# Temporal Instability of Space Charge Wave in Semiconductor Plasma Embedded with Nanoparticle Cluster

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# Abstract

While studying space charge wave propagation through semiconductor plasma medium, in this paper we have reported the induction of two additional modes of propagation due to the presence of nanoparticle (NP) cluster. We have used the hydrodynamic model of plasmas for semiconductor medium embedded with nanoparticle cluster, to derive the required dispersion relation and studied the temporal instability associated with space charge waves. We found that the space charge wave spectra is strongly modified in presence of nanoparticle cluster within n-type semiconductor plasma medium through the electron cloud plasma frequency of nanoparticle cluster and a non dimensional parameter l characteristics of the nanoparticle cluster which is a function of nanoparticle density N and their radius r. We have found that the two newly induced modes are always copropagating in nature in the parameter regime under study. These two new modes show decaying behaviour in their absorption spectra. The spectra of other two preexisting modes are found to be modified significantly in presence of nanoparticle cluster. Interestingly, the presence of nanoparticle cluster converts the character of second preexisting mode from decaying to amplifying and defines a threshold amplitude of the applied electric field required to obtain growth of the wave. It is hoped that the reported modified spectral behaviour of this newly thought media makes it very suitable for the fabrication of oscillators of desired frequency and for the use as a probe for solid state diagnostics.

# Introduction

Nanoscience and nanotechnology are recent revolutionary developments of science and engineering that are evolving at a very fast pace. It has made great contributions to next generation optical sensors and nanodevices [1,2]. They are driven by the desire to

fabricate materials with novel and improved properties that are likely to impact all areas of physical and chemical sciences and other interdisciplinary fields of science and engineering. Nanoparticle (NP) clusters show behavior that is intermediate between that of macroscopic solid and atomic or molecular system because of their size dependent properties [3].

The presence of NP cluster in a medium makes perceptible modifications in the spectral properties that this medium supports. These modifications are due to three major inherent properties of nanoparticles: high surface to volume ratio [4-7], quantum size effect [8] and electrodynamic interactions [9]. Nanoparticles or NP clusters exhibits optical properties of great aesthetic, technological and intellectual value [10].

Motivated by above discussion and the previous reports of the present authors [11, 12] it is hoped that the study of space charge wave (the most fundamental wave) spectra in semiconductor plasma medium diffused with NP cluster may also result some pleasant modifications in spectral characteristics of supporting modes. Authors also hoped that this study may provide some useful outcomes that will lend a hand in fabrication of advanced microwave and optical devices. With this opinion in mind, in the present paper authors have reported their study on characteristic of space charge wave propagating through a semiconductor plasma medium which is doped with a NP cluster.

#### **Theoretical Formulation**

A NP cluster with number density N, electron density  $n_{0n}$ , and radius r is assumed to be impinged within a n-type semiconductor plasma medium. The medium is acted upon by an external dc electric field  $(\vec{E}_0 = -E_0\hat{z})$  due to which free electrons of medium feels a drift  $\vec{B}_{0z}(= \theta_0 \hat{z})$ . A perturbation of the kind  $\exp[i(\omega t - kz)]$  is assumed to be imposed on the medium. Because of which free electrons of the system starts exhibiting oscillations

at a frequency known as electron plasma frequency  $\omega_{pe} \left[ = \left( n_{0e} e^2 / m_e \varepsilon_0 \varepsilon_L \right)^{1/2} \right]$ . Under

favourable physical conditions this perturbation starts propagating as a wave termed as space charge or plasma wave.

We have considered the hydrodynamic model for semiconductor plasma ( $kL \ll 1$  in which k is wave number and L is mean free path of the carrier) to derive a general dispersion relation for the propagation of space charge wave. Here, for the geometry under consideration the one dimensional momentum transfer equation for free electrons of the semiconductor medium may be written as-

$$\frac{\partial \mathcal{G}_{lz}}{\partial t} + \mathcal{G}_{0z} \frac{\partial \mathcal{G}_{lz}}{\partial z} = -\frac{e}{m} E_{lz} - v_e \mathcal{G}_{lz} + \frac{\mathcal{G}_{\theta}^2}{\rho_0} \frac{\partial \rho_l}{\partial z}$$
(1)

The explicit expression for first order perturbed velocity of free electrons in terms of electric field intensity becomes.

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$$\mathcal{G}_{1z} = -i\frac{e}{m}\frac{\omega - k\mathcal{G}_{0z}}{(\omega - k\mathcal{G}_{0z})(\omega - k\mathcal{G}_{0z} - i\nu_e) - k^2\mathcal{G}_{\theta}^2}E_{1z}$$
(2)

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The conduction current density may be obtained as-

$$J_{1z} = -en_0 \vartheta_{1z} = -i\omega\varepsilon_0 \varepsilon_L \frac{\omega_{pe}^2}{(\omega - k\vartheta_{0z})(\omega - k\vartheta_{0z} - iv_e) - k^2 \vartheta_{\theta}^2} E_{1z}$$
(3)

On the other hand the displacement  $\triangle$  of the electron cloud of NP cluster under the influence of wave rf electric field may be expressed as

$$\frac{d^2\Delta}{dt^2} + \frac{\omega_{pn}^2}{3}\Delta = -\frac{eE}{m} \quad , \tag{4}$$

Here,  $\omega_{pn} \left[ = \left( n_{0n} e^2 / m \right)^{1/2} \right]$  denotes the plasma frequency of electron cloud present within NP cluster.

As we have assumed that the perturbation varies as  $\exp[i(\omega t - kz)]$ , the first order perturbed velocity of electron cloud within NP cluster becomes

$$\vec{\vartheta}_{np} = \frac{i\omega eE_{1z}}{m\left(\omega^2 - \frac{\omega_{pn}^2}{3}\right)}$$
(5)

Now the conduction current density for this electron cloud may be expressed as-

$$\vec{J}_{np} = -\frac{4\pi l}{3} e n_{0n} \vec{\mathcal{G}}_{np} \tag{6}$$

Here,  $l = Nr^3$  is the non dimensional parameter which is a function of nanoparticle density *N* and its radius *r*.

Substituting equation (5) in equation (6), we obtain

$$\vec{J}_{np} = -\frac{4\pi l}{3} i\omega \frac{\omega_{pn}^{2}}{\left(\omega^{2} - \frac{\omega_{pn}^{2}}{3}\right)} E_{1z}$$
(7)

Using equation (3) and (7), we obtain the resultant current density  $\vec{J}_1 = \vec{J}_{1z} + \vec{J}_{np}$  as

$$\vec{J}_{1} = -i\omega \left[ \frac{\omega_{pe}^{2}}{(\omega - k\vartheta_{0z})(\omega - k\vartheta_{0z} - iv_{e}) - \vartheta_{\theta}^{2}k^{2}} + \frac{4\pi l}{3} \frac{\omega_{pn}^{2}}{\left(\omega^{2} - \frac{\omega_{pn}^{2}}{3}\right)} \right] E_{1z}$$

$$\tag{8}$$

The self consistent solution of the Maxwell's equation in non-magnetic media leads to wave equation

$$\vec{k} \times \left(\vec{k} \times \vec{E}_{1z}\right) + \omega^2 \mu_0 \varepsilon_0 \varepsilon_L E_{1z} = i\omega \mu_0 \vec{J}_1 \tag{9}$$

Under the present field geometry, we may rewrite the wave equation for space charge wave  $(\vec{k} \times \vec{E}_{1z} = 0)$  as

$$\omega \varepsilon_0 \varepsilon_L E_{1z} = i \vec{J}_1 \tag{10}$$

Using equations (8) and (10), we obtain the general dispersion relation for space charge wave in n-type semiconductor plasma medium diffused with a NP cluster as-

$$\varepsilon(\omega,k) = \left[1 - \frac{\omega_{pe}^2}{\left(\omega - k\vartheta_{0z}\right)\left(\omega - k\vartheta_{0z} - i\nu_e\right) - \vartheta_{\theta}^2 k^2} - \frac{4\pi}{3}l \frac{\omega_{pn}^2}{\left(\omega^2 - \frac{\omega_{pn}^2}{3}\right)}\right] = 0$$
(11)

The above dispersion relation reveals that the space charge wave spectra shall be strongly modified in presence of NP cluster within the medium through the electron cloud plasma frequency of NP cluster  $\omega_{pn}$  and a non-dimensional parameter *l* characteristics of the NP cluster and is a function of nanoparticle density *N* and its radius *r*.

#### **Results and discussion**

The principal aim of the paper is to study the temporal behavior of the space charge modes propagating through n-type semiconductor plasma embedded with a NP cluster. To do so, now we will rewrite the dispersion relation [equation (11)] in the form of a polynomial in terms of complex angular frequency  $\omega$  as-

$$A_4\omega^4 + A_3\omega^3 + A_2\omega^2 + A_1\omega + A_0 = 0$$
(12)

where,

$$A_{0} = -\frac{\omega_{pn}^{2}}{3} (4\pi l + 1) [k^{2} (\vartheta_{0z}^{2} - \vartheta_{\theta}^{2}) + ik \vartheta_{0z} v_{e}] + \frac{\omega_{pn}^{2}}{3} \omega_{pe}^{2}$$

$$A_{1} = \frac{\omega_{pn}^{2}}{3} (4\pi l + 1) (2k \vartheta_{0z} + i v_{e})]$$

$$A_{2} = k^{2} (\vartheta_{0z}^{2} - \vartheta_{\theta}^{2}) - \omega_{pe}^{2} - \frac{\omega_{pn}^{2}}{3} (4\pi l + 1) + ik \vartheta_{0z} v_{e}$$

$$A_{3} = -(2k \vartheta_{0z} + i v_{e})$$

$$A_{4} = 1$$

The above polynomial, a fourth order in  $\omega$  with complex coefficients infers the possibility of four different modes of propagation of space charge wave.

Now if we rewrite the above polynomial in absence of NP cluster in the medium, it will reduce to a second order polynomial in  $\omega$  as-

$$B_2\omega^2 + B_1\omega + B_0 = 0 \tag{13}$$

where,

$$B_{0} = \left[k^{2}\left(\vartheta_{0z}^{2} - \vartheta_{\theta}^{2}\right) - \omega_{pe}^{2} + ik\vartheta_{0z}v_{e}\right]$$
$$B_{1} = -(2k\vartheta_{0z} + iv_{e})$$
$$B_{2} = 1$$

This establishes the fact that when space charge wave propagates through n-type semiconductor plasma medium, it finds only two modes of propagation. Hence, it is clear that the presence of NP cluster in the medium is responsible for two novel modes of propagation of space charge wave. It is also hoped that the behavior of two preexisting modes will be influenced by the presence of NP cluster in the medium.

Now to achieve some qualitative appreciation of the results so obtained, we consider here n-Ge crystal at room temperature as our medium. The relevant physical parameters used are:

$$m_e = 1.588m_0$$
,  $\varepsilon_L = 15.8$ ,  $n_{0e} = 2 \times 10^{24} m^{-3}$  and  $v_e = 3.076 \times 10^{11} s^{-1}$ 

We have solved the polynomial [equation (12)] numerically for l = 0.001 and studied the dispersive and amplification characteristics of all the four possible modes. The qualitative results are illustrated in Figures 1 to 4, out of which Figures 1 and 2 display

the spectral modifications of two preexisting modes. Figures 3 and 4 illustrate the nature of two modes induced due to the presence of NP cluster.

Curves in Figures 1(a) and 1(b) depict the nature of first mode with applied dc electric field in presence as well as in absence of NP cluster. From Figure 1(a), it may be inferred that in presence of NP cluster this mode is always contrapropagating with decreasing phase velocity upto  $E_0 \approx 4.45 \times 10^8 Vm^{-1}$ . Beyond this value of electric field this mode becomes non-propagating as its phase velocity acquires zero value.



Figure 1(a): Variation of phase constant of I-mode with applied electric field  $E_0$ 



Figure 1(b): Variation of gain coefficient of I-mode with applied electric field  $E_0$ 

On the other hand in absence of NP cluster within the medium, this mode is initially contrapropagating in nature with phase velocity decreasing with  $E_0$  and approaches zero value at  $E_0 \approx 4.39 \times 10^8 Vm^{-1}$ . Interestingly enough for a dc electric field

 $E_0 > 4.39 \times 10^8 Vm^{-1}$ , this mode changes its propagation characteristics by becoming copropagating in nature with phase velocity increases with  $E_0$ . The amplification characteristic of I mode is depicted in Figure 1(b). This figure reveals that the I mode is decaying in nature in both types of media (with and without NP cluster). Qualitatively the decay constant is found independent of applied electric field  $E_0$  when the medium is free from NP cluster. Surprisingly, the presence of NP cluster in the medium becomes responsible for rapid decrement in decay constant of the I mode with increasing electric field upto  $E_0 \approx 1.94 \times 10^9 Vm^{-1}$ . If one applies dc electric field of magnitude more than  $1.94 \times 10^9 Vm^{-1}$  the mode becomes evanescent mode i.e., a propagating mode with constant amplitude.

The nature of variation of II preexisting mode are depicted in Figures 2 (a) and 2 (b). It may be inferred from Figure 2 (a) that in absence of NP cluster this mode is a copropagating mode with phase velocity increasing with applied electric field  $E_0$ . The presence of NP cluster in the medium converts this mode to a non-propagating aperiodic



Figure 2(a): Variation of phase constant of II-mode with applied electric field  $E_0$ 



Figure 2(b): Variation of gain coefficient of II-mode with applied electric field  $E_0$ 

mode up to a dc electric field  $E_0 \approx 2.9 \times 10^8 Vm^{-1}$ . If we increase the value of  $E_0$  further the phase velocity of this mode increases with  $E_0$  but propagates in opposite direction. The phase velocity of this contrapropagating mode achieves its maximum value at  $E_0 \approx 4.2 \times 10^9 Vm^{-1}$ . Beyond this value of  $E_0$  the magnitude of phase velocity starts decreasing asymptotically and becomes zero at  $E_0 \approx 4.44 \times 10^8 Vm^{-1}$ . If one tunes the magnitude of  $E_0 > 4.44 \times 10^8 Vm^{-1}$ , the propagation character of this mode crosses over to copropagating nature. This copropagating mode has a phase velocity which is an increasing function of applied dc electric field. This increasing phase velocity touches a maxima at  $E_0 \approx 4.64 \times 10^8 Vm^{-1}$ , further increase in  $E_0$  reduces the phase velocity exponentially and then becomes independent of  $E_0$ . From figure 2(b) one may yield the information that the second mode always shows decaying behavior in absence of NP cluster. Its decay constant does not depend on the magnitude of  $E_0$  and is equal to  $1.538 \times 10^6 s^{-1}$ . Even in presence of NP cluster for lower values of electric field, this mode still shows decaying nature but with decay constant increasing with  $E_0$  upto  $E_0 \approx 4.3 \times 10^8 Vm^{-1}$ . If one applies electric field more than  $4.3 \times 10^8 Vm^{-1}$  the decay constant suddenly starts decreasing very rapidly and becomes zero at  $E_0 \approx 4.59 \times 10^8 Vm^{-1}$ . For  $E_0 > 4.59 \times 10^8 Vm^{-1}$  this mode starts growing with increasing growth rate and touches a maximum value at  $E_0 \approx 4.77 \times 10^8 Vm^{-1}$ . Hence, we may term  $E_0 \approx 4.59 \times 10^8 Vm^{-1}$  as the threshold field required for amplification of II mode in presence of NP cluster.

The characteristic of two novel modes induced due to presence of NP cluster within the semiconductor plasma medium are depicted in Figures 3 and 4. The curves in Figures 3(a) and 4(a) yield that both the novel modes are copropagating in nature, whereas Figures 3(b) and 4(b) reflect their decay character.



Figure 3(a): Variation of phase constant of III-mode with applied electric field  $E_0$ 



Figure 3(b): Variation of gain coefficient of III-mode with applied electric field  $E_0$ 



Figure 4(a): Variation of phase constant of IV-mode with applied electric field  $E_0$ 



Figure 4(b): Variation of gain coefficient of IV-mode with applied electric field  $E_0$ 

With the increment in  $E_0$  the phase velocity of III mode [fig 3(a)] initially increases very slowly upto  $E_0 \approx 3 \times 10^8 Vm^{-1}$ . For  $3 \times 10^8 Vm^{-1} < E_0 < 5 \times 10^8 Vm^{-1}$ , it shows a very sharp incremental behavior. For the electric field greater than this range, it is found that the phase velocity of III mode becomes independent of  $E_0$ . The amplification characteristic of this mode is depicted in Figure 3(b) which reflects that upto  $E_0 \approx 4.07 \times 10^8 Vm^{-1}$ , this induced mode is evanescent in nature. If one applies an electric field in the range  $4.07 \times 10^8 Vm^{-1} < E_0 < 4.97 \times 10^8 Vm^{-1}$ , the decay constant of the said mode increases suddenly and then becomes nearly constant. The applied electric field increases the phase velocity of the fourth mode linearly as depicted in Figure 4(a). Figure 4(b) infers that this decaying mode has a constant decay coefficient approximately equal to  $1.538 \times 10^{11} s^{-1}$ . Hence the applied electric field has no effect on the amplification characteristic of this second novel mode.

## Conclusion

From the above report one may conclude that by diffusing a NP cluster in a normal semiconductor medium, one may get a very interesting complex medium for the study of space charge wave propagation characteristic. The reported modified behavior of the medium makes it very suitable for the fabrication of oscillators and for the use as a probe for solid state diagnostics. We found that the presence of NP cluster in the medium not only induces two new channels of propagation but also strongly modifies the spectral behavior of other two preexisting modes. The presence of NP cluster converts the character of second preexisting mode from decaying to amplifying; a very interesting observation that may be used for the fabrication of solid state switches. In addition, the medium may be used for designing the oscillator of a desired frequency.

## References

- [1] U. Kreibig and M. Vollmer: Optical Properties of Metal Clusters, (Springer, Berlin, 1995).
- [2] E. Hutter and J. Fendler, 2004, "Exploitation of localized surface plasmon resonance," Adv. Mater., Vol. 16, No. 19, pp. 1685-1706.
- [3] S. K. Ghosh and T. Pal, 2007, "Interparticle coupling effect on the surface plasmon resonance of gold nanoparticles; From theory to applications," Chem. Rev., Vol. 107, No. 11, pp. 4797-4862.
- [4] G. Schmid: Clusters and Colloids From Theory to Applications, (VCH Weinheim, Germany, 1994).
- [5] S. K. Ghosh, S. Kundu, M. Mandal and T. Pal, 2002, "Silver and gold nanocluster catalyzed reduction of methylene blue by arsine in a micellar medium," Langmuir, Vol. 18, No. 23, pp. 8756-8760.

- [6] S. K. Ghosh, T. Pal, S. Kundu, S. Nath and T. Pal, 2004, "Fluorescence quenching of 1-methylamicropyrene near gold nanoparticles: Size regime dependence of the small metallic particles," Chem. Phys. Lett., Vol. 395, No. 4, pp. 366-372.
- [7] S. K. Ghosh, A. Pal, S. Kundu, S. Nath, S. Panigrahi and T. Pal, 2005, "Dimerization of eosin on nanostructured gold surfaces: Size regime dependence of the small metallic particles," Chem. Phys. Lett., Vol. 412, No. 1, pp. 5-11.
- [8] R. Kubo, 1962, "Electronic properties of metallic fine particles," J. Phys. Soc. Jpn., Vol. 17, No. 6, pp. 975-986.
- [9] J. D. Jackson: Classical Electrodynamics, (Wiley, New York, 1975)
- [10] H. A. Atwater, 2007, "The promise of plasmonics," Sci. Am., Vol. 296, No. 4, pp. 56-63.
- [11] S. Ghosh and P. Dubey, 2014, "Temporal instability of longitudinal electrokinetic wave in semiconductor plasma doped with a metal nanoparticle," Int. J. Phy. Mathe. Sci., Vol. 4, No. 4, pp. 45-55.
- [12] S. Ghosh and P. Dubey, 2015, "Space charge wave spectra in semiconductor plasma in presence of nanoparticle cluster," AIP Conference Proceedings, Vol. 1670, pp. 030011.