. Aziz, T.Y. Na, Transient response of fins by coordinate perturbation expansion, Int. J. He- at and Mass Transfer 23 (1980) 1695–1698.

[9] A. Aziz, A.D. Kraus, Transient heat transfer in extended surfaces, Appl. Mech. Rev. 48 (3) (1995) 317–349.

[10] A. Campo, A. Salazar, Similarity between unsteady-state conduction in a planar slab for short times and steady-state conduction in a uniform, straight fin, Heat Mass Transf. 31 (5) (1996) 365–370.

[11] A.K. Saha, S. Acharya, Parametric study of unsteady flow and heat transfer in a pin-fin heat exchanger, Int. J. Heat Mass Transf. 46 (20) (2003) 3815–3830.

[12] T.H. Hsu, C.K. Chen, Transient analysis of combined forced and free-convection conduction along a vertical circular fin in micro polar fluids, Numer. Heat Transf. A 19 (2) (1991) 177–185.

[13] M. Benmadda, M. Lacroix, Transient natural convection from a finned surface for thermal storage in an enclosure, Numer. Heat Transf. A 29 (1) (1996) 103– 114.

[14] D.K. Tafti, L.W. Zhang, G. Wang, Time-dependent calculation procedure for fully developed and developing flow and heat transfer in louvered fin geometries, Numer. Heat Transf. A 35 (3) (1999) 225–249.

[15] A.K. Saha, S. Acharya, Unsteady simulation of turbulent flow and heat transfer in a channel with periodic array of cubic pin-fins, Numer. Heat Transf. A 46 (8) (2004) 731–763

[16] M. Tutar, A. Akkoca, Numerical analysis of fluid flow and heat transfer characteristics in three-dimensional plate fin-and-tube heat exchangers, Numer. Heat Transf. A 46 (3) (2004) 301–321.

[17] I. Mutlu, T.T. Al-Shemmeri, Steady-state and transient performance of a shrouded Longitu- dinal fin array, Int. Commun. Heat Mass Transf. 20 (1993) 133–143.

[18] K.D. Cole, C. Tarawneh, B. Wilson, Analysis of flux base fins for estimation of heat transfer coefficient. Int. J. Heat and Mass Transfer (2009).

[19] K.D. Cole, Computer programs for temperature in fins and slab bodies with themethod of Green's functions, Comput. Appl. Eng. Educ. 12 (3) (2004) 189–197.