International Journal of Physics and Applications.
ISSN 0974-3103 Volume 3, Number 1 (2011), pp.105-115
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A Review of the Chaotic Behaviour of a Diode Resonator

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Abstract

One of the classic problems in the research of nonlinear dynamics has been the diode resonator. By making a collection of previous work with the driven diode resonator, we have investigated origins of period doubling and chaotic behaviour. By using a model of the diode that includes the forward bias voltage, reverse recovery time, and junction capacitance, the nonlinear behaviour of the circuit is simulated by Multisim. Our work establishes that the nonlinearities of the reverse recovery time must also be considered for a complete understanding of period doubling route to chaos in this circuit. In addition to this comparative investigation, we present our contribution to referred implementation. A new high amplitude oscillation is observed at specific driving amplitude, and the circuit led to a non-periodic mode of operation with rich chaotic content at large driving signal amplitudes.

Keywords: Chaos, Nonlinear dynamics, R-L Varactor circuit, Bifurcation.

Introduction
Simple nonlinear electrical resonators have proven to be a valuable means to investigate the universal features of nonlinear dynamical systems both theoretically and experimentally. Most probably, the diode resonator [1- 3] consisting of a resistor, an inductor, and a varactor diode driven by a periodic signal source was the first chaotic electronic circuit that was extensively studied. It is the simplest chaotic non-

autonomous circuit that shows period doubling and chaos as a function of sinusoidal driving amplitude and frequency. The varactor diode function both as a nonlinear resistor and a nonlinear charge storage device, and the resulting circuit is a nonlinear resonator with complex dynamics. The various causes of nonlinearities that bifurcates the circuit into period doubling or chaos have also been reported [4- 6]. The driven RL varactor was shown to follow a well defined route to chaos in good agreement with the predictions of theory as in iterated and unimodel maps [7-11]. We must have a clear understanding about the physical conditions present in the nonlinear system that leads to a simple model suitable for mathematical modelling. The previous work on the diode resonator contributes the nonlinearity put-in by the voltage-dependent capacitance of the varactor for period doubling and chaotic behaviour. However, Hunt commented that [12] another property of the diode, namely, the reverse recovery time as the parameter responsible for the nonlinear behaviour. We establish that both factors are relevant for the diode resonator to exhibit chaotic behaviour. We believe that the capacitance variation of the varactor is unimportant with regard to the salient features of the response. It is assumed that the varactor diode will behave as an ideal diode with the following characteristics; (i) the diode will conduct only if the external forward bias voltage exceeds a finite forward bias voltage V_f , and the voltage drop across the diode remains at V_f during its conduction period (ii) the diode doesn't conduct if the external voltage is less than V_f, and it acts as a capacitor with a fixed capacitance value C (iii) when the diode is switched abruptly from on to off state, it does not turn off immediately, but continues to conduct for a time interval equal to the reverse recovery time τ_{rr}.

The paper is organized as follows. In the next section, the topology of the R-L Varactor is presented and an idealized mathematical model is derived from the circuit. In section 3, we briefly outline the nonlinear behaviour of the storage time. Multisim simulation results of the driven R-L Varactor at varying amplitude of the input signal and in the sub-resonance region are elucidated in sections 4 and 5 respectively. Finally, the concluding remarks are given in section 6.

Driven R-L Varactor Diode

Consider the R-L Varactor circuit given in Fig. 1 driven by a sinusoidal voltage source V(t) = E Sin2πft. The dynamics of the circuit is described by the equation

$$L \overset{\circ\circ}{q} + R \overset{\circ}{q} + V_C \;=\; E \sin(2\pi f t) \qquad (1)$$

where Vc is the voltage across the varactor diode. The circuit was simulated by using component values L= 400μH, R= 47Ω, and with a silicon varactor diode (type FMMV 109) which is the nonlinear element. The capacitance variation of varactor diode is given by

$$C = C_0/[1+V_C/0.6]^{0.5} \qquad (2)$$

Here V_C is the voltage across the diode under reverse bias which is given by V_C = q/C and C_0 = 62pF. In forward biased condition, the varactor diode functions like a normal silicon diode. When the driving voltage is very low, the circuit behaves like a high Q resonant circuit with a resonant frequency given by

$$f = \frac{1}{2\pi\sqrt{LC_0}} = 1.01\text{MHz} \tag{3}$$

If the circuit contains a large value of resistance R, it is said to be dissipative. In this case, the period doubling routes to chaos are in harmony with the theory of Feigenbaum [13] and the appearance of the periodic windows follows the predictions of Metropolis, Stein, and Stein [14]. The resulting attractor of this system is then quite similar to the logistic equation $x_{n+1} = 4\,\mu\, x_n(1- x_n)$. On the other hand, if the value of the resistance is small, then the attractor is more complex, but quite regular in its structure. It is argued that the modulation parameter λ should satisfy the recurrence relation

$$(\lambda_{n+1} - \lambda_n) / (\lambda_{n+2} - \lambda_{n+1}) = \delta \tag{4}$$

where δ is universal convergence rate. For a quadratic equation $\delta = 4.669\ldots$

In a driven R-L varactor circuit, three types of nonlinearities are present; (i) The nonlinear I-V characteristics of the diode. Most researchers consider it to be unimportant for chaotic formation [15]. (ii) The nonlinear forward-bias capacitance associated with the diffusion of carriers near the junction. Now the diode can be modelled as a parallel combination of nonlinear resistor and a nonlinear capacitor as shown in Fig. 2. Period doubling will occur as a result of this capacitance when its value reaches four times the zero bias value [16]. It is because the resonant frequency of the RLD circuit is inversely proportional to the square root of the capacitance (see Eq. 3). This is considered as the first step in a period doubling route to chaos. (iii) The finite diffusive dynamics of charge in the pn-junction and the associated memory of previous forward current contribute to the third nonlinearity. The main source of chaos in a driven RLD circuit was proposed due to this finite memory of forward bias current [17].

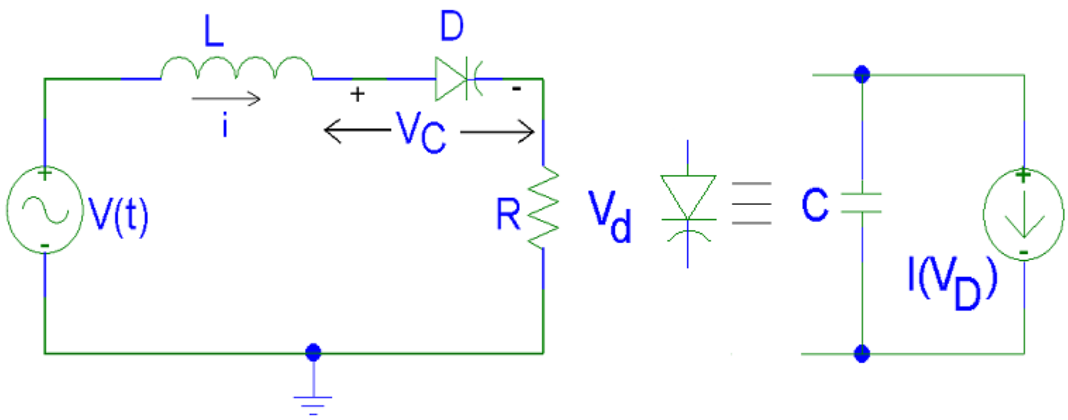


Figure 1 & 2: 1: Schematic diagram for a driven R-L Varactor circuit. **2:** Equivalent model of a varactor diode.

Nonlinear Behaviour of the Storage Time

The nonlinear behaviour of the storage time can be explained using the idealized representation of a switching circuit shown in Fig. 3(a). Prior to switching, the diode is taken to be forward biased with a steady state forward current of I_f. At $t = 0$, the circuit is switched to position b. Current through the diode is abruptly switched to $-I_r$, remains there for a limited period of time before eventually decaying to the steady state value as shown in Fig. 3(b). The period of time during which it remains constant is known as storage time τ_s. Other timing parameters of the circuit are recovery time τ_r, and the total time or reverse recovery time τ_{rr}. The root cause of delay in switching between the on and off states can be explained as follows; the forward biasing of a diode causes a build up of storage of excess minority carriers in the quasi-neutral regions immediately adjacent to the depletion region. When the diode is reverse biased, there is a deficit of minority carriers in the near vicinity of the depletion region. To progress from on to off state, the excess minority carriers must be removed from the two sides of the junction. It accounts for the storage time τ_s . This takes place either by eliminating carriers by recombination or by reducing the excess carriers by carrier flow out of the region. Neither mechanism can safely remove the charge at a sufficiently rapid rate to be considered instantaneous. Hence a delay is observed in going from the on state to the off state. An approximate expression for the storage time τ_s can be analyzed using charge dynamics of the pn junction and is given by

$$\tau_s = \tau_m \ln [1+ I_m/I_r] \tag{5}$$

where I_r is the reverse current through the diode during the storage phase. With the amplitude of forward bias current, the reverse recovery time of the diode increase according to the equation

$$\tau_{rr} = \tau_m[1-\exp(- |I_m| /I_C)] \tag{6}$$

where I_m is the most recent maximum forward current, τ_m and I_C are lifetime and current scales particular to each diode.

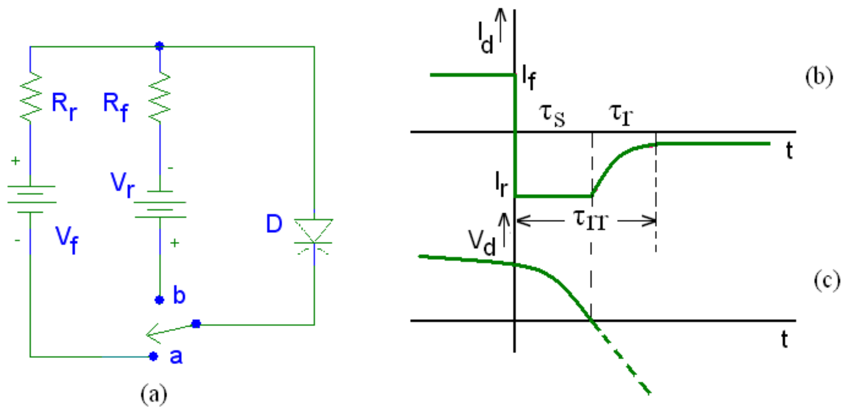
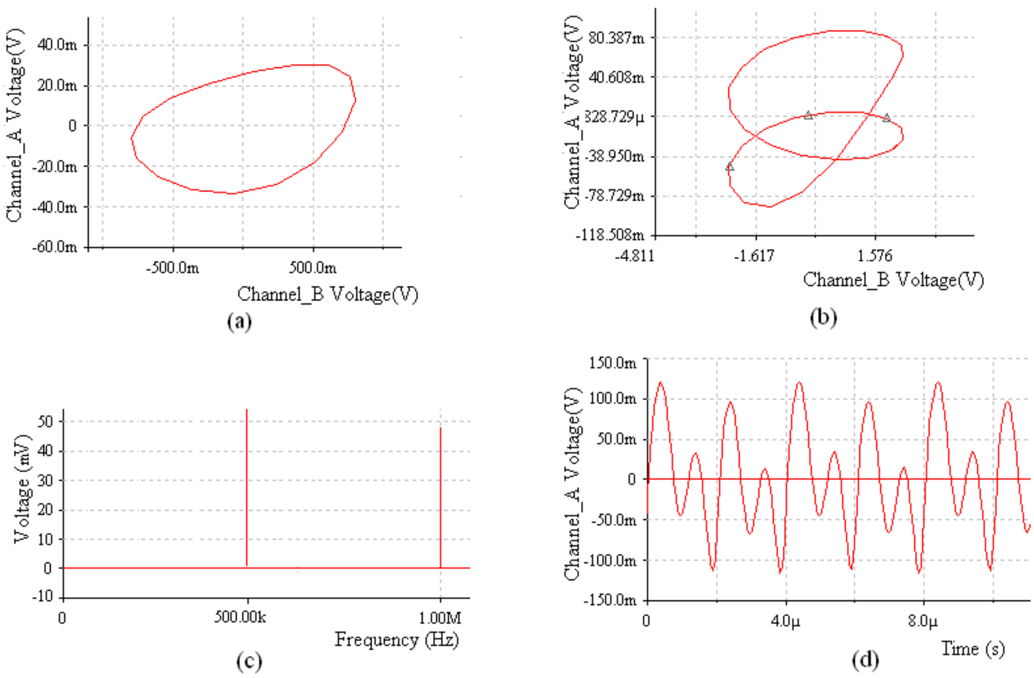
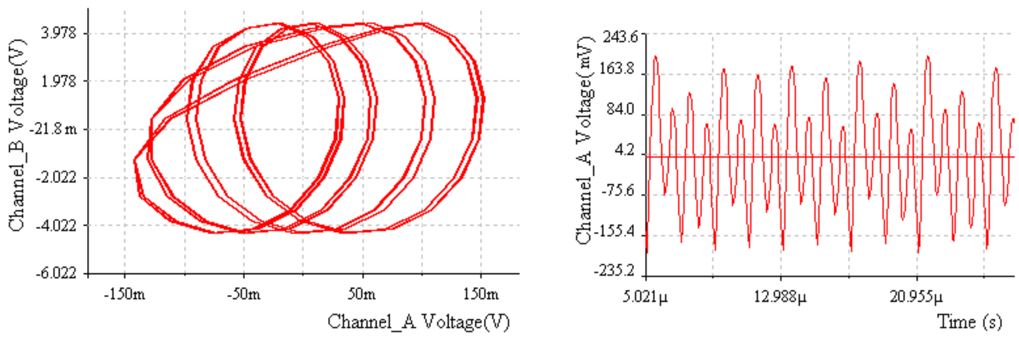


Figure 3: Diode switching time (a) Circuit diagram (b) Current-time transient (c) Voltage-time transient.

Behaviour in the Resonance Region

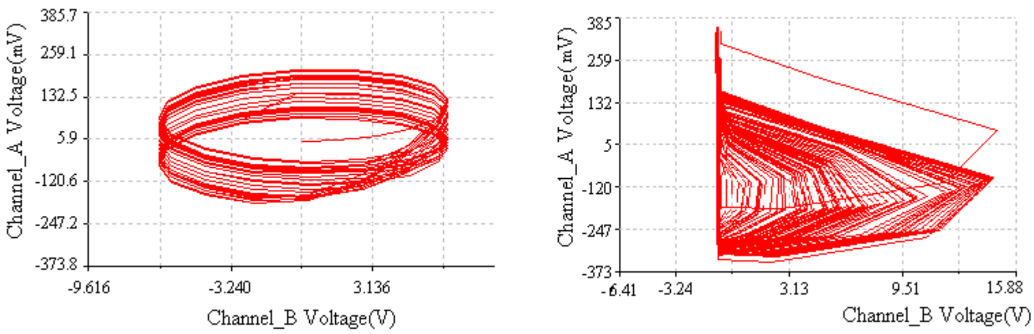
The circuit shown in Fig. 1 was simulated using Multisim with component values L= 400μH, R= 47Ω, and a silicon varactor diode (type FMMV 109) to give a resonant frequency of 1.01MHz. The dynamical nature of this circuit was studied by driving the circuit at this frequency and gradually increasing the drive voltage. Assuming that the varactor nonlinearity is fixed, various control parameters of the circuit are the drive frequency, the drive amplitude, and the series resistance. The response of the circuit was simulated by displaying the output voltage versus the drive signal. At low voltages, the circuit displayed the frequency multiplication peculiar to all nonlinear circuits. For input voltage less than 2V, output waveform is periodic, having a frequency equal to the resonant frequency and V_o Vs V_i appeared like a circle (Fig. 4(a)). The first sub-harmonic appeared at a drive voltage of 2.35V, the phase spectrum of which is shown in Fig. 4(b). The Fourier spectrum of the output has two harmonic components, one at 500 kHz and the other at 1MHz as illustrated in Fig. 4(c). Further increase of input voltage resulted in period 4 waveform (Fig. 4(d)) for an amplitude of 3.3V. Figs. 4(e) and 4(f) correspond to phase spectrum and time base waveform for inputs 4.35 and 5.58V respectively. When the amplitude of input voltage is increased to a large value, the system undergoes various dynamic states, wherein one can observe high amplitude oscillations, non-periodic waveforms with rich dynamic contents, and a number of periodic windows in between the chaotic behaviour. This is illustrated in Figs. 5(a) to 5(c). By measuring the difference in threshold voltages between period doublings and by using Eq. 4 the value of δ is calculated. It is found to be 4.669 which are in good agreement with the theoretical value.



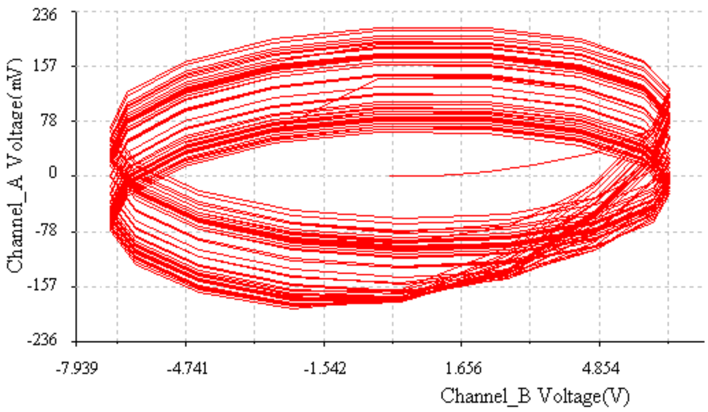


(e) (f)

Figure 4: Simulation results at 1MHz (a) Phase spectra for Vin = 0.8V (b) for Vin = 2.35V (c) Harmonic components for Vin = 2.35V (d) Second period doubling at Vin = 3.3V (e) Phase spectrum for Vin = 4.35V (f) Output for Vin = 5.58V



(a) (c)



(b)

Figure 5: Phase spectra showing Vo Vs Vi for (a) Vin = 6.5V (b) Vin =10.2V (c) Diode voltage versus V_i for Vin = 10.2V

The following explanation would give an insight into the dynamical nature of a driven R-L Varactor circuit. When forward biased, the un-recombined electrons and holes cross the junction, which diffuse back to their origin as the diode changes its state-of-bias. The diode, therefore acts like a capacitor which continues to charge and discharge with variations in the width of depletion region. More the amount of forward voltage, the greater the amount of charges that cross the junction and the longer the system needs to return to its reverse bias equilibrium. If the reverse current is unable to reach equilibrium before reaching the forward bias, then the next cycle will depend upon the previous cycle, and it is equivalent to different parameters at each cycle's initial conditions. Thus, the reverse recovery and the nonlinear capacitor model will have memory built into them which makes the circuit chaotic due to period doubling or bifurcation of the output signal. For a sinusoidal signal, the power spectrum of the output signal will contain the fundamental input frequency and some high order harmonics due to the system's nonlinearities. If the same system has to undergo chaos by period doubling, then additional frequency components, known as sub-harmonics and ultra-sub-harmonics, should appear [15, 18]. Further amplitude increase will result in the formation of non-periodic waveforms, leading progressively into higher periodicity until there are no more stable states, and eventually chaos prevails.

Behaviour in the Sub Resonance Region

In this study, the amplitude of the external voltage is served as the control parameter by keeping the signal frequency at 450 kHz. The circuit response was studied as before. The results are shown in Figs. 6 to 9. For signal amplitude in the range of 0 to 5.8V, the circuit current is of low amplitude and we get the phase plot as a circle (Fig. 6a). As the amplitude of the signal is increased we get first period doubling (Fig. 6b), followed by a second period doubling (not shown in the figure) as in the case with f =1MHz. But at a particular driving amplitude of Vin = 15.8V, the system exhibits a new high amplitude oscillation. The corresponding output waveform and phase spectra are shown in Figs. 7a & 7b.

The waveform shows that it is periodic with a frequency of 2.5 kHz much lower than the drive frequency. Further increase of signal amplitude results in the formation of non periodic waveforms and shows chaotic nature. The typical waveforms are shown in Figs. 8(a) – (c) for a signal amplitude of 17.2V. The continuous broadband spectrum shown in Fig. 8(c) establishes the chaotic behaviour of the circuit. The dynamic content of the circuit increases with increase in amplitude of the drive signal as illustrated by a sample waveform noted for Vin = 25.8V (Figs. 9a& 9b). An interesting phenomenon observed here is that after a period of time the circuit attains a periodic nature. The phase spectrum corresponding to this region of operation is a circle as shown in Fig. 9c.

The reason for this nonlinear behaviour of the circuit is due to large diffusion capacitance caused by the diffusion of minority carriers under forward bias and transition capacitance caused by the depletion region of reverse bias. In addition, a third capacitance also comes into picture due to the injection of minority carriers, the effect of which dominates for large values of drive voltage. At higher signal amplitudes, the R-L Varactor acts as a low pass filter having two resonant frequencies of which the higher one is attenuated and lower one is locked by the circuit. This is the reason for the exponential decay of the extra peaks appearing well above the periodic background. It may be noted that, for diodes having large breakdown voltages such high amplitude oscillations are not obtained thereby establishing the possibility of third capacitance at breakdown as the cause for high amplitude oscillation.

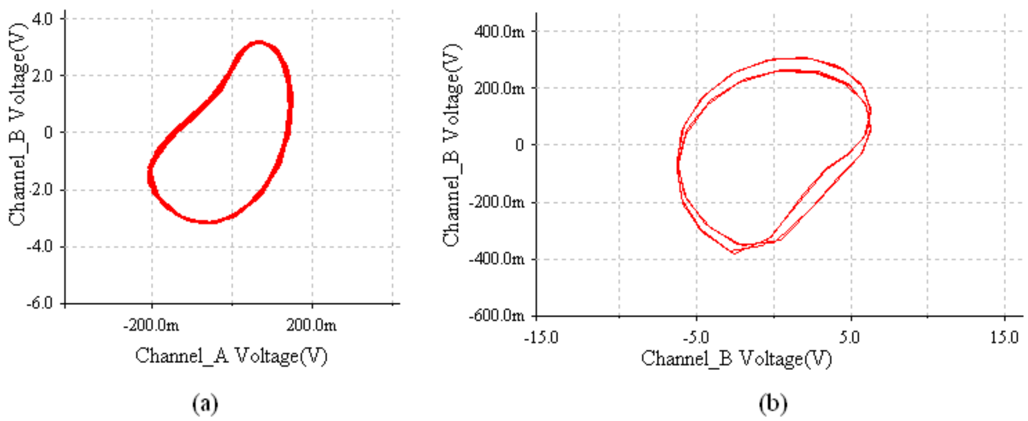


Figure 6: Simulation results at 450 kHz for (a) Vin = 3.3V (b) Vin = 6.3V.

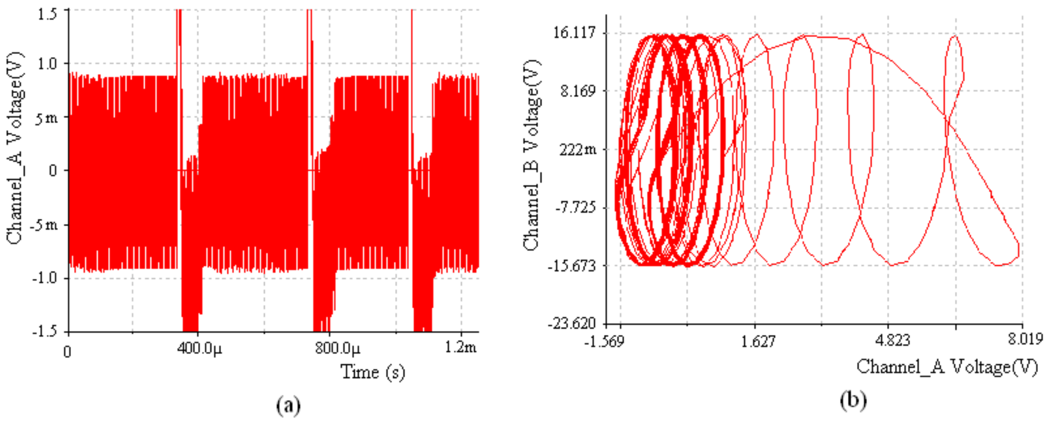


Figure 7: Appearance of low frequency signal (a) for Vin = 15.8V (b) Corresponding phase spectrum.

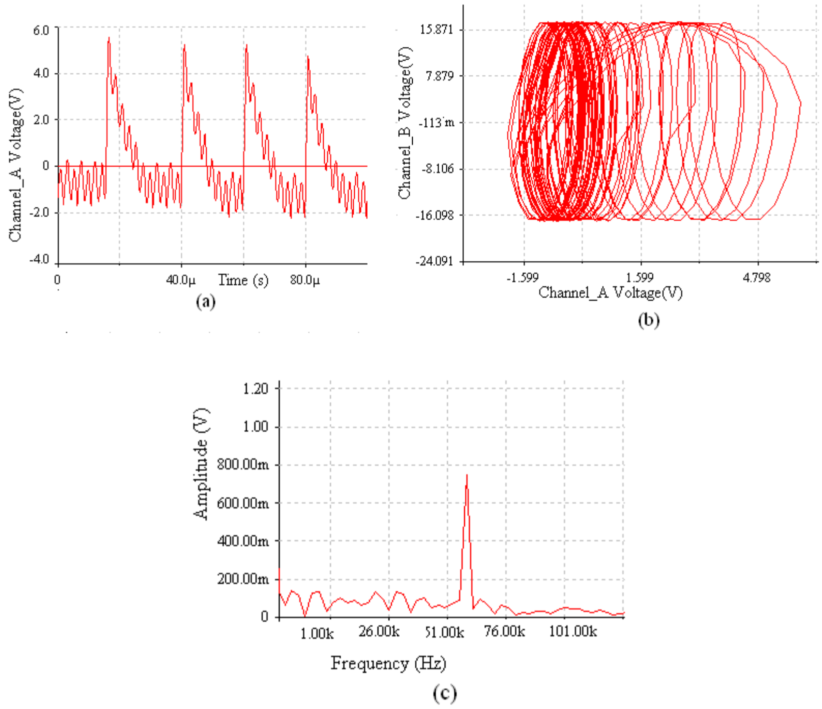


Figure 8: Chaotic bands observed for Vin = 17.2V (a) Output waveform (b) Phase spectrum (c) Power spectrum.

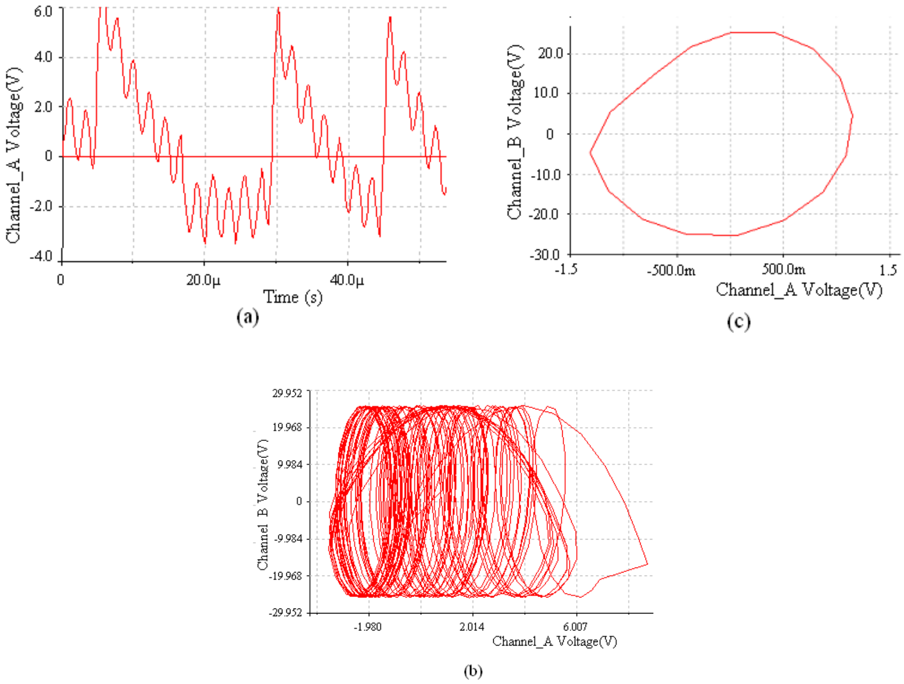


Figure 9: Behaviour of the circuit at 25.8V (a) Output waveform up to 3.55ms (b) Phase spectrum up to 3.55ms (c) Phase spectrum after 3.55ms.

Conclusions

We have demonstrated that a driven R-L-Resonator system has a complicated and multi-dimensional self replicating attractor. This structure repeats itself indefinitely as the control voltage is increased to arbitrarily large values and a new branch is added whenever a period doubling route to chaos takes place. To conclude, a thorough understanding of chaos in the R-L Varactor circuit must include the reverse recovery effect and all its nonlinearities. The rich dynamic behaviour and the simplicity of the system discussed here allow direct applications of other theories of sub-harmonic generation and chaotic behaviour in nonlinear systems.

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